Exercise sheet 8
Systems Biology class 2014

May 13, 2014

Print and return to during classes, tutorials or office hours until May 23th 2014.

1 Pareto optimality

In the class, we saw that in the presence of trade-offs between tasks, optimal phenotypes are weighted averages of phenotypes optimized for single tasks (archetypes). For 2 tasks, we get a line, for 3 tasks, a triangle, for 4 tasks, a tetrahedron, etc.. The present question aims to utilize the above conclusions for analyzing data on bird-toes. In the attached PDF, you can see made-up data of toe-parameters of 20 bird-species. The measured bird's toe is composed of 3 bones (phalanges). A talon (nail) is located at the tip of the extreme bone. The lengths of the three bones are denoted by l_1, l_2, l_3 (see toe sketch on PDF). Each bird is represented by two traits: the ratio between the lengths of the second and the first bones l_2/l_1, and the ratio between the lengths of the third and the first bones l_3/l_1. The species of the bird is mentioned for some of the birds.

1. How many tasks do you think that the bird's toe is facing? Why?
2. Estimate the location of the archetypes
3. Using your knowledge on the bird-species, can you guess which tasks are being optimized? Explain
4. Sketch schematically the toes that correspond to the three archetypes.
5. Can you think why these designs are optimal for the tasks you proposed?

2 A 1-dimensional Pareto front

Consider a system that is defined by one trait, x. This system optimizes two tasks. The performance function of these tasks are $P_1 = a_1 - b_1(x - x_1)^2$, $P_2 = a_2 - b_2(x - x_2)^2$ $(b_1, b_2 > 0)$. Consider fitness functions that combine the two performances in a linear fashion: $F = w_1P_1 + w_2P_2$, where $0 \leq w_1, w_2$

1. What are the archetypes in this system?
2. Find the optimal solution for given weights w_1, w_2, by calculating $\partial F/\partial x$ and equating to zero.
3. Are the solutions you found weighted averages of the archetypes? How does it relate to what we saw in class?

3 Projections of Pareto fronts

When talking about Pareto optimality, we think about a biological system that is described by n traits and need to perform k tasks. Ideally, when we study a system, we want to have data on all relevant traits. However, this is not always possible. In this question we ask how much information can we get by measuring only part of the traits. Consider a system that is fully determined by three traits. Please answer the following questions and provide schematic sketches.

1. Assume the system performs two tasks (k = 2). What do you expect the Pareto front to look like according to what we learned in class? If you measure only 2 of the traits, how would the Pareto front look like? Will you deduce the correct number of tasks from this data? What happens if you measure only one trait?

2. Assume the system performs three tasks (k = 3). What do you expect the Pareto front to look like according to what we learned in class? If you measure only 2 of the traits, how would the Pareto front look like? Will you deduce the correct number of tasks from this data? What happens if you measure only one trait?

3. Assume the system performs four tasks (k = 4). What do you expect the Pareto front to look like according to what we learned in class? If you measure only 2 of the traits, how would the Pareto front look like? Hint: There are 2 different cases, according to the position of the 2-trat plane relative to the Pareto front. Will you deduce the correct number of tasks from this data? What happens if you measure only one trait?

For further reading see *Evolutionary Trade-Offs, Pareto Optimality, and the Geometry of Phenotype Space* O. Shoval, H. Sheftel, G. Shinar, Y. Hart, O. Ramote, A. Mayo, E. Dekel, K. Kavanagh, U. Alon, Science 2012