Ergodic Theory: Home Assignment 6

Definition 1 *Let $(X, \mathcal{B}, \mu)$ be a Borel measure space. Let $f : X \to \mathbb{R}$ be a measurable function. $\sigma(f) := \{f^{-1}[E] : E \in \mathcal{B}\}$- the smallest sigma-algebra with respect to which f is measurable.*

Let $X = [-1, 1]$, $\mathcal{B}$ the Borel sigma-algebra, and λ the Lebesgue measure. Let $g(x) := \cos(x)$, $h(x) := x$, and $f(x) := |x|$. Let $\mathcal{F} := \{A \in \mathcal{B} : -A = A\}$.

1. Calculate $\mathbb{E}(h|\sigma(f))$.

2. Calculate $\mathbb{E}(g|\mathcal{A})$.

Good luck!