1. Prove that the SUSY transformation of the components of a chiral superfield

\[ \Phi(x, \theta, \bar{\theta}) = \phi(y) + \sqrt{2} \theta \psi(y) + \theta \theta F(y), \]

\[ \delta \Phi = (\xi Q + \bar{\xi} \bar{Q}) \Phi, \]

is given by

\[ \delta \phi = \sqrt{2} \xi \alpha \psi \alpha, \]

\[ \delta \psi \alpha = \sqrt{2} \xi \alpha F + \sqrt{2} i \sigma_{\alpha \beta} \bar{\xi} \beta \partial_{\mu} \phi, \]

\[ \delta F = -\sqrt{2} i \partial_{\mu} \psi \sigma_{\mu} \bar{\xi}. \]

2. The O’Raifeartaigh model consists of three chiral superfields with a canonical Kähler potential

\[ K = \Phi^\dagger \Phi \]

and a superpotential

\[ W = \mu^2 \Phi_1 + m \Phi_2 \Phi_3 + g \Phi_1 \Phi_2^2, \]

where \( \mu^2, m \) and \( g \) are real and positive.

(a) Compute the scalar potential and the ground state(s) of the theory.

(b) In the case \( m^2 > 2g\mu^2 \), calculate the spectrum of bosons and fermions in the ground state(s). Is supersymmetry broken? If so, identify the Goldstino fermion.

3. Using the form of the vector superfield in abelian gauge theories in Wess-Zumino gauge,

\[ V(x, \theta, \bar{\theta}) = -\theta \sigma^{\mu} \bar{\theta} v_\mu + i \theta \theta \bar{\theta} \lambda - i \theta \theta \bar{\theta} \lambda + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta}, \]

show that

\[ W_\alpha \equiv -\frac{1}{4} \bar{D} D D_\alpha V = -i \lambda_\alpha(y) + \left[ \delta^\beta_\alpha D(y) - \frac{i}{2} (\sigma^{\mu} \sigma^{\nu})_\alpha^{\beta} F_{\mu \nu}(y) \right] \theta_\beta + \theta \theta \sigma_{\alpha \beta} \partial_\mu \tilde{\lambda}_\gamma(y), \]

where \( y^\mu = x^\mu + i \theta \sigma^{\mu} \bar{\theta} \).

4. The Fayet-Iliopoulos model is defined by

\[ S = \int d^4x d^2 \theta d^2 \bar{\theta} \left( \Phi^e_+ e^{2V} \Phi^- + \Phi^e_- e^{-2V} \Phi^+ + 2 \xi V \right) + \int d^4 x d^2 \theta \left( \frac{1}{4g^2} W^\alpha W \alpha + m \Phi^+ \Phi^- \right) + c.c., \]

where the parameters \( g, \xi \) and \( m \) are real, and \( m > 0 \).

(a) Find the vacua of the theory.

(b) For the case \( \xi > m^2/g^2 \), compute the spectrum of particles.
5. In the last TA session we will discuss methods for distinguishing between SUSY and UED at the LHC, based on spin determination of the intermediate particles in cascade decays. Assume that we want to use these methods at the Tevatron or at a future $e^+e^-$ collider.


(b) Regarding the latter method, can you think of a qualitative difference between the Tevatron and an $e^+e^-$ collider? Recall that the spin information is extracted using invariant mass distributions.

(c) Demonstrate your answer explicitly for the following example: Consider the production of $\tilde{t}\tilde{t}$ (or the equivalent UED states), where one stop decays through $\tilde{t} \rightarrow t\tilde{N}_2 \rightarrow (bW^+)(\ell^+\tilde{\ell}^+) \rightarrow (b\ell^+\nu)(\ell^+\ell^-\tilde{N})$ and for the other $\tilde{t} \rightarrow t\tilde{N}_1 \rightarrow (bW^-)\tilde{N}_1$, with a hadronic decay of $W^-$. Assume that the masses of all new particles have already been measured and that the $b$ momentum is always known, and ignore any detector effects.

(d) For the cascade decay described above, write explicitly a corresponding cascade for UED.