This chapter’s plan:

• Intro’, extra dim why what?
• Large extra dimensions.
• Flat (small) extra dim’: low energy description.
• Tools for model building.
• Warped extra dim’: stabilization, phenomenology.


G. Kribs, hep-ph/0605325, TASI 2004 lectures on the phenomenology of extra dimensions;

Ex dims why?

• Solves problems within the SM, e.g: hierarchy, flavor puzzle, dark matter, strong CP, SUSY breaking ...

• Seems to be a viable options and appears naturally in string theory.

• Provide a weekly coupled dual description strongly coupled gauge theories.
• The idea is very old introduced in the 1920’s by Kaluza and Klein (KK) who tried to unify EM + Gravity.
• 1980’s lead to revitalization, due realization that a consistent string theory will include extra dimensions.
• Must hide them in to be consistent with observation (as we’ll see later).
• A plausible solution is if the other dims’ are small/compact (an interesting non-flat exception is Randall-Sundrum II (RSII)).
• Thus, we need to understand how to derive an effective description higher dimension theoires => Low E effective 4D theory.
Matching higher dim’ theory a 4D effective theory

call the fundamental (higher dimensional) Planck scale of the theory $M_*$

assume $n$ extra dimensions, and that the radii $r$.

Line element is given by: $ds^2 = g_{MN} dx^M dx^N$

using the $(+, -, -, ..., -)$ sign convention for the metric.

The Christoffel symbol is given by: $\Gamma^A_{MN} \sim A^{AB} \partial_M g_{NB}$  with mass dim’ $[\Gamma] = 1$.

\[
R_{\sigma\nu} = R^\rho_{\rho\sigma\nu} = \Gamma^\rho_{\nu\sigma,\rho} - \Gamma^\rho_{\rho\sigma,\nu} + \Gamma^\rho_{\rho\lambda} \Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda} \Gamma^\lambda_{\rho\sigma}.
\]

the Ricci tensor will carry dimension two, $[R_{MN}] = 2$, and similarly the curvature scalar $[R] = 2$.  

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Matching higher dim’ theory a 4D effective theory

Generalized Einstein Hilbert action is:

\[ S_{4+n} \sim \int d^{4+n} x \sqrt{g^{(4+n)}} R^{(4+n)}. \]

\[ S_{4+n} = -M_*^{n+2} \int d^{4+n} x \sqrt{g^{(4+n)}} R^{(4+n)}. \]

\( M_*^{n+2} \) is the \( 4 + n \) fundamental scale.

We need to find how to decompose the above action to a 4D effective one:

\[ S_4 = -M_{Pl}^2 \int d^4 x \sqrt{g^{(4)}} R^{(4)}. \]
Finding the effective 4D Planck mass

We will for now assume, that spacetime is flat, and that the $n$ extra dimensions

So the metric is given by

$$ds^2 = (\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu - r^2 d\Omega_{(n)}^2,$$

Switching on only fluctuations along 4D slice:

$$\sqrt{g^{(4+n)}} = r^n \sqrt{g^{(4)}}, \quad R^{(4+n)} = R^{(4)},$$

where these latter quantities are to be calculated from $h$. Therefore we get

$$S_{4+n} = -M_{*}^{n+2} \int d^{4+n}x \sqrt{g^{(4+n)}} R^{(4+n)} = -M_{*}^{n+2} \int d\Omega_{(n)} r^n \int d^4 x \sqrt{g^{(4)}} R^{(4)}.$$

The factor $\int d\Omega_{(n)} r^n$ is nothing but the volume of the extra dimensional space which we denote by $V_{(n)}$. For toroidal compactification it would simply be given by $V_{(n)} = (2\pi r)^n$.

we find the matching relation for the gravitational couplings

$$M_{Pl}^2 = M_{*}^{n+2} V_{(n)} = M_{*}^{n+2}(2\pi r)^n.$$
What if all the fields are in the bulk?

Let us now repeat the same matching procedure for the gauge couplings.

\[ S^{(4+n)} = - \int d^{4+n}x \frac{1}{4g_*^2} F_{MN} F^{MN} \sqrt{g^{(4+n)}}. \]

Again performing the integral over the extra dimension we find:

\[ S^{(4)} = - \int d^4x \frac{V(n)}{4g_*^2} F_{\mu\nu} F^{\mu\nu} \sqrt{g^{(4)}}. \]

Thus the matching of the gauge couplings is given by

\[ \frac{1}{g_{eff}^2} = \frac{V(n)}{g_*^2}. \]

Note, that it is clear from this equation, that the coupling constant of a higher dimensional gauge theory is not dimensionless, but rather it has dimension \([g_*) = -n/2\). As a consequence it is not a renormalizable theory, but can be thought of as the low-energy effective theory of some more fundamental theory at even higher energies.

The simplest assumption is that the same physics that sets the strength of gravitational couplings would also set the gauge coupling, and thus

\[ g_* \sim \frac{1}{M_*^{\frac{n}{2}}}. \]
Then we would have the following two equations:

\[ \frac{1}{g_4^2} = V(n) M_*^n \sim r^n M_*^n \]
\[ M_{Pl}^2 = V(n) M_*^{n+2} \sim r^n M_*^{n+2}, \]
from which it follows that

\[ r \sim \frac{1}{M_{Pl} g_4^n} \frac{n+2}{n}. \]

This would imply that in a “natural” higher dimensional theory \( r \sim 1/M_{Pl} \)! In this case there would be no hope of finding out about the existence of these tiny extra dimensions in the foreseeable future. This is what the prevailing view has been until the 90’s about extra dimensions. However, we should note that these arguments crucially depended on the ASSUMPTION that every field propagates in all dimensions.
Arkani-Hamed, Dimopoulos and Dvali realized that if the SM fields are localized on a 4D boundary (or brane) then the above tension could be avoided! 

check, how large a radius one would need, if in fact $M_*$ was of the order of a TeV. Reversing the expression $M^2_{Pl} \sim M_*^{n+2} r^n$ we would now get 

$$\frac{1}{r} = M_* \left( \frac{M_*}{M_{Pl}} \right)^\frac{2}{n} = (1\text{TeV}) 10^{-\frac{32}{n}},$$

where we have used $M_* \sim 10^3 \text{ GeV}$ and $M_{Pl} \sim 10^{19} \text{ GeV}$. To convert into conventional length scales one should keep the conversion factor 

$$1 \text{GeV}^{-1} = 2 \cdot 10^{-14} \text{cm}$$

in mind. Using this we finally get 

$$r \sim 2 \cdot 10^{-17} 10^{\frac{32}{n}} \text{ cm}.$$ 

For $n = 1$ this would give the absurdly large value of $r = 2 \cdot 10^{15} \text{ cm}$, which is grater than the astronomical unit of $1.5 \times 10^{13} \text{ cm}$. This is clearly not possible: there can’t be one flat large extra dimension if one would like to lower $M_*$ all the way to the TeV scale. However, already for two extra dimensions one would get a much smaller number $r \sim 2 \text{ mm}$. This is just borderline excluded by the latest gravitational experiments performed in Seattle. Conversely, one can set a bound on the size of two large extra dimensions from the Seattle experiments, which gave $r \leq 0.2 \text{ mm} = 10^{12} \text{ 1/GeV}$. This results in $M_* \geq 3 \text{ TeV}$. 

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The hierarchy problem in disguise

\[ ds^2 = (\eta_{\mu\nu} + h_{\mu\nu}) dx^\mu dx^\nu - r^2 d\Omega^2_{(n)}, \]

can be viewed as a physical d.o.f scalar which has to be stabilized.

\[ r = \langle r \rangle \]
\[ \langle r \rangle \sim (1\text{TeV})10^{-16} \ll \text{TeV}. \]

The hierarchy is still there:
\[ (M_* \langle r \rangle)^{-n} \sim \left( \frac{\text{TeV}}{M_{\text{Pl}}} \right)^2. \]
End of 1st lecture
Towards low E EFT: Kaluza-Klein decomposition

Real Scalar field: (starting from the quadratic action)

\[ S_{5D} = \int d^4 x \int dy \left[ (\partial^M \Phi) (\partial_M \Phi) - M^2 \Phi \Phi \right] \]

- Compactness on a circle (\( S^1 \)):
  
  \(-\infty < y < \infty \) with \( y \equiv y + 2\pi R \) or
  
  \( 0 \leq y \leq 2\pi R \) \( (y = 0 \) same as \( y = 2\pi R \) \)

Periodic boundary condition: \( \Phi(y = 2\pi R) = \Phi(y) \Rightarrow \)

\[
\Phi = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{n=+\infty} \phi^{(n)}(x) e^{iny/R}
\]

Substitute into \( S_{5D} \), use orthogonality of profiles:

\[ S_{4D} = \int d^4 x \sum_n \left[ (\partial_\mu \phi^{(n)}) (\partial^\mu \phi^{(n)}) - \left( M^2 + \frac{n^2}{R^2} \right) \phi^{(n)} \phi^{(n)} \right] \]
Tower of $4D$ fields: Kaluza-Klein (KK) modes

$\phi^{(n)}$ with mass$^2$: $m_n^2 = M^2 + n^2/R^2$

($n^2/R^2$ from $\partial_5$ acting on profile)

Lightest or zero-mode ($n = 0$) has mass $M$

(massless only for $M = 0$)

KK modes start at $\sim 1/R$: compactification scale

(for $M \ll 1/R$)

Generalize to $\delta$ circles of same radius:

$m_n^2 = M^2 + \sum_{i=1}^{\delta} n_i^2/R^2$
Low E 4D observer?

- Signature of extra dimension from 4D point of view:

  appearance of infinite tower of KK modes

Lightest (zero)-modes $\equiv$ SM

+ Heavier ones (KK's) (with identical quantum numbers!)

\[
\phi^{(n)}
\]
What about interval (circle can’t yield chiral theory)

Can use boundary conditions (BCs) to eliminate modes.

For example we can KK-decompose in term of modes consistent with Dirichlet or Neumann BCs:

\[ \Phi(x, y) = \frac{1}{\sqrt{2\pi R}} \phi^{(0)} + \sum_{n=1}^{\infty} \frac{1}{\sqrt{\pi R}} \phi_{\pm}^{(n)} \cos \frac{ny}{R} \left( \sin \frac{ny}{R} \right) \]

with \( \phi_{\pm}^{(n>0)} = \frac{1}{\sqrt{2}} (\phi^{(n)} \pm \phi^{(-n)}) \)

Each “tower” now has half of the modes + 0-mode.

The Dirichlet tower/case has no zero mode!
Orbifold language

Circle is (smooth) manifold: no special points

“Mod out” manifold by discrete symmetry \( \rightarrow \) orbifold

- \( S^1 / \mathbb{Z}_2 \): discrete identification:
  \[ y \leftrightarrow -y \text{ in addition to } y \equiv y + 2\pi R \]

Physical/fundamental region/domain:

\[ y = 0 \text{ to } y = \pi R \]

(or \( y = 0 \) to \( y = 2\pi R \): \( y = \pi R \) to \( y = 2\pi R \) not independent)

- Endpoints \((y = 0, \pi R)\) do not transform under \( \mathbb{Z}_2 \)
  
  (fixed points)

  not identified with each other either by \( S^1 \) or \( \mathbb{Z}_2 \)

  (unlike \( y = 0, 2\pi R \) on circle)

Can translate previous results in term of function with well define transformation under the orbifold sym’.

In this simple case:

\( \mathbb{Z}_2 \) even and odd functions.

\[ \phi_{\mp}^{(n>0)} = 0 \text{ for } P = \pm 1 \]
Fermions on a circle

Representation of 5D Clifford algebra:  \( \{ \Gamma_M, \Gamma_N \} = 2\eta_{MN} \)

provided by Dirac matrices  \( \Gamma_\mu = \gamma_\mu \quad \Gamma_5 = -i\gamma_5 \)

Smallest (irreducible) representation has 4 components (cf. 2-component Weyl spinor in 4D)

\[
S_{5D} = \bar{\Psi} \left( i\partial_M \Gamma^M - M \right) \Psi
\]

Plug  \( \Psi_{\alpha=1-4} = \sum_n \psi^{(n)}_{\alpha} e^{iny/R} \)

\[
S_{4D} = \sum_n \bar{\psi}^{(n)} \left( i\gamma_\mu \partial^\mu - M \pm in/R \right) \psi^{(n)}
\]

Tower of Dirac (4-component) spinors from 4D point
Fermions on a circle, doubling/chirality problem

Tower of Dirac (4-component) spinors

\[ m_n^2 = M^2 + \frac{n^2}{R^2} \]

For \( M = 0 \), non-chiral massless modes:

\[ \psi^{(0)}_{\alpha=1-4} \sim [\psi^{(0)}_L (\alpha = 1, 2), \psi^{(0)}_R (\alpha = 3, 4)] \]

in Weyl representation of Dirac matrices:

\[ \gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \sigma_\mu & 0 \end{pmatrix} \]

\[ \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]

\[ \sigma_\mu = (\sigma_{i=1,3}, 1) \]

\( L \) and \( R \) identical under gauge symmetry \( \rightarrow \)

can’t be SM (chiral fermions):

LH (RH) doublets/singlets under \( SU(2)_L \)
CHIRALITY FROM ORBIFOLD

- Choose $\Psi_L$ even $\rightarrow \Psi_R$ odd:

$$\bar{\Psi} \Gamma^5 \partial_5 \Psi \ni \Psi_L^\dagger \partial_5 \Psi_R$$

For $M = 0$:

$$\Psi_{L,R} \sim \sum_n \psi_{L,R}^{(n)} \cos ny/R (\sin ny/R)$$

- Massless-mode only for $\Psi_L$ (even)

Or equivalently on interval, the KK towers from Neumann BCs would contain zero modes. The towers from Dirichlet would start at level one with no massless states.
CHIRALITY FROM ORBIFOLD

\[
\begin{align*}
\Psi_L & : n \geq 0 \quad n < 0 \\
\Psi_R & : n \geq 0 \quad n < 0
\end{align*}
\]

\[
\begin{align*}
(b) \ S^1/Z_2 \quad \Psi_L & (\text{even}) & \cos \left( \frac{n \pi}{R} \right) \quad \sin \left( \frac{n \pi}{R} \right) \\
\Psi_R & (\text{odd}) & \cos \left( \frac{n \pi}{R} \right) \quad \sin \left( \frac{n \pi}{R} \right)
\end{align*}
\]
More general boundary conditions (-scalars-)

\[ S = \int d^4x \int_0^{\pi^R} dy \left( \frac{1}{2} \partial^N \phi \partial_N \phi - V(\phi) \right) + \int_{y=0} d^4x \frac{1}{2} \phi^2 M_1^2 + \int_{y=\pi_R} d^4x \frac{1}{2} \phi^2 M_2^2. \]

\[ \delta S = - \int d^4x \int_0^{\pi^R} dy \delta \phi \left( \Box_5 \phi - \frac{\partial V}{\partial \phi} \right) + \delta S_{(4D)} - \int d^4x \left( \delta \phi (\partial_5 \phi + M_1^2 \phi) \right)_{|_{\pi_R}} - \delta \phi (\partial_5 \phi + M_2^2 \phi)_{|_{0}} = 0. \]

The variation of the bulk terms will give the usual bulk equation of motion

\[ \partial_M \partial^M \phi - \frac{\partial V}{\partial \phi} = 0 \quad \Rightarrow \quad \partial_5^2 \phi = M_N^2 \phi. \]

\[ \delta \phi (\partial_5 \phi + M_i^2 \phi)_{|_{0,\pi_R}} = 0. \]

\[ (\partial_5 \phi + M_i^2 \phi)_{|_{y=0,\pi_R}} = 0; \]

\[ M_i \to 0, \infty \text{ interpolates between Neumann & Dirichlet BCs.} \]
consider a 5D scalar field decomposed into modes as \( \Phi(x, y) = \sum_n \phi^{(n)}(x)f_n(y) \).

5D action:

\[
S_{5D} = \int d^4x \int dy \left[ (\partial^M \Phi) (\partial_M \Phi) - M^2 \Phi \Phi \right]
\]

We require that, after integrating over the extra dimension, we get

\[
S_{4D} = \int d^4x \sum_n \left[ \left( \partial_\mu \phi^{(n)} \right) \left( \partial^\mu \phi^{(n)} \right) - \left( M^2 + \frac{n^2}{R^2} \right) \phi^{(n)} \phi^{(n)} \right]
\]

so that we can interpret \( \phi^{(n)} \)'s as particles (KK modes) from the 4D point of view.

This requirement gives us the following two equations:

(i) orthonormality condition

\[
\int dy f_n^*(y) f_n(y) = 1
\]

(ii) differential equation:

\[
\partial_y^2 f_n(y) - M^2 f_n^2(y) = -m_n^2 f_n(y)
\]
in terms of the two-component spinors contained in the 5D Dirac spinor as

\[ \Psi = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^\alpha \end{pmatrix} \]

\[ S = \int d^5 x \left( -i \bar{\chi} \sigma^\mu \partial_\mu \chi - i \psi \sigma^\mu \partial_\mu \bar{\psi} + \frac{1}{2} (\psi \, \nabla_5 \chi - \bar{\chi} \, \nabla_5 \bar{\psi}) + m(\psi \chi + \bar{\chi} \bar{\psi}) \right) \]

Varying the action with respect to \( \bar{\chi} \) and \( \psi \) we obtain the standard bulk equations

\[-i \bar{\sigma}^\mu \partial_\mu \chi - \partial_5 \bar{\psi} + m \bar{\psi} = 0 \]
\[-i \sigma^\mu \partial_\mu \bar{\psi} - \partial_5 \chi + m \chi = 0 \]

**KK decomposition:**

\[ \chi = \sum_n g_n(y) \, \chi_n(x), \]
\[ \bar{\psi} = \sum_n f_n(y) \, \bar{\psi}_n(x), \]
where $\chi_n$ and $\psi_n$ are 4D two-component spinors which form a Dirac spinor of mass $m_n$ and satisfy the 4D Dirac equation:

\[-i\bar{\sigma}^\mu \partial_\mu \chi_n + m_n \bar{\psi}_n = 0,\]
\[-i\sigma^\mu \partial_\mu \bar{\psi}_n + m_n \chi_n = 0.\]

\[g'_n + m g_n - m_n f_n = 0, \quad g'' + (m_n^2 - m^2)g = 0, \quad g_n(y) = A_n \cos k_n y + B_n \sin k_n y,\]
\[f'_n - m f_n + m_n g_n = 0. \quad f'' + (m_n^2 - m^2)f = 0, \quad f_n(y) = C_n \cos k_n y + D_n \sin k_n y.\]

(we define $\cos k_n L = \cosh k_n L$ for $k_n^2 = m^2 - m_n^2 > 0$, $\cos k_n L = \cos k_n L$ for $k_n^2 = m_n^2 - m^2 > 0$ and similarly for $\sin k_n L$)

Const’ solution is not possible, inconsistent with simple BC. However note that for the following case the 2 EOMs decouples:

\[f'(y) = m f(y), \quad g'(y) = -m g(y)\]
Naively by deriving the KG equation from the Dirac equation, it seems that the KK reduction would be similar to the scalar case (no massless mode):

\[
(\partial_M + m)(\partial^M - m)\left( \begin{array}{c} \chi_{\alpha} \\ \bar{\psi}^{\dot{\alpha}} \end{array} \right) = (\partial^2 - m^2)\left( \begin{array}{c} \chi_{\alpha} \\ \bar{\psi}^{\dot{\alpha}} \end{array} \right),
\]

However, for the solution where they decouple we find:

\[
- i\sigma^\mu \partial_\mu \chi - \partial_5 \bar{\psi} + m\bar{\psi} = 0
- i\sigma^\mu \partial_\mu \bar{\psi} + \partial_5 \chi + m\chi = 0
\]

\[
\partial_5 f^{(0)}_\chi = -mf^{(0)}_\chi \Rightarrow f^{(0)}_\chi \propto \exp[-my]
\]

vanishes, massless 4D Dirac Eq.
More general boundary conditions (fermions)

The boundary conditions project out one of the 0-modes

\[ \delta S_{\text{bound}} = \frac{1}{2} \int d^4 x \left[ \delta \chi \psi - \delta \psi \chi - \delta \bar{\psi} \bar{\chi} + \delta \bar{\chi} \bar{\psi} \right]^L_0 = 0. \]

see that the most general solution to the vanishing of the boundary variation is when, on the boundary, the two fields \( \psi \) and \( \chi \) are proportional to each other:

\[ \psi_{0,L} = \left( \frac{\partial_5 \psi}{m} + c \bar{\chi} \right)_{0,L} \]
Solution and zero modes (fermions)

How is this solution fits into the BC?

In orbifold language the bulk mass is parity odd.

It has therefore to have a jump at the boundary.

Zero mode exist only for parity even fermion:

\[
\begin{align*}
  f_{L0}(y) &= Ne^{My} \quad (0 \leq y \leq \pi R) \\
  &= Ne^{-My} \quad (0 \geq y \geq -\pi R)
\end{align*}
\]

Profile of odd mass term (dashed line) and fermion zero-mode (solid line). Here and henceforth, we set radius of extra dimension, \( R = 1 \) in all figures.
Summary 0-modes  (fermions)

We saw boundary condition can be shown to match:

$$\partial_5 \chi^{(0)}_0 \pm = \pm m \chi^{(0)}|_0 \Rightarrow \chi^{(0)} \propto e^{\int dy m(y)}.$$ 

The KK state will maintain a typical spectra.
A fermion with (-+,+-) boundary conditions would not contain a zero mode.

This leads to fermion localizations.

We can use this feature in order to induce low energy approximate symmetries with no fundamental ones!

We can get hierarchies without symmetries in some cases. Only due to geometry.
Split fermion and solution to the flavor puzzle

Let us assume that all the SM fermions and gauge fields are allowed to live in the bulk. Thus for each SM bulk field we expect to find a KK tower.

**suppose the SM Higgs field is localized at** $y = \pi R$

**add the following coupling of 5D fermions to it:**

$$S_{5D} \equiv \int d^4x dy \delta(y - \pi R) H \Psi_L \Psi'_R \lambda_{5D}$$

where $\Psi$ and $\Psi'$ are two different 5D fermion fields which are $SU(2)_L$ doublets and singlets with $M, M'$ being their 5D masses, respectively. Note that $\Psi_L$ and $\Psi'_R$ are chosen to be even under $Z_2$ so that they give the LH and RH zero-modes, respectively. Since $\Psi_R$ and $\Psi'_L$ vanish at the $y = \pi R$

Let us look at this model in more detail
Split fermion and solution to the flavor puzzle

\[ \mathcal{L}^m = \left( Q_i, U_i, D_i \right) M^{ij}_{Q,U,D} \left( Q^j, U^j, D^j \right) + H Y^{ij}_{U,D} \bar{Q}^i \left( U^j, D^j \right) \delta(y - \pi R) \]

Where the flavor parameters in the form of the two 5D Yukawa matrices and three bulk masses are assumed to be, generic, of anarchical form.

\[ f_{Q,U,D}(y) \sim e^{m_{Q,U,D}(y - \pi R)} \quad \text{for positive bulk mass and} \]

\[ f_{Q,U,D}(y) \sim e^{-m_{Q,U,D}y} \quad \text{for negative mass, assuming } m_i R > 1 \]

and \( m_X \) being an eigenvalue of \( M_X, X \in Q, U, D \).
Split fermion and solution to the flavor puzzle

\[ Y_{U,D}^{(4D)} = \text{diag} \left[ f_Q^1(\pi R), f_Q^2(\pi R), f_Q^3(\pi R) \right] \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix} \text{diag} \left[ f_{U,D}^1(\pi R), f_{U,D}^2(\pi R), f_{U,D}^3(\pi R) \right] \]

Since the top Yukawa is \( \sim 1 \) then both \( m_Q^3 \) and \( M_U^3 \) needs to be positive.

Hence \( f_Q^3(\pi R) \sim f_U^3(\pi R) \sim 1 \)

However, the rest of the quark Yukawa are smaller hence they are probably localized towards the other boundary. The resulting 4D Yukawa matrices are of hierarchical, quasi-diagonal form
each $2 \times 2$ block has the feature that $(M_{12}, M_{21})/(M_{22}) \ll 1$ and $M_{11} \ll M_{22}$

$$Y_{U,D}^{(4D)} = \text{diag} \left[ e^{\Delta_{Q}^{13}}, e^{\Delta_{Q}^{23}}, 1 \right] \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & j \end{pmatrix} \text{diag} \left[ e^{\Delta_{U,D}^{13}}, e^{\Delta_{U,D}^{23}}, 1 \right]$$

with $\Delta_{X}^{ij} = \pi \left( m_{X}^{i} - m_{X}^{j} \right) R$. Thus, up to order one (complex) coefficients

the effective 4D Yukawa matrices can be rewritten as

$$Y_{U,D}^{(4D)} \sim \begin{pmatrix} e^{\Delta_{U,D}^{13} + \Delta_{Q}^{13}} & e^{\Delta_{U,D}^{13} + \Delta_{Q}^{13}} & e^{\Delta_{Q}^{13}} \\ e^{\Delta_{U,D}^{13} + \Delta_{Q}^{23}} & e^{\Delta_{U,D}^{23} + \Delta_{Q}^{23}} & e^{\Delta_{Q}^{23}} \\ e^{\Delta_{U,D}^{13}} & e^{\Delta_{U,D}^{23}} & 1 \end{pmatrix}$$
Split fermion and solution, quark masses

One can see that the mass hierarchy can be easily via order $1/R$ masses, for the up-type quark:

$$\frac{m_{u,c}}{m_t} \sim 10^{-2} \sim e^{\Delta_{U,D}^{13,23} + \Delta_Q^{13,23}} \Rightarrow \Delta_{U,D}^{13,23} + \Delta_Q^{13,23}$$

$$= \pi \left( m_{U,D}^{1,2} - m_{U,D}^{3} + m_{Q}^{1,2} - m_{Q}^{3} \right) R = \ln \left( \frac{m_{u,c}}{m_t} \right) \sim -6, -11$$

The above relation, for the SM masses, fixes the sum of the 5D mass difference,

we can use the hierarchies in the CKM angles $\theta_{12,23,13} \sim \lambda_C, \lambda_C^2, \lambda_C^3$ ($\lambda_C \sim 0.23$)

 to fix the mass differences for the, quark doublet, 5D masses as follows.
Split fermion and solution, mixing angles

Recall that the objects \( Y_{U,D}^{(4D)} Y_{U,D}^{(4D)\dagger} \) transform as octet and singlet of SM flavor group and diagonalized only by LH rotation matrices, \( V_{u_L,d_L} \) with \( V_{\text{CKM}} = V_{u_L} V_{d_L} \dagger \).

In the above case the structure of \( Y_{U,D}^{(4D)} Y_{U,D}^{(4D)\dagger} \) is given by

\[
Y_{U,D}^{(4D)} Y_{U,D}^{(4D)\dagger} \sim \begin{pmatrix}
e^{2\Delta_{Q}^{13}} & e^{\Delta_{Q}^{13} + \Delta_{Q}^{23}} & e^{\Delta_{Q}^{13}} \\
e^{\Delta_{Q}^{13} + \Delta_{Q}^{23}} & e^{2\Delta_{Q}^{23}} & e^{\Delta_{Q}^{23}} \\
e^{\Delta_{Q}^{13}} & e^{\Delta_{Q}^{23}} & 1
\end{pmatrix}.
\]
Thus, to get the correct mixing angles requires

$$\theta_{12,23,13} \sim e^{\Delta_{Q}^{13}} - \Delta_{Q}^{23}, e^{\Delta_{Q}^{23}}, e^{\Delta_{Q}^{13}} = e^{\Delta_{Q}^{12}}, e^{\Delta_{Q}^{23}}, e^{\Delta_{Q}^{13}}$$

which implies

$$\Delta_{Q}^{12,23,13} \sim \pi \left( m_{Q}^{1,2,1} - m_{Q}^{2,3,3} \right) R = \ln \left( \lambda_{C}, \lambda_{C}^{2}, \lambda_{C}^{3} \right) \sim -1.5, -3, -4.5$$

we have 5 ratios for the quark masses and 3 CKM angles with fix 8 mass differences

This roughly fixes the 9 bulk masses with one non-trivial relation $\theta_{12}\theta_{23} \sim \theta_{13}$

looks very similar to the Froggatt- Nielsen case with one important

difference, overlooked in most of the literature on the subject, to described latter.
Gauge Bosons, first on a circle

\[ S_{5D} = \int d^4xdy \frac{1}{4} F_{MN} F^{MN} \]
\[ = \int d^4xdy \frac{1}{4} (F_{\mu\nu} F^{\mu\nu} + F_{\mu5} F^{\mu5}) \]
\[ A_M = A_\mu , A_5 \]

Plug \( A_{\mu,5} = \sum_n A_{\mu,5}^{(n)} f_{\mu,5n}(y) \) into \( S_{5D} \)

\(~\text{scalar, up to Lorentz index and gauge fixing}\)

- Unification of spins:
  
  massless 4D scalars from gauge fields

Extra long range force (if \( A_5^{(0)} \) massless) \( \rightarrow \) ruled out

acquires mass from loop corrections \( \rightarrow \)

not a robust problem

(Unlike fermions)
Gauge Bosons, interval

Get rid of zero-mode using orbifold:

\[ \mathcal{F}_{\mu 5} = \partial_{\mu} A_5 - \partial_5 A_{\mu} \]

Two choices:

(i) \( A_{\mu} \) even (zero-mode \( \equiv \) SM gauge boson) \( \rightarrow \)
\( A_5 \) odd (no zero-mode)

or

(ii) \( A_{\mu} \) odd,
\( A_5 \) even (zero-mode is Higgs?)
Gauge Bosons, KK decomposition

For $A_{\mu,5}$ even (odd):

$$f_{\mu 0} = \frac{1}{\sqrt{2\pi R}} \text{ (flat)}$$

$$f_{\mu n(y)} = \frac{1}{\sqrt{\pi R}} \cos ny/R$$

$$f_{5 n(y)} = \frac{1}{\sqrt{\pi R}} \sin ny/R$$

$A_{\mu}^{(n\neq 0)}$ "eats" $A_5^{(n)}$ to form massive spin-1 gauge boson

(a la longitudinal $W \sim$

unphysiscal component of Higgs):

$$\mathcal{F}_{\mu 5}^2 \equiv \partial_{\mu} A_5 \partial_5 A^{\mu}$$

$$\sim \sum_n A_{\mu}^{(n)} \partial_{\mu} A_5^{(n)} \partial_y f_{\mu n(y)}$$

(like $W_\mu \partial^\mu H \langle H \rangle$ in SM)
Interaction with matter

0-mode has universal couplings (guaranteed by 4D gauge invariance):

\[ \int d^4x dy \bar{\Psi} \Gamma^M (\partial_M + g_5 A_M) \Psi \ni \sum_n \bar{\psi}_L^{(n)} A^{(0)}_\mu \gamma^\mu \psi_L^{(n)} \times \]

\[ \int dy f_{L n}^2 \frac{g_5}{\sqrt{2\pi R}} \]

\[ = \ldots g_4 \text{ (for all } n) \]

with

\[ g_4 \text{ (or } g_{SM}) = \frac{g_5}{\sqrt{2\pi R}} \]

KK states have non-universal couplings:

\[ g(n, M) = g_5 \int dy (N e^{-M y})^2 \times f_{\mu, n}(y) \]

\[ \equiv g_4 \times a(n, M), \quad a \sim O(1) \]
The Xtra Dim’ Flavor Problem

With split fermions, we have additional sources of flavor breaking (on top of the Yukawas).

It comes in the form of 5D bulk masses.

Missalignment between the 5D masses and the Yukawas yields extra sources of CP flavor violation.

Flavor diagonal, but non-universal couplings

in interaction/weak basis:

\[ g_4 \left( \bar{d}_L \bar{s}_L \right) \left( \begin{array}{cc} a_d & 0 \\ 0 & a_s \end{array} \right)_{L} \gamma^{\mu} A^{(n)}_{\mu} \left( \begin{array}{c} d_L \\ s_L \end{array} \right) \]

→ Flavor violation after rotation to mass basis:

...\[ g_4 D^L_D \text{diag} (a_d, a_s) D^L_D \cong g_4 (a_s - a_d) (D^L_D)_{12} \times \]

\[ \bar{d}_L \text{mass} \gamma^{\mu} A^{(n)}_{\mu} s_L \text{mass} \]
The Xtra Dim’ Flavor Problem

The most dangerous contributions is to CPV in the Kaon system $\epsilon_K$, (from LLRR operator, $Q_4^{q_i q_j} = \bar{q}_{jR} q_{iL} \bar{q}_{jL} q_{iR}^\beta$):

$$\frac{g_3^2}{M_{KK}^2} (a_s - a_d)_L (a_s - a_d)_R \text{Im}[(D_L)_{12} (D_R)_{12}^*].$$

For Higgs on the brane case it is roughly given by:

$$\frac{g_3^2}{M_{KK}^2} e^{-R(\Delta m_{21}^d + \Delta m_{21}^Q)} \sim \frac{g_3^2}{M_{KK}^2} \frac{m_d}{m_s} \sim \frac{0.25^2}{M_{KK}^2}$$

What’s the bound on the KK scale (or $1/R$)?

Roughly $6 \times 10^4$ TeV!

This brings back the fine tuning problem!
Symmetry breaking via boundary conditions

Can break symmetries consistently via a-symmetric BCs!

Sherk-Schwarz mechanism (1970):

We can give the bosons and fermions different boundary conditions in such a way that only the fermions would have a zero mode. However this would be hard breaking (UV sensitive).

We can do it collectively by breaking part of the bulk SUSY on each side of the boundary. Then it’s non-local breaking and is therefore finite.
So far Xtra dim’ left several problem unanswered.

The hierarchy problem is unaddressed.

The flavor puzzle may be solved but generically leading to a flavor and CP problems.

Why is the Higgs localized on a brane?

Issues resolved within warped models.
Warped Extra Dim'

Bulk cosmological constant (CC) + brane tensions

(CC’s):

\[ S_{5D} = \int d^4x \, dy \sqrt{-\det G} \left( M_5^3 \mathcal{R}_5 - \Lambda \right) \]

\[ S_{brane \, 1, \, 2} = \int d^4x \sqrt{-\det g_{1, \, 2}} T_{1, \, 2} \]

where \( g_{\mu\nu \, 1, \, 2}(x) = G_{\mu\nu}(x, y = 0, \pi R) \)

With 2 fine-tunings: \( T_1 = -T_2 = 24k M_5^3 \),

where \( \Lambda = 24k^2 M_5^3 \), obtain flat, but \( y \)-dependent 4D metric:

\[ (ds)^2 = e^{-2ky} \eta_{\mu\nu} (dx)^\mu (dx)^\nu + (dy)^2 \]

Slice of AdS\(_5\): warp factor, \( e^{-ky} \)
4D/zero-mode graviton:  \( g^{(0)}_{\mu\nu}(x) \approx \eta_{\mu\nu} + h^{(0)}_{\mu\nu}(x) \)

Plug into 5D action:

\[
S_{4D, \text{eff}} = M_{Pl}^2 \int d^4x \sqrt{-\det g^{(0)} R_4[g^{(0)}]}
\]

\[
M_{Pl}^2 \sim \frac{M_5^3}{k} \left(1 - e^{-2k\pi R}\right)
\]

\[
\sim \frac{M_5^3}{k} \quad \text{for } kR \gg 1
\]

Choose \( k \approx M_5 \) (higher curvature terms small) →

\( (k \approx) M_5 \approx M_{Pl} \approx 10^{18} \text{ GeV} \)

- 4D graviton localized near \( y = 0 \) (Planck/UV brane):
  
  \[ \text{profile} \sim e^{-2ky} \]
Solving the hier’ problem

4D Higgs localized on \( y = \pi R \) brane (TeV/IR brane):

\[
S_{\text{Higgs}} = \int d^4x \sqrt{-\det g_{\text{ind.}}} \left[ g^{\mu\nu}_{\text{ind.}} \partial_\mu H \partial_\nu H - \lambda \left( |H|^2 - v_0^2 \right)^2 \right]
\]

using \( g^{\text{ind.}}_{\mu\nu} = G_{\mu\nu}(y = \pi R) = g^{(0)}_{\mu\nu} e^{-2k\pi R} \)

\[
S_{\text{Higgs}} = \int d^4x \sqrt{-\det g^{(0)}} \left[ e^{-2k\pi R} g^{(0)}_{\mu\nu} \partial_\mu H \partial_\nu H - e^{-4k\pi R} \lambda \left( |H|^2 - v_0^2 \right)^2 \right]
\]
Solving the hier’ problem

Rescale for canonical normalization:

\[ H \equiv \hat{H} e^{k\pi R} \rightarrow \]

\[ S_{\text{Higgs}} = \int d^4 x \sqrt{\text{det} g^{(0)}} [g^{(0)}_{\mu\nu} \partial_\mu \hat{H} \partial_\nu \hat{H} - \lambda (|\hat{H}|^2 - v_0^2 e^{-2k\pi R})^2] \]

Weak scale/Higgs vev “warped-down” to \( \sim \) TeV from Planck scale if

\[ k\pi R \sim \log (M_{Pl}/\text{TeV}) \]

True for any mass scale parameter in the fundamental (5D) theory!
Radius Stabilization [Goldberger-Wise, (99)]

\[ ds^2 = e^{-2k|\phi|T(x)}g_{\mu\nu}(x)dx^{\mu}dx^{\nu} - T^2(x)d\phi^2, \quad (T = r_c) \]

Kaluza-Klein reduction of the five-dimensional Einstein-Hilbert action for this metric leads to the following effective action for the massless modes \( T(x) \) and \( g_{\mu\nu}(x) \):

\[ S = 2M^3 \int d^4x d\phi \sqrt{-g} e^{-2k|\phi|T} [6k|\phi|\partial_\mu T \partial^\mu T - 6k^2|\phi|^2 T \partial_\mu T \partial^\mu T + TR], \]

After the \( \phi \) integration \( S = \frac{2M^3}{k} \int d^4x \sqrt{-g} \left( 1 - (\varphi/f)^2 \right) R + \frac{1}{2} \int d^4x \sqrt{-g} \partial_\mu \varphi \partial^\mu \varphi, \)

\[ \varphi = f \exp(-k\pi T) \text{ with } f = \sqrt{24M^3/k}. \]

\( \varphi \) is massless!

The 5D interval is not stabilized, can’t yet considered as a solution to the hierarchy.
include a scalar field with bulk action

\[ S_b = \frac{1}{2} \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{G} \left( G^{AB} \partial_A \Phi \partial_B \Phi - m^2 \Phi^2 \right), \]

also include interaction terms on the branes

\[ S_h = - \int d^4x \sqrt{-g_h} \lambda_h \left( \Phi^2 - v_h^2 \right)^2, \quad S_v = - \int d^4x \sqrt{-g_v} \lambda_v \left( \Phi^2 - v_v^2 \right)^2, \]

Note that \( \Phi \) and \( v_{v,h} \) have mass dimension \( 3/2 \), while \( \lambda_{v,h} \) have mass dimension \( -2 \).

The terms on the branes cause \( \Phi \) to develop a \( \phi \)-dependent vacuum expectation value \( \Phi(\phi) \) which is determined classically by solving the differential equation

\[
0 = -\frac{1}{r_c^2} \partial_\phi \left( e^{-4\sigma} \partial_\phi \Phi \right) + m^2 e^{-4\sigma} \Phi + 4 e^{-4\sigma} \lambda_v \Phi \left( \Phi^2 - v_v^2 \right) \frac{\delta(\phi - \pi)}{r_c} \\
+ 4 e^{-4\sigma} \lambda_h \Phi \left( \Phi^2 - v_h^2 \right) \frac{\delta(\phi)}{r_c}, \quad (T = r_c)
\]

where \( \sigma(\phi) = kr_c|\phi| \). Away from the boundaries at \( \phi = 0, \pi \), this equation has the general solution

\[ \Phi(\phi) = e^{2\sigma}[A e^{\nu \sigma} + B e^{-\nu \sigma}], \quad \text{with} \quad \nu = \sqrt{4 + m^2/k^2}. \]
Putting this solution back into the scalar field action and integrating over \( \phi \) yields an effective four-dimensional potential for \( r_c \) which has the form

\[
V_\Phi(r_c) = k\epsilon v_h^2 + 4ke^{-4kr_c\pi}(v_v - v_h e^{-\epsilon kr_c\pi})^2 \left(1 + \frac{\epsilon}{4}\right) - k\epsilon v_h e^{-(4+\epsilon)kr_c\pi}(2v_v - v_h e^{-\epsilon kr_c\pi})
\]

where terms of order \( \epsilon^2 \) are neglected (but \( \epsilon kr_c \) is not treated as small). Ignoring terms proportional to \( \epsilon \), this potential has a minimum at

\[
k r_c = \left(\frac{4}{\pi}\right) \frac{k^2}{m^2} \ln \left[\frac{v_h}{v_v}\right].
\]

With \( \ln(v_h/v_v) \) of order unity, we only need \( m^2/k^2 \) of order 1/10 to get \( kr_c \sim 10 \). Clearly, no extreme fine tuning of parameters is required to get the right magnitude for \( kr_c \). For instance, taking \( v_h/v_v = 1.5 \) and \( m/k = 0.2 \) yields \( kr_c \sim 12 \).
**RS KK decomposition**

\[ ds^2 = e^{-2\sigma} \eta_{\mu \nu} dx^\mu dx^\nu + dy^2, \quad \sigma = k|y|, \quad M_P^2 = \frac{M_5^3}{k} \left( 1 - e^{-2\pi k R} \right). \]

or

\[ ds^2 = \left( \frac{R}{z} \right)^2 (\eta_{\mu \nu} dx^\mu dx^\nu - dz^2), \]

where \( R = 1/k \) (do not confuse this with the size of the extra dimension in proper \( y \) coordinates), and the variable \( z \) runs between \( R \) and \( R', R'/R = e^{kr} = 10^{16} \).

\[
S_5 = - \int d^4x \int dy \sqrt{-g} \left[ \frac{1}{4g_s^2} F_{MN}^2 + |\partial_M \phi|^2 + i\Psi \gamma^M D_M \Psi + m_\phi^2 |\phi|^2 + im_\Psi \bar{\Psi} \Psi \right],
\]

\[ D_M = \partial_M + \Gamma_M, \text{ where } \Gamma_M \text{ is the spin connection } \quad \Gamma_\mu = \frac{1}{2} \gamma_5 \gamma_\mu \frac{d\sigma}{dy} \quad \text{and} \quad \Gamma_5 = 0. \]

acts as additional bulk mass \( k/2 \).

For the fermion \( \Psi \), the \( \mathbb{Z}_2 \) transformation is given by \( \Psi(-y) = \pm \gamma_5 \Psi(y) \).
The equations of motion for the gauge, scalar and fermion fields are respectively given by

\[
\partial_M \left( \sqrt{-g} g^{MN} g^{RS} F_{NS} \right) = 0, \\
\frac{1}{\sqrt{-g}} \partial_M \left( \sqrt{-g} g^{MN} \partial_N \phi \right) - m^2 \phi = 0, \\
\left( g^{MN} \gamma_M D_N + m_\Psi \right) \Psi = 0.
\]

one can write a general second-order differential equation

\[
\left[ e^{2\sigma} \eta^{\mu\nu} \partial_\mu \partial_\nu + e^{s\sigma} \partial_5 (e^{-s\sigma} \partial_5) - M^2_{\phi} \right] \Phi(x^\mu, y) = 0,
\]

\[
\Phi = \{ V_\mu, \phi, e^{-2\sigma} \Psi_{L,R} \}, \quad s = \{2, 4, 1\} \text{ and } M^2_{\phi} = \{0, ak^2, c(c \pm 1)k^2\}. 
\]
Let us decompose the 5D fields as

\[
\Phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \Phi^{(n)}(x^\mu) f_n(y),
\]

where the Kaluza-Klein modes \(f_n(y)\) obey the orthonormal condition

\[
\frac{1}{2\pi R} \int_{-\pi R}^{\pi R} dy \, e^{(2-s)\sigma} f_n(y) f_m(y) = \delta_{nm}.
\]

the Kaluza-Klein eigenmodes \(f_n(y)\) satisfy the differential equation

\[
\left[ e^{s\sigma} \partial_5 (e^{-s\sigma} \partial_5) + M_\Phi^2 \right] f_n = e^{2\sigma} m_n^2 f_n.
\]

\[
f_n(y) = \frac{e^{s\sigma/2}}{N_n} \left[ J_\alpha \left( \frac{m_n}{k} e^\sigma \right) + b_\alpha(m_n) Y_\alpha \left( \frac{m_n}{k} e^\sigma \right) \right],
\]

The Bessel functions \(J_\alpha\) and \(Y_\alpha\) are of order \(\alpha = \sqrt{(s/2)^2 + M_\Phi^2/k^2}\).

In the limit \(m_n \ll k\) and \(kR \gg 1\) an approximate expression for the normalization constant is

\[
N_n \approx \frac{e^{\pi kR}}{\sqrt{2\pi k R}} J_\alpha \left( \frac{m_n}{k} e^{\pi kR} \right) \approx \frac{e^{\pi kR/2}}{\sqrt{\pi^2 R m_n}}.
\]
RS KK decomposition

\[
\left( \frac{df_n}{dy} - r \sigma f_n \right) \bigg|_{0, \pi R} = 0 ,
\]

where the parameter \( r \) has the values \( r = \{0, b, \mp c\} \) for \( \Phi = \{V_\mu, \phi, e^{-2\sigma} \Psi_{L, R}\} \), respectively. These two conditions gives rise to the two equations

\[
b_\alpha(m_n) = \frac{(-r + s/2) J_\alpha(m_n/k) + m_n J'_\alpha(m_n/k)}{(-r + s/2) Y_\alpha(m_n/k) + m_n Y'_\alpha(m_n/k)} ,
\]

\[
b_\alpha(m_n) = b_\alpha(m_n e^{\pi k R}) .
\]

These two conditions determine the values of \( b_\alpha \) and \( m_n \). In the limit that \( m_n \ll k \) and \( kR \gg 1 \) the Kaluza-Klein mass solutions for \( n = 1, 2, \ldots \) and \( \alpha > 0 \) are

\[
m_n \simeq (n + \frac{\alpha}{2} - \frac{3}{4}) \pi k e^{-\pi k R}.
\]

KK scale is always of order of TeV, reachable but dangerous!
0-mode fermions

\[ \hat{f}_0(c, z)^2 = z^{-3} f_0^2 = \frac{k(1-2c)}{e^{-k\pi r_c(1-2c)}-1} \left( k z \right)^{\frac{1}{2}} - c, \quad c = m\Psi/k, \quad k z = e^{-k r_c \phi} \]

It is useful to find the asymptotic dependence of \( f_{x^i} \) on \( c_{x^i} \)

\[
f_{x^i}^2 \sim \begin{cases} 
\frac{1}{2} - c_{x^i} & \text{for } c_{x^i} < \frac{1}{2} - \epsilon \\
\frac{1}{2k\pi r_c} & \text{for } c_{x^i} \to \frac{1}{2} \\
(c_{x^i} - \frac{1}{2}) e^{k\pi r_c(1-2c_{x^i})} & \text{for } c_{x^i} > \frac{1}{2} + \epsilon,
\end{cases}
\]

the effective 4D Yukawa matrices \( \lambda_{u,d}^{4D} \) are

\[
\lambda_{u,d}^{ij} = 2 \lambda_{u,d 5D}^i k f_Q^i f_{u^j,d^j}
\]

Can easily solve the flavor puzzle!
0-mode fermions, solution of the flavor puzzle

<table>
<thead>
<tr>
<th>Flavor</th>
<th>$c_Q$, $f_Q$</th>
<th>$c_u$, $f_u$</th>
<th>$c_d$, $f_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.64, 0.002</td>
<td>0.68, $7 \times 10^{-4}$</td>
<td>0.65, $2 \times 10^{-3}$</td>
</tr>
<tr>
<td>II</td>
<td>0.59, 0.01</td>
<td>0.53, 0.06</td>
<td>0.60, 0.008</td>
</tr>
<tr>
<td>III</td>
<td>0.46, 0.2</td>
<td>-0.06, 0.8</td>
<td>0.58, 0.02</td>
</tr>
</tbody>
</table>

$f_0^2 = \frac{1}{2\pi r_c}$ \implies g = g_5/\sqrt{2\pi r_c}$
KK excitation

All the states are Bessel functions sharply (exponentially) localized near the IR brane with width of $\sim 1/k$

$$f_n \sim e^{kr_c(\phi - \pi)}$$

Thus we can approximate them as localized on the IR brane.

$$S_{\psi}^{\text{int}} \approx -\int d^4x g \overline{\psi} \gamma^\mu \left( A_\mu^{(0)}(x) + \sqrt{2\pi kr_c} \sum_{n=1}^{\infty} A_\mu^{(n)}(x) \right) \psi$$

Taking $kr_c \approx 11.27$, we obtain $\sqrt{2\pi kr_c} \approx 8.4$. Therefore, the excited KK modes couple to the 3-brane fermions about 8 times more strongly than the zero mode, which is identified with the usual ‘photon’

Thus all the couplings can be deduced once we know the zero modes wave function on the TeV brane!
Effective Lagrangian and spurion analysis

\[ \mathcal{L}_G = g \sqrt{k \pi r_c} G^{(n)} \bar{q}_i q_i \left( \frac{1}{k \pi r_c} - 2f^2 x_i \right). \]

RS GIM protection: much better than the flat extra dimension case. Since the non-univ. couplings to the first two gen’ is very small and there’s partial alignment with the 4D Yukawa matrices both are are controlled by hierarchical f’s!
Connection with strong dynamics & AdS/CFT

\[
\mathcal{L}_{4D} = \mathcal{L}_{CFT} + Z_0 \bar{\psi}^{(0)}_L i \gamma^\mu \partial_\mu \psi^{(0)}_L + \frac{\omega}{\Lambda^{c+\frac{1}{2}}-1} (\bar{\psi}^{(0)}_L \phi_R + h.c.) ,
\]

RS is probably dual to composite Higgs models vs. Higgsless which is dual to Technicolor models.