Ergodic Theory: Home Assignment 5

1. Show that the assumptions of the Stone-Weierstrass theorem are satisfied for the set of trigonometric polynomials in the set of the continuous functions on the circle. (It is recommended regardless to be familiar with the proof of the theorem.)

2. Let $(X, \mathcal{C}, T, \mu)$ be a Bernoulli scheme with a set of states $S = \{a_i\}_{i=1}^{N}$, where μ is the Bernoulli measure associated with a positive probability vector $\underline{p} = (p_1, ..., p_N)$. We have seen the following fact in class for any two cylinders $[\underline{a}], [\underline{b}]$:

$$\forall k > |\underline{a}|, \mu\left([\underline{a}] \cap T^{-k}\left[[\underline{b}]\right]\right) = \mu([\underline{a}]) \cdot \mu([\underline{b}]).$$

   Using this fact, prove that $\forall E \in \mathcal{C}$ s.t. $\mu(E \triangle T^{-1}[E]) = 0$, $\mu(E) \in \{0, 1\}$. Hint: approximate each measurable set by a finite disjoint union of cylinders.

Good luck!