Exercise sheet 1
Systems Biology class 2014

March 19, 2014

Return by email to jean.hausser@weizmann.ac.il until March 30th 2014 at the latest with `[SB14] Exercise sheet 1` in the subject of the email.

1 Origin of the production-removal equation

In the class, we introduced the production-removal equation:

$$\frac{dX}{dt} = \beta - \alpha X \tag{1}$$

This exercise explains why the rate of protein removal, α, is equal to the sum of the degradation rate α_{deg} and the dilution rate α_{dil}, $\alpha = \alpha_{deg} + \alpha_{dil}$.

Consider a collection of cells, which contain a certain protein. Let P be the total number of molecules of this protein, summing up over all cells. The concentration of the protein is $X = P/V$, where V is the total volume of all cells. The protein is produced at a rate β per unit cell volume. It is actively degraded at rate α_{deg}. Cells grow exponentially, such that cell volume produces new cell volume at rate α_{dil}.

1. The differential equation describe the change in the total cell volume V is $\frac{dV}{dt} = \alpha_{dil} V$. Interpret this equation in words.
2. Write a differential equation for the rate of change the total protein: $\frac{dP}{dt} = ...$
3. Write a differential equation for the rate of change of protein concentration $X = P/V$. Compare to Equation 1.

2 Change in degradation/dilution

Protein X is at steady state, produced at a constant rate β, with degradation/dilution rate α_0. The degradation/dilution rate suddenly changes to a new, lower value, α_1. Solve for the dynamics $X(t)$, plot the dynamics, and calculate the response time.
Hint: the response time $t_{1/2}$ is defined as the time necessary to reach half the regulation. Formaly, $X(t_{1/2}) = \frac{X_0 + X_{ss}}{2}$.

3 Long lived mRNA

Consider a gene which is constantly activated, producing mRNA at rate β_m. The mRNA is long lived (not degraded), and produces β_p proteins per unit time. The protein is also long lived (not degraded).

1. Solve for the protein concentration as a function of time, starting from zero initial mRNA and protein, for the case where cells do not grow ($\alpha_{dil} = 0$).
2. At time t_1, a drug is added to the cells that blocks transcription completely. At a later time, t_2, another drug is added that stops translation completely. Assuming that cells do not grow ($\alpha_{dil} = 0$), solve and plot the resulting protein concentration as a function of time.
3. *optional*: solve for the protein concentration as a function of time, starting from zero initial mRNA and protein, for the case where cells grow at rate α_{dil}. Plot the protein concentration as a function of time.