A SCHEMATIC MODEL OF QUARKS AND LEPTONS ☆

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We consider a scheme in which all quarks and leptons are composites of only two types of fundamental spin 1/2 objects with electric charges 1/3 and 0. The concepts of color and flavor acquire meaning only at the level of the composite systems. Gauge bosons such as $W^{\pm}$ or gluons connect only composite states, and are not fundamental. The scheme accounts for several regularities of the observed pattern of quarks and leptons. However, we cannot offer any convincing dynamics, leaving many important questions unanswered.

The discovery of at least five flavors of quarks, each appearing in three colors, and the observation of at least five (and probably six) types of leptons, raise the possibility that these particles conceal some further substructure [‡1]. It is simply unlikely that more than twenty building blocks of matter are fundamental.

The observed similarity between quarks and leptons suggests that, if there is a substructure, both types of particles are constructed from the *same* basic entities. This is particularly indicated by the relationship between the electric charge quantizations for leptons and quarks. The neutrality of the hydrogen atom reflects a mysterious connection between the charges of quarks and leptons. Such a connection would arise naturally if they consist of the same objects. A related empirical fact, which should be naturally explained by a successful scheme, is the vanishing sum of electric charges of quarks and leptons in each "generation" (e.g., ν_e, e^-, u, d).

Additional motivation for an underlying structure is offered by the observed pattern of "generations" of quarks and leptons [2]. Here we have two independent facts, both hinting at a common substructure: within each generation, quarks and leptons appear in an analogous way and each generation reproduces all the properties of its predecessors, except for the masses. It is possible to envisage some fundamental entities whose combinations create one generation of quarks and leptons, while the next generations are simply higher-order excitations of the same system.

Our final motivation is much more speculative. Ambitious attempts to unify all fundamental interactions, including gravity, have led to the construction of a class of extended supergravity theories [3], based on the SO(N) groups. The largest such theory which accommodates the $J = 2$ graviton without introducing $J = 5/2$ fields is based on SO(8), yielding 28 vector bosons and 56 fermions. However, even these large multiplets fail to accommodate the $W^{\pm}$-bosons and the μ, τ and b fermions [3]. If the overall unification is carried out at the level of the substructure of quarks and leptons, the number of fundamental fermions and vector bosons would be much smaller, possible allowing for a more realistic extended supergravity scheme. In such a model, μ, τ, b and even $W^{\pm}$ would not be fundamental.

Several different arguments lead us to believe that *any* possible substructure may be observed only at extremely short distances and correspondingly large momenta. The well-known evidence for the "point-like"

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[‡1] For an earlier consideration of a substructure, see e.g. ref. [1]. The fundamental building blocks in these papers are called respectively, prequarks (or preons), subquarks, maons, alphons and quinks.

behavior of leptons and quarks indicates that such a substructure must correspond to distances well below 10^{-16} cm. The present accuracy of QED tests, the success of the Weinberg–Salam model, the approximate scaling of deep inelastic structure functions and the approximate constancy (and correct value) of $R_{e^+e^-}$, all indicate that leptons and quarks are "point-like" at least down to 10^{-16} cm.

If second-generation fermions are excitations of the same bound system which forms the first-generation fermions, transitions such as $\mu \to e + \gamma$, $s \to d + \gamma$ might be allowed, but suppressed by factors representing the tiny radii of such systems or the heavy constituent masses. The present upper limit on the rate for $\mu \to e + \gamma$ indicates that the characteristic distances involved may actually be much smaller than 10^{-16} cm, and probably below 10^{-24} cm. The details of such calculations depend, of course, on the unknown dynamics.

Finally, if quarks and leptons are "made" of the same objects, baryon and lepton number violations are very likely. The present limit on the proton's lifetime indicates that this can probably happen only at distances below 10^{-29} cm, or momenta somewhere above 10^{15} GeV [4]. Unification with gravity hints, of course, at the Planck mass, corresponding to distances around 10^{-34} cm.

Having realized that any substructure must involve distances which are many orders of magnitude below our present understanding and intuition, we should not exclude the possibility that the relevant dynamics is different from anything we have seen, so far. It would be premature to insist, for example, that presently established ideas of gauge theories are sufficient for *fully* explaining the interactions of the new hypothetical building blocks. In fact, the correct dynamics at very short distances may be radically different, and is likely to involve some entirely new principles. However, when viewed at present energies and distances, in which quarks, leptons and ordinary gauge bosons are "point-like", it should somehow reproduce currently accepted theories such as SU(2) × U(1) and QCD.

Several authors [1] have already discussed possible substructures. All published schemes have two common features: the fundamental building blocks were assigned either color or flavor and the standard gauge bosons remained fundamental. We wish to consider here a scheme which differs from these ideas in both respects. We suggest that both color and flavor may be generated through combinations of the fundamental building blocks, and cannot be attributed to the building blocks themselves. We further suggest that at least some of our present gauge bosons (such as $W^\pm$ and the gluons) act only on composite states and are therefore not fundamental. If one accepts a composite electron, why should $W^\pm$ remain fundamental?

In a composite model of quarks and leptons, the fundamental electric charge must, presumably, be one third of the electron charge. The most economical set of building blocks consists of two $J = 1/2$ objects: one charged ($Q = 1/3$) and one neutral. This is precisely the full content of our scheme. We denote the charged particle by T and the neutral one by V. We name these particles "rishons" [‡2]. Their antiparticles are $\bar{T}$ ($Q = -1/3$) and $\bar{V}$ ($Q = 0$).

The simplest composite fermion can be constructed from three rishons. There are eight combinations:

(i) *TTT*. This is a $Q = +1$ fermion. We identify it as the positron e^+.

(ii) *TTV*, *TVT*, *VTT*. These form three $Q = +2/3$ fermions. We suggest that the dynamics is such that the three states are degenerate. We identify them as the three color states of the u-quark. If these states are degenerate, we may clearly replace them by any other three orthogonal combinations and we have an overall "color" SU(3) symmetry among them.

(iii) *TVV*, *VTV*, *VVT*. Three degenerate $Q = +1/3$ fermions, identified as the three color states of the $\bar{d}$-antiquark.

(iv) *VVV*. A neutral fermion, identified as ν_e.

Note that the three-rishon states create a quark, an antiquark, a lepton and an antilepton. The antiparticles of the above states are clearly: $\bar{\nu}_e(\bar{V}\bar{V}\bar{V})$, d $(\bar{V}\bar{V}\bar{T}, \bar{V}\bar{T}\bar{V}, \bar{T}\bar{V}\bar{V})$, $\bar{u}$ $(\bar{T}\bar{T}\bar{V}, \bar{T}\bar{V}\bar{T}, \bar{V}\bar{T}\bar{T})$, e^- $(\bar{T}\bar{T}\bar{T})$.

The concept of color relates to different internal arrangements of rishons in a quark. Leptons have no color because they have only one allowed arrangement (TTT or VVV). The rishons themselves cannot be assigned color degrees of freedom. Quarks have three colors (rather than two or four) because there are

[‡2] "Rishon" means first, primary (in Hebrew). The choice of T and V originates from Genesis 1, 2 where before the creation of matter everything was "Tohu va-Vohu" (unformed and void; chaos). The reader might simply remember that the charge of T is one Third while the charge of V Vanishes.

three ways to arrange three rishons in a quark (and because two rishons cannot make a fermion).

In the limit in which the net number of T's and V's are conserved, we have two additive quantum numbers:

$$n(\mathrm{T}) - n(\bar{\mathrm{T}}) = 3Q ,$$

$$n(\mathrm{V}) - n(\bar{\mathrm{V}}) = 3Q - 3(B - L) ,$$

where Q, B, L are the usual electric charge, baryon number and lepton number. We immediately see that baryon and lepton number are not conserved, but their difference is conserved, as long as the net number of rishons is conserved. However, we may wish to consider parity violating V–$\bar{\mathrm{V}}$ mixing, in which case, $B - L$ will not be exactly conserved.

Baryon-number violating processes are allowed. An important example is: $\mathrm{u} + \mathrm{u} \to \bar{\mathrm{d}} + \mathrm{e}^+$, or, equivalently: (TTV) + (TTV) → (TVV) + (TTT). This is precisely the process which is responsible for proton decay in grand unification schemes [5] [‡3] such as SU(5).

The relation between quark and lepton charges is natural in this scheme. The empirical observation concerning the vanishing sum of charges of ν_e, e^-, u, d is now explained by the assertion that two of them (ν_e, u) contain rishons while the other two (d, e^-) contain anti-rishons. The total content of ν_e, e^-, u, d includes equal numbers of rishons and antirishons (6T+ 6V+6$\bar{\mathrm{T}}$+6$\bar{\mathrm{V}}$), guaranteeing a vanishing sum of electric charges. The same argument correctly predicts that the sum of $B - L$ values for each generation also vanishes. We were not previously aware of this simple regularity.

It is interesting to note that, at the level of rishons, matter and antimatter may be equally abundant in the universe. A hydrogen atom contains an equal number of rishons and antirishons.

The second and third generations of quarks and leptons are presumably constructed in an analogous way to the first generation. Each generation must contain the same set of states, at higher energy values. The calculation of the energy levels (quark and lepton masses) and of transition rates among them, depend on the unknown dynamics.

We now proceed to discuss the role played by the usual gauge bosons. Since the fundamental unit of electric charge is 1/3, the $\mathrm{W}^\pm$ cannot act between single rishon states. In fact, the simplest boson with the quantum numbers of W^+ ($Q = 1, B - L = 0$) corresponds to a state of the form (TTTVVV). Such a boson, when acting on a fermion which consists of three antirishons, will yield a state of three rishons, e.g.:

$$\mathrm{W}^+|\bar{\mathrm{V}}\bar{\mathrm{V}}\bar{\mathrm{V}}\rangle = |\mathrm{TTT}\rangle , \quad \mathrm{W}^+|\bar{\mathrm{T}}\bar{\mathrm{V}}\bar{\mathrm{V}}\rangle = |\mathrm{VTT}\rangle .$$

Thus W^+ can convert $\bar{\nu}_e \to \mathrm{e}^+$, $\mathrm{d} \to \mathrm{u}$, $\bar{\mathrm{u}} \to \bar{\mathrm{d}}$, $\mathrm{e}^- \to \nu_e$, as required. The W^- boson carries the quantum numbers of ($\bar{\mathrm{T}}\bar{\mathrm{T}}\bar{\mathrm{T}}\bar{\mathrm{V}}\bar{\mathrm{V}}\bar{\mathrm{V}}$). Both W^+ and W^- can connect only composite states. The $\mathrm{W}^\pm$-bosons are presumably singlets under color and universality is likely to hold for their couplings to leptons and quarks, provided that their couplings are symmetric between T and V.

Gluons change a VTT state into TVT, etc. They rearrange the rishons in a quark and can act only on composite states. Some of the lepto-quarks assumed in grand unification theories [5] are also carrying the quantum numbers of a six-rishon state. For instance, the $Q = 4/3$ leptoquark of SU(5) corresponds to (TTTTVV).

If $\mathrm{W}^\pm$ (and Z) are composite, it is reasonable to guess that several sets of W's and Z's may exist. Parity may be violated at the short distances relevant to our scheme (perhaps through V–$\bar{\mathrm{V}}$) mixing. Alternatively, it may be conserved at small distances and broken spontaneously at larger distances. In both cases we may have additional weak bosons above the 80–90 GeV masses predicted for the usual W and Z.

No symmetry principle prevents $\mathrm{W}^\pm$ from connecting, say, the ground state of TTV (namely, the u-quark) to an excited state of $\bar{\mathrm{V}}\bar{\mathrm{V}}\bar{\mathrm{T}}$ (namely, the s-quark). Such transitions may be suppressed by small overlap integrals between ground and excited states, but they are not forbidden. Their relative strengths define the Cabibbo angle and its generalizations.

In the absence of a clear understanding of parity violation in weak processes, we can offer only a wild speculation concerning the Weinberg angle. Assuming that the short-distance dynamics somehow leads to an $\mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \times \mathrm{U(1)}$ theory at, say, TeV energies, θ_W is defined by:

$$J_{\mathrm{em}} = \sin\theta_\mathrm{W}(W_\mathrm{L}^3 + W_\mathrm{R}^3) + \sqrt{\cos 2\theta_\mathrm{W}}\, B ,$$

where W_L^3, W_R^3 and B are the neutral generators of $\mathrm{SU(2)_L}$, $\mathrm{SU(2)_R}$ and U(1), respectively. Since all of these neutral currents carry no net charges, we may talk about their *effective* couplings to our fundamental

[‡3] The original suggestion of baryon non-conservation in grand-unification theories is due to Pati and Salam [6].

objects. We presumably have

$$J_{em} = 2^{-1/2}(\bar{T}_L\gamma_\mu T_L + \bar{T}_R\gamma_\mu T_R).$$

Since all W's couple symmetrically to T and V we must have:

$$W_L^3 = 2^{-1/2}(\bar{T}_L\gamma_\mu T_L + \bar{V}_L\gamma_\mu V_L)\,,$$

$$W_R^3 = 2^{-1/2}(\bar{T}_R\gamma_\mu T_R + \bar{V}_R\gamma_\mu V_R)\,.$$

B is right–left symmetric and orthogonal to W_L^3, W_R^3. Hence:

$$B = \tfrac{1}{2}(\bar{T}_L\gamma_\mu T_L + \bar{T}_R\gamma_\mu T_R - \bar{V}_L\gamma_\mu V_L - \bar{V}_R\gamma_\mu V_R).$$

Substituting these expressions into the defining equation for θ_W we find

$$\sin^2\theta_W = \tfrac{1}{4}\,.$$

This pleasant result should be taken seriously only when and if we fully understand the transition from the unknown dynamics of our scheme to the presently accepted gauge theory of electroweak interactions.

Needless to say, we have major difficulties, all relating to the lack of a dynamical theory.

(i) A full theory must explain why we do not have low-lying fermions composed of two rishons and one antirishon, why we do not have low-lying $J = 3/2$ quarks and leptons, why the three different arrangements of TTV are degenerate, and what is the quantum number which distinguishes between generations.

(ii) The question of quark confinement is still open. Why do we have free TTT states but not free TTV? All *observed* particles in nature consist only of combinations of TTT, $T\bar{T}$, VVV, $V\bar{V}$ (e.g., π^+ is 3T + 3V, p is 3T + $T\bar{T}$ + 2$V\bar{V}$).

(iii) The mass spectrum of quarks and leptons remains unexplained. The mass splittings between generations are extremely small when compared to momenta which are conjugate to the relevant radii. How do we generate splittings of order GeV at such short distances?

(iv) The neutrino masses continue to puzzle us. However, neutrinos are the *only* particles which consist solely of neutral rishons. Is there a mass-generating mechanism which is somehow linked with electric charge? The detailed mechanism of parity violation is equally mysterious.

We conclude by returning to our original list of motivations. The scheme proposed here is extremely economical. It suggests that all of matter consists of only two fundamental entities. The related quantization of quark and lepton charges is explained, the contents of each generation appears naturally and the similarity between generations is obtained. The concepts of color and flavor are meaningful only at the level of composite states. We may even speculate that an SO(4) extended supergravity scheme [7] may be considered with one graviton, four gravitinos, six vector particles (charges ±1/3, ±1/3, 0, 0), four $J = 1/2$ rishons, (T, V, $\bar{T}$, $\bar{V}$) and two $J = 0$ particles. Finally, we do not yet have any convincing dynamics to offer, leaving many, many open questions.

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