Ergodic Theory: Home Assignment 1

Choose between the following two questions:

1. Prove that every countable subset of the real line has Lebesgue measure 0.

2. Write $\mathcal{I} := \{[a, b) : [a, b) \subseteq [0, 1)\}$ (recall the remark from class: $\varnothing \in \mathcal{I}$, for $[a, a), a \in [0, 1)$, and so $\mathcal{I}$ is a semi-algebra). Define the Lebesgue measure when restricted to $\mathcal{I}$, by $\lambda([a, b)) := b - a$. Prove that $\lambda|_{\mathcal{I}}$ is σ-additive. Guidance can be found in the course notes of Measure Theory.

Good luck!