LIGHT NEUTRINOS AS COSMOLOGICAL DARK MATTER.
A CRUCIAL EXPERIMENTAL TEST

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Cosmological dark matter allegedly dominates the energy of the universe. Among all dark matter candidates, the light neutrino is the only particle actually known to exist in nature. The most likely light neutrino candidate is \( \nu_e \), with mass \( m(\nu_e) \approx 15-65 \) eV. The only practical way to show that \( m(\nu_e) \) is in that range, is to search for \( \nu_e-\nu_e \) oscillations reaching values of \( \sin^22\theta_{ee} \) as low as \( 4 \times 10^{-4} \). This calls for an improvement of the best existing experiment by one order of magnitude. A dedicated accelerator experiment with an emulsion followed by a spectrometer, detecting at least 40000 neutrino interactions, can settle the issue. Such an experiment does not seem impossible. A positive result would prove that most of the energy of the universe consists of \( \nu_e \) particles.

It is widely believed that the universe is filled with cosmological dark matter. The dark matter accounts for most of the energy of the universe. It probably leads to a “flat” universe, i.e. neither “open” nor “closed”. A flat universe has an energy density \( \rho_0 \) which equals the “critical density” \( \rho_c = 3H^2/8\pi G \), where \( H \) and \( G \) are, respectively, the Hubble parameter and Newton’s constant. Theoretically, we prefer \( \rho_0 = \rho_c \) (in other words \( \Omega \equiv \rho_0/\rho_c = 1 \)). Experimentally, it seems that \( \Omega \) may still be somewhat larger or smaller than one, but is nevertheless dominated by the cosmological dark matter.

There are many candidates for the cosmological dark matter \(^1\), the leading among them being weakly interacting neutral particles which do not emit observable radiation and are difficult to detect in terrestrial experiments. Among these candidates, the three leading classes are

(i) Light neutrinos with masses around 15–65 eV. The dominant neutrino could be, in principle, \( \nu_e, \nu_\mu, \nu_\tau \), or a hypothetical fourth neutrino \( \nu_\alpha \).

(ii) WIMPs (weakly interacting massive particles) with masses of several GeV’s. These could be a heavy neutrino around 4–8 GeV, a photino, another supersymmetric particle (if it is lighter than the photino) or other neutrino-like or photino-like objects.

(iii) Axions or other Goldstone particles.

There are other, even more exotic, candidates. There are also other kinds of dark matter such as the dark matter inside galaxies. We do not discuss these here.

Among all the above candidates, all class (ii) and class (iii) particles may or may not exist. At present, there is no shred of evidence for the existence of any of them. On the other hand, the three light neutrinos of class (i) definitely exist, although we do not know if any of them have masses in the range 15–65 eV.

To our best knowledge, the only argument against the light-neutrino dark-matter hypothesis is based on attempts to understand galaxy formation \(^2\). Some such calculations have indicated that light neutrinos may not have the right “clumping” properties. More recent calculations, in which cosmic strings are “thrown in”, yield more optimistic results. We cannot express any opinion on this issue, except to suggest that the theory of galaxy formation, even

\(^1\) For various reviews of dark matter, see e.g. ref. [1].

\(^2\) For a review of galaxy formation and dark matter, see e.g. ref. [2]. For a discussion of the influence of cosmic strings, see e.g. ref. [3].
according to its most enthusiastic practitioners, is far from reaching a stable, mature, status. It can hardly be used as a decisive argument for or against a specific dark matter candidate.

Any unbiased observer who has not been “brain-washed” by recent speculations concerning supersymmetry, axions or galaxy formation would undoubtedly conclude that the leading “suspect” in the dark matter puzzle must be the light neutrino, the only candidate actually known to exist in nature. Among the three known neutrinos, the tau neutrino $\nu_{\tau}$, is the most likely candidate. It is, therefore, extremely important to search for neutrinos in general, and tau neutrinos in particular, at the relevant mass range of 15–65 eV.

In this note we consider the phenomenological situation related to this problem. We argue that a conceptually simple neutrino oscillation experiment can play a crucial role in establishing the tau neutrino as the dark matter of the universe. A positive result will solve the cosmological dark matter problem! A negative result in such an experiment, will make the light-neutrino dark-matter hypothesis extremely unlikely.

We first note that the total energy density of the universe can be written as

$$\rho_0 = \Omega \rho_c = \frac{3H^2}{8\pi G} = \Omega h^2 \cdot 11 \text{ keV cm}^{-3}$$

where $h$ is defined by $H = h \cdot 100 \text{ km s}^{-1} \text{ Mpc}$. The accepted observational bounds [4] on $\Omega$ and $h$ are $\Omega < 2$ and $\frac{1}{2} < h < 1$. However, $\Omega$ and $h$ are related to the present age of the universe $t_0$. For instance, for $\Omega < 2$ and $t_0 > 10^{10} \text{ yr}$, we obtain $h < 0.57$ and $\Omega h^2 < 0.65$. For $\Omega = 1$ (the preferred theoretical value) and $t_0 = 1.5 \times 10^{10} \text{ yr}$ in a matter-dominated universe, we obtain $\Omega h^2 = 0.2$. We can safely assume

$$0.15 \leq \Omega h^2 \leq 0.65.$$

The number density $n_\nu$ of any flavor of light stable neutrinos is related to the known number density of photons $n_\gamma$ by

$$n_\nu = \frac{3}{11} n_\gamma \approx 110 \text{ cm}^{-3}.$$

Hence, if $\rho_0$ is entirely dominated by one flavor of light neutrinos, we must have [5]

$$m_\nu = 100 \Omega h^2 \text{ eV}.$$

For $\Omega = 2$ and $t_0 > 10^{10} \text{ yr}$ we obtain the most conservative upper limit $m_\nu < 65 \text{ eV}$. For the “favored” values of $\Omega = 1$, $t_0 = 1.5 \times 10^{10} \text{ yr}$ we obtain $m_\nu = 20 \text{ eV}$. For other reasonable values of $\Omega$, $h$ and $t_0$ we always obtain masses around 15–65 eV.

Which of the three known neutrinos might have a mass around 15–65 eV? The upper limits on $m(\nu_e)$, obtained from direct measurements and from SN1987A are around 10–20 eV. The two other neutrinos, $\nu_\mu$ and $\nu_\tau$, could be heavier than 65 eV only if they decayed fast enough. There are very good reasons to believe that this is not the case [5]. We therefore assume here that $m(\nu_e) < 65 \text{ eV}$, $m(\nu_\mu) < 65 \text{ eV}$.

It is probable that $m(\nu_\tau) \gg m(\nu_\mu) \gg m(\nu_e)$. This would be the case in most models [6] and particularly in almost any theory in which neutrino masses are obtained via the “see-saw” mechanism [7]. We therefore assume here that $\nu_e$ is much lighter than $\nu_\tau$. The most likely ratio in a “see-saw” mechanism is

$$\frac{m(\nu_\mu)}{m(\nu_\tau)} \approx \left[ \frac{m(\mu)}{m(\tau)} \right]^2 \approx 3.5 \times 10^{-3},$$

and we may probably safely assume [6] that the above mass ratio is somewhere between $10^{-1}$ and $10^{-3}$. However, as long as it is well below one, our arguments are essentially independent of the precise ratio.

If both $\nu_\mu$ and $\nu_\tau$ are lighter than 65 eV and if $m(\nu_\tau) \gg m(\nu_\mu)$, the tau neutrino becomes the leading dark matter candidate.

We are discussing here a $\nu_\tau$ mass value which is six orders of magnitude below the best direct limit [8] $m(\nu_\tau) < 35 \text{ MeV}$. The only way to probe this mass region are neutrino oscillations involving $\nu_\tau$. If $\nu_\mu$, $\nu_\tau$, $\nu_e$ have nonvanishing masses, it is essentially inevitable that neutrino oscillations occur. Such oscillations between two species $\nu_i$ and $\nu_j$ depend only on $\Delta m^2 = m_i^2 - m_j^2$ and on $\sin^2 2\theta_{ij}$ where $m_i$, $m_j$ are the masses and $\theta_{ij}$ is the mixing angle. Since we assumed that $m(\nu_\tau) \gg m(\nu_\mu) \gg m(\nu_e)$, and we are interested in the range 15 eV $\leq m(\nu_\tau) \leq 65$ eV, we must consider only $\nu_e$, $\nu_\tau$, $\nu_\mu$ and $\nu_\tau$, $\nu_\mu$, oscillations and we know that, to a good approximation, $\Delta m^2 \approx \left[ m(\nu_\tau) \right]^2 \approx (200–4500) \text{ eV}^2$.

What can we say about the $\nu_\tau$, $\nu_\mu$, and $\nu_\tau$, $\nu_\mu$, mixing angles $\theta_{\nu e}$ and $\theta_{\nu \tau}$? The angle $\theta_{\nu e}$ mixes non-adjacent generations. It is analogous to $\theta_{13}$ in the quark...
sector, which is known to be smaller (but probably not much smaller) than $10^{-2}$. If $\theta_{\mu\nu} \approx \theta_{\tau\nu}$, we expect $\sin^2 2\theta_{\nu} \leq 4 \times 10^{-4}$. The best $\nu_e - \nu_e$ oscillation data \footnote{For a recent review of neutrino oscillations, see e.g. ref. \cite{9}.} (as well as the best $\nu_e$ “disappearance” data) reach only much larger values of $\sin^2 2\theta_{\nu}$, and therefore tell us nothing about $m(\nu_\tau)$.

This leaves us with $\nu_e - \nu_\mu$ oscillations as the last resort. The angle $\theta_{\mu\nu}$ mixes adjacent generations. It is analogous to $\theta_{\tau\nu}$ in the quark sector. Experimentally, $\sin^2 \theta_{\tau\nu} = 0.043 \pm 0.008$. If we had $\theta_{\mu\nu} = \theta_{\tau\nu}$ we would expect $\sin^2 2\theta_{\nu} \approx 0.005$–0.010. In the quark sector, we have another mixing angle which connects neighbouring generations: the original Cabibbo angle, obeying $\sin^2 \theta_{\nu} \approx 0.22$ or $\sin^2 2\theta_{\nu} \approx 0.18$. We do not really know why $\theta_{\nu} > \theta_{\tau\nu}$. We also do not know the actual value of $\theta_{\mu\nu}$, but on the basis of the above analogy to the quark sector, it might be anywhere, say, between 0.03 and 0.22. The pattern of the charged lepton mass ratios is not very different from that of the quark mass ratios. Most theoretical models expect mixing angles to be somehow related to fermion mass ratios. We may therefore “guess” that the $\theta_{\mu\nu}$ is not far from the above range, possibly below it, but not too far below. Since $\theta_{\nu}$ is probably near 0.01, and the mixing of “distant” generations is expected to be smaller, we propose a very conservative lower bound $\theta_{\mu\nu} \geq 0.01$. This would mean $\sin^2 2\theta_{\mu\nu} \geq 4 \times 10^{-2}$. This bound seems safe although, in principle, arbitrarily small values of $\theta_{\mu\nu}$ cannot be excluded.

What we need is, therefore, a $\nu_e - \nu_\mu$ oscillation experiment probing the region of $\Delta m^2$ between 200 and 4500 eV$^2$ and reaching $\sin^2 2\theta_{\nu}$ values which are at least as low as $4 \times 10^{-4}$, preferably even lower.

The relevant range in $\Delta m^2$ is easily accessible. How far can we go in the other crucial variable, $\sin^2 2\theta_{\mu\nu}$? The best $\nu_\mu$ “disappearance” experiments reach only \cite{9} $\sin^2 2\theta_{\nu} \approx 0.05$, far above the required range. By far the best $\nu_e - \nu_\mu$ data comes from \cite{10} Fermilab experiment E531, using a hybrid combination of an emulsion and a spectrometer. This experiment, at the 90% confidence level, reached $\sin^2 2\theta_{\nu} \approx 4 \times 10^{-3}$, just enough to exclude $\theta_{\mu\nu} = \theta_{\tau\nu}$. What we now need is an improved experiment that can reach at least down to $\sin^2 2\theta_{\nu} \approx 4 \times 10^{-4}$, hopefully below it. Such an experiment will provide us with an excellent probe of the possibility that the cosmological dark matter is due to tau-neutrinos.

The E531 experiment \cite{10} was not originally designed to search for $\nu_e$ oscillations. It was a by-product of a charm lifetime experiment. It still achieved, by far, the best $\nu_e - \nu_\mu$ oscillation data. In that experiment, approximately 4000 neutrino interactions were detected. A $\tau$ candidate was defined as an event with a kink (having $p_\tau > 125$ MeV) or a three-prong secondary vertex, no prompt muon (to eliminate standard $\nu_\mu$-$\mu$ events), a negative charged track (to eliminate charm events) and a minimum momentum for the $\tau$ ($p_\tau \sim 2.5$ GeV, to avoid confusion with other background). With these cuts, most $\tau$ events should survive, but no candidate events were found. The experiment, with these cuts, had no background at all. On the basis of zero $\tau$ candidates and 1870 ordinary charged current events with an identified $\mu$, the range of $\sin^2 2\theta_{\mu\nu} \leq 4 \times 10^{-4}$ was obtained.

Improving the bound by at least an order of magnitude would require a new dedicated experiment using similar techniques. The emulsion seems necessary in order to observe $\tau$ tracks with a typical length of a few hundred microns. The spectrometer is needed in order to point towards the suspected vertex. Conceptually, the simplest method would be to repeat the essential features of experiment E531 with a larger number of events. One needs at least 20000 charged current neutrino interactions with identified muons, preferably more. Depending on the efficiency and the acceptance for muon identification, this would require a total of at least 30000 and probably 40000 neutrino interactions.

This can be achieved by any combination of more emulsion, higher beam intensity and longer running time. Assuming that the transverse size of the detector covers most of the width of the neutrino beam, the number of neutrino interactions can be roughly estimated by the following crude formula:

$$\frac{N_{\nu\text{ events}}}{1000} \approx \eta \frac{E_\nu}{100 \text{ GeV}} \frac{n_p}{10^{18}} \frac{M_{\text{target}}}{1 \text{ ton}},$$

where $E_\nu$ and $n_p$ are, respectively, the energy and the number of protons on target and $M_{\text{target}}$ is the active target mass. The coefficient $\eta$ is always of order one and it contains all the details of the beam, detector, etc. In a sample of CERN and Fermilab experiments over the last few years, $\eta$-values between 0.6 and 3.5
are obtained. For our purposes, we need to generate a factor of 40 on the left-hand side of our equation.

For a single realistic run at Fermilab with 800 GeV protons and $10^{18}$ protons on target, we therefore have

$$\frac{N_{\nu\text{-events}}}{1000} = 8\eta \frac{M_{\text{target}}}{1 \text{ ton}}.$$  

For $\eta = 1$ we therefore need, say, two runs with at least 2.5 tons of emulsion. The situation for the CERN SPS is somewhat better. Because of the higher beam intensity and the higher repetition rate of the machine, and in spite of the lower energy, one obtains for a typical realistic run $E_{\nu} = 400$ GeV, $n_{\nu} = 6 \times 10^{18}$, yielding

$$\frac{N_{\nu\text{-events}}}{1000} = 24\eta \frac{M_{\text{target}}}{1 \text{ ton}}.$$  

With $\eta = 1$, two such runs with 800 kg (or 200 $\ell$) of emulsion would do the job. Some of the above numbers could be modified by factors of two, depending on the quality of the neutrino beam, the length of the run, the percentage of machine protons dedicated to the experiment, the distance of the detector, the acceptance and efficiency, etc. In fact, we believe that by optimizing all of these parameters, it may be possible to obtain the required sensitivity with a somewhat smaller amount of emulsion, possibly below 100 $\ell$. For $\eta \approx 3$ (a value which has been achieved in past experiments), one needs approximately 70 $\ell$.

With so many events, scanning the emulsion becomes a difficult and lengthy procedure. Almost all scanned events would involve a muon which is detected by the spectrometer and traced back to a primary vertex in the emulsion. Rejecting these events is a fairly rapid procedure. Selecting the serious candidates and scanning them is the heart of the experiment. A dedicated $\nu_{\tau}$ experiment which is not a byproduct of something else, may allow a more efficient procedure of selecting candidate events before the cuts.

It may be worthwhile to concentrate on specific decay modes of $\tau$ (e.g., single hadron or three prongs or electron) and in this way considerably reduce the necessary amount of scanning. The price paid would, of course, be the necessity of having a higher total number of events and therefore a proportionately larger amount of emulsion.

It seems that the best method would be to concentrate on events containing an energetic negative electron and no muon. Such events would include 17% of all $\tau$-leptons, necessitating a total number of events which is six times larger, i.e., a total of 250000 neutrino interactions. However, such a procedure would eliminate all normal charged current events and almost all neutral current events. The main physics background here would come from $v_e$ contamination in the neutrino beam, usually estimated at 1%. This would yield approximately 1500 $v_e$-initiated charged current events. Most of the scanned events would be of this type. If the electron comes from the primary vertex in the emulsion, the event should be rejected. If a kink is observed for an $e^-$ it is a $\tau^-$ candidate. In spite of the sixfold increase in the total number of neutrino interactions, the absolute number of scanned events will be reduced by more than an order of magnitude, relative to the case in which one searches for all $\tau$ decay modes.

The total amount of emulsion needed for performing this version of the experiment at CERN will have to be of the order of 500 $\ell$ (assuming $\eta \approx 3$). The typical effective transverse area of the neutrino beam at a distance of 1 km is a few squared meters (say, 3 m$^2$), leading to a total emulsion thickness of the order of 15 cm or five radiation lengths. In order to overcome showers, conversions and other facts of life, it would be advantageous to use several layers of emulsion (say, each with a depth of 1 cm) separated by tracking chambers which can help identify the electrons and distinguish them from various types of background. The combined electronic information from the detector behind the emulsion and the chambers between the emulsion plates could help identify true electron events, reducing the total number of scanned events to a few thousands, a number similar to that of experiment E531. Scanning will consist of searching for the relatively simple signature of a kink involving a short track of a few hundred microns followed by a single negative electron.

It is conceivable that the experiment can also be performed with other detectors containing a track-sensitive target. It might be interesting to pursue this possibility. However, the requirement of hundreds of kilograms of active target and the necessity of observing $\tau$-tracks of a few hundred microns are not easily reconciled in other methods. A particularly attractive possibility along these lines is the idea of using scin-
oscillating optical fibres in order to detect τ-tracks in a neutrino beam [11].

It is, in principle, also possible to detect τ-leptons without explicitly observing their tracks, using much larger active targets and higher event rates. However, at the level of sensitivity required here, background becomes an extremely serious problem in such experiments.

If τ events are discovered, we must be certain that they come from νμ’s which oscillated into ντ’s rather than from a ντ-contamination which exists in the neutrino beam as a result of direct hadronic decays. The prime candidate for such decays is the τ meson, known as F or Dτ. The decay of F is the dominant mechanism for producing ντ in beam dump experiments. However, for the type of experiment discussed here, at a distance of, say, 1 km, the number of τ events originating from F-decay is expected to be negligible. It may become the limiting factor if the ντ−ντ oscillation experiment is ever pushed to even lower values of sin²2θτμ. The background due to “direct ντ” can, in principle, be measured by turning down, removing or diverting the focused neutrino beam. At lower energies (such as at CERN), the F background is smaller than at higher energies (such as at Fermilab).

We conclude that the proposed experiment is difficult, but not impossible. The potential reward is, in our opinion, extremely significant.

If the experiment is performed and oscillations are found, it will provide us with information on m(ντ). A precise determination of m(ντ) may require additional, more complicated, experiments at different distances and/or energies. However, the existence of any νμ−ντ oscillations in an experiment of the type discussed here, would indicate that m(ντ) is at least a few eV’s, making it a very likely candidate for the dark matter. If m(ντ) is found to be in the appropriate mass range, it is probably the cosmological dark matter of the universe and it becomes the dominant contributor to its energy!

If the result is negative down to sin²2θτμ ≈ 4×10⁻⁴ and if, like E531, the experiment is sensitive to m(ντ) values as low as a few eV, we face two possibilities: The most likely one is that m(ντ) is at, or below, few eV and it does not form the cosmological dark matter of the universe. In that case, m(ντ) is most likely to be at, or below, 10⁻² eV, just the range required for explaining the solar neutrino puzzle by νμ−νe oscillations [12]. The dark matter could then be a fourth light neutrino ντ, in the 15–65 eV mass range or, more likely, an axion or a WIMP.

The second possibility (in the case of a negative result) is that ντ is still around 15–65 eV, but for some peculiar reason θτμ < 0.01, well below the analogous quark angles and possibly even below the angle θwμ. This would be a very small angle and it is not suggested by any known model. However, such a situation cannot be ruled out and the only way to cope with it would be to push the experiment even further, to lower values of sin²2θτμ.

If m(ντ) is in the 15–65 eV range, m(νμ) is likely to be approximately around 0.1 eV. In such a case, νμ−ντ oscillations at Δm²≈ 10⁻² eV² become relevant. Such experiments are being now contemplated. However, even if νμ−ντ oscillations are discovered at m(νμ)≈ 0.1 eV, we still cannot be sure that ντ is the cosmological dark matter. Only a direct observation of ντ−νμ oscillations will be convincing.

Needless to say, the purpose of this note is not to design an experiment. Any experimental method which would lead to the necessary values of sin²2θτμ and to the discovery of ντ at the dark-matter mass range, will be welcome. The above discussion serves only to emphasize the great importance of the proposed measurement and to indicate that the experiment appears to be feasible.

We summarize: among all dark matter theories, only the light neutrino possibility is based on a particle which is known to exist; the most likely light neutrino as a dark matter candidate is ντ; if ντ is the cosmological dark matter, we must have m(ντ)≈ 15–65 eV; the only practical way to prove this mass is to search for νμ−ντ oscillations at 200<Δm²<4500 eV² down to low values of sin²2θτμ; a conservative estimate requires θτμ≥0.01 or sin²2θτμ≥4×10⁻⁴; this calls for an improvement of the best existing experiment by at least one order of magnitude; a dedicated accelerator experiment with an emulsion followed by a spectrometer, detecting at least 40000 neutrino interactions, should settle the issue; such an experiment does not seem impossible.

We urge experimentalists to perform this crucial experiment, hoping that it can prove that the cosmological dark matter of the universe consists of tau-neutrinos. A positive result will, of course, also be the

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first experimental observation of a \( \nu \), the first observation of neutrino oscillations and the first evidence for non-vanishing neutrino masses. It should be exciting to be the first to observe a new particle and, at the same time, to show that it dominates the mass of the universe!

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