A Lightning Introduction to String Theory

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As we have known for more than 40 years, the Standard Model is renormalizable, which means there is no mathematical reason for new physics if this is all there is. (Ignoring the QED Landau pole.)

It has two fine tuned relevant operators, namely, $\int \Lambda d^4 x$ and $\int m^2_H |H|^2$, while all the marginal operators look natural. That $\Lambda \sim 10^{-120} M^4_{Pl}$ and $m^2_H \sim 10^{-34} M^2_{Pl}$ is very interesting and may require an explanation, but it poses no danger to the consistency of the model.
The only irrelevant operator we know of is associated to the spin-2 gravity field $h_{\mu\nu}$, whose Lagrangian takes the schematic form

$$\mathcal{L} = \int d^4x \left( h \partial^2 h + \frac{1}{M_{Pl}} h \partial h \partial h + \cdots \right)$$

and that’s therefore the only (almost) unambiguous hint for new physics at short distances.
We have seen that many times before: we should try to add new particles and either delay (like rho mesons do) or altogether remove (like the $W, Z$ bosons and Higgs field do) the sickness of the theory.
So why not just add a few new particles (say, massive particles at the Planck scale) and c’est tout?

However, there is not enough freedom to do that, since all couplings are fixed by the mass. Nobody has ever succeeded in writing such a UV completion.

For experts: there are claims in the literature of a candidate renormalizable theory with $\mathcal{N} = 8$ SUSY, but there is no strong evidence for this claim (and anyway it has little to do with nature).
And we have lots of indirect arguments that it cannot be done:

- If it were a local QFT, we would expect events with high energy to localize to \( r \sim 1/E \), but actually they spread out to bigger and bigger distances of the order \( r \sim E/M_{Pl}^2 \), forming a horizon and a black hole.

- Black holes have an area law entropy \( S \sim A/G_N \). For local systems in some generic thermal state we would expect a volume law.

- Gravity has other “holographic” features (see later talks).
So not only gravity presents us with the only irrelevant operator we certainly know exists, it is also a very exciting one, not accountable for by adding a finite number of new particles.

I would take the risk and say that unraveling the consequences of this irrelevant operator has been the biggest challenge in theoretical physics for 80 years or so.
Because we can’t just add a few particles and get away with it, we need a new framework (beyond Wilsonian QFT)!

Often, when seeking a new framework it is worth looking at the existing framework from a new perspective.

For example, to understand QM it is best to first master classical mechanics in the language of Lagrange, Hamilton and Jacobi.
I will therefore start by presenting to you a way of looking at particle physics which is not usually emphasized.

\[ G(x, y) = \int d^4 p \frac{e^{ip(x-y)}}{p^2} \]

Let us introduce a new parameter, \( \tau \), which we can think about as the property time

\[ \frac{1}{p^2} = \int_0^\infty d\tau e^{-\tau p^2} \]

Thus

\[ G(x, y) = \int d^4 p d\tau e^{ip(x-y)-\tau p^2} \]

We can think of \( L = p^2 \) as a Lagrangian.
We can repeat this trick for an arbitrary Feynman diagram. Take as an example

\[ \int d^4x_5 d^4x_6 d\tau_1 \ldots d\tau_5 \exp(ip_i x_i - \tau_i H_i). \]

UV limits of the diagram correspond to \( \tau \to 0 \) and infrared limits to \( \tau \to \infty \).
We can think of the above structure as some graph with length $\tau_i$ associated to each edge. I will call it a metric graph. Let us denote the space of all metric graphs by $\Gamma$. We can introduce a measure on this space as above and integrate. So QFT is nothing more than

$$
\int d\Gamma e^{ip_i x_i} e^{-S} \equiv \langle e^{ip_i x_i} \rangle
$$

where $S[\gamma \in \Gamma] = \int_\gamma L$ is the action with Lagrangian $L \sim \dot{x}_\mu \dot{x}^\mu$. 
In QFT we need to divide by symmetry factors. Here, this is implicit in the measure $d\Gamma$, since we sum over all metric graphs that are different from each other with the natural isomorphism between metric graphs (edge $\rightarrow$ edge, vertex $\rightarrow$ vertex, preserving the structure).

Different choices of coupling constants correspond to how we sum over different graphs, i.e. which relative weight.

We can also invent graphs with different types of lines, corresponding to different species of particles.
The richness of QFT thus stems from the fact that $\Gamma$ contains many disconnected components that can be related to each other by singular moves in the space of graphs.

![Graphs](image)

We can associate to these different disconnected components of $\Gamma$ different weights, different particles species etc.
Quantum gravity is the path integral over all metric spaces modulo all coordinate transformations. In one dimension, quantum gravity is therefore the sum over all metric graphs modulo graph isomorphism.
Therefore, we see that

Quantum Field Theory = 1-dimensional Quantum Gravity coupled to the matter fields $x_\mu(t)$!

The matter fields have the action we discussed above, $\int x_\mu \dot{x}^\mu$. 
This is not a way one is usually taught to think about QFT, but it is perfectly correct and natural from many points of view.

Note that 1-dimensional quantum gravity is by itself completely well defined and has no intrinsic inconsistencies like gravity in 4d because the metric tensor $g_{tt}$ has no local degrees of freedom. We can always bring the metric to the form $ds^2 = l^2 d\tau^2$. 
From this vantage point, we can ask an extremely natural question: so what if we study Quantum Gravity in two dimensions coupled to $\int d^2 z \partial X_\mu \bar{\partial} X^\mu$?

This is no longer a local QFT. Instead of summing over graphs with all possible graph structures, we sum over surfaces with all possible topologies.
Why not keep going higher? Because the sum over surfaces seems to make mathematical sense only in $d \leq 2$, so we only have QFT for $d = 1$ and the new mathematical structure just described for $d = 2$. The $d = 2$ theory is called String Theory.
The surface has no vertices or special points on it – all the points are locally exactly the same. So there are no coupling constants in String Theory!

Also, because all the points are locally the same, there are no “string species” – there is just one string. (Say, we fix the space to be asymptotically flat.)

One can argue without much effort, but with some mathematical preliminary about surfaces that I don’t have time to explain, that any such theory of surfaces gives rise to dynamical gravity in the “target space” $X_\mu$.

If the surface is very elongated, we can neglect the width and consider it as a Feynman diagram. So String Theory includes QFT as a limit.

Therefore, String Theory includes QFT, it always includes gravity, it has no coupling constants, and no freedom in choosing several types of surfaces/strings. These are all extremely desirable properties.
In QFT, the limit of small $\tau_i$ leads to UV divergences and renormalization. But in String Theory such limits are not singular, because for two dimensional surfaces when one edge becomes very small, we can always rotate it by 90 degrees and then it seems that one edge is actually very long! So $\text{UV} = \text{IR}$. This is another clear difference from local QFT. String Theory is thus a finite theory of Quantum Gravity.
When one studies the theory, one finds that it can only live in 10d, i.e. we must have precisely 10 $X_\mu$ fields, $\mu = 0, \ldots, 9$.

There are many ways to go down to $d = 4$, e.g. by the Kaluza-Klein mechanism, but that introduces lots of freedom from the 4d point of view. So the theory in 4d is not unique, it depends on which Kaluza Klein reduction we choose – maybe one day we will find a dynamical way to fix that. It is not currently clear.

String Theory includes supersymmetry (which can be broken spontaneously) in all the known consistent examples.
In summary, String Theory has extremely beautiful formal properties and it looks like a very natural extension of QFT. It is a consistent theory which includes a lot of old and new physics, it makes contact with highly nontrivial modern mathematics, and provides a conceptual framework to study Quantum Gravity.

However, direct contact with nature has been severely obstructed by the fact that we need to reduce along 6 compact directions, and computations are extremely hard without supersymmetry. Perhaps in the future people will figure out dynamical mechanisms which specify along which compact 6-manifold we need to reduce and how to break supersymmetry. As of June 2014, it looks pretty hard.
In addition to the extremely fruitful contact that String Theory made with mathematics (leading directly or indirectly to numerous Fields medals) there are various applications to QFT ranging from phenomenological ideas that were inspired by String Theory to recent applications in Hydrodynamics and Turbulence. These are some of the topics of the following lectures.