Problem set 1: Stellar structure

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1. (a) Use the virial theorem to estimate the mass $M(\mu)$ of a star for which the plasma particle pressure and the radiation pressure are comparable, assuming that the plasma particles are a (non-relativistic) ideal gas of mean molecular weight μ. What is $M(\mu = m_p/2)$?

   (b) Calculate $(\partial \log e/\partial \log \rho)_S$ for a non-relativistic ideal gas of mean molecular weight μ in thermal equilibrium with radiation. Express the results in terms of the ratio β between the photon and particle number density.

2. (a) Derive an approximate expression for the luminosity $L(M, R, \mu, K)$ of a low mass star of mass M, radius R and mean molecular weight μ, assuming that the opacity is well approximated by Kramers' $\kappa = K\rho T^{-7/2}$. For $\kappa(\rho = 1\text{g/cm}^3, T = 1\text{keV}) = 1\text{cm}^2/\text{g}$, how does the luminosity predicted by this expression (for $M = M_\odot$, $\mu = m_p/2$) compare with the solar luminosity (you may assume H fusion threshold at $T = 1$ keV)?

   (b) How much brighter than the sun would a $M = 0.5M_\odot$ Helium main sequence star be (you may assume it is composed of fully ionized He and that the He fusion threshold is 10 keV).

3. Consider a H sphere of mass M and radius R in hydrostatic equilibrium contracting due to emission of radiation. What is the minimum mass M that is required to achieve H ignition? [Hint: Consider the point where electron degeneracy pressure sets in, and compare L to the *pp* fusion energy generation rate.]

4. Consider a ("stripped") star of mass $M \sim 1M_\odot$ composed of fully ionized ^{12}C. Estimate its luminosity L, temperature T, radius R and effective temperature $T_{\text{eff.}}$ ($L = 4\pi r^2 \sigma T^4_{\text{eff.}}$), assuming that the energy

source is purely ^{12}C to ^{24}Mg fusion (use $S = 10^{-22}\text{cm}^2\text{keV}$, give the M dependence and the values for $M = 1M_\odot$).

5. (a) Express the criterion for stability against convection in terms of the temperature and pressure gradients, dp/dr and dT/dr, for an ideal gas equation of state, $p = (\gamma - 1)e$ for which $(\partial \log p/\partial \log \rho)_S = \gamma$.

   (b) Generalize the criterion for stability against convection for the case where the composition of the star depends on radius. Express you result using $(\partial p/\partial \rho)_{S,X_i}$ and $(\partial p/\partial X_i)_{S,\rho}$ where X_i is the mass fraction of element i.