3 Stellar structure, evolution and end states

3.1 Binaries [1hr]

Accurate determination of stellar masses: binaries. Most stars in binaries. Types: visual, astrometric, eclipsing, spectroscopic. Recent review by Torres, Andersen & Gimenez (2010 A&ARv 18, 67) gives 94 detached non-interacting eclipsing systems with mass and radius of both stars be known within errors of 3% accuracy or better.

m_1, m_2 binary, rest frame $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = 0$, choose $m_1\mathbf{r}_1 + m_2\mathbf{r}_2 = 0$. Defining

$$m \equiv \frac{m_1 m_2}{m_1 + m_2}, \quad M \equiv m_1 + m_2, \quad \mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{v} \equiv \dot{\mathbf{r}}, \tag{4}$$

the equations of motion become $\ddot{\mathbf{r}} = -GMm\hat{\mathbf{r}}/r^2$ and

$$\mathbf{r}_1 = \frac{m}{m_1}\mathbf{r}, \quad \mathbf{r}_2 = -\frac{m}{m_2}\mathbf{r}, \quad r_1 + r_2 = r, \quad v_1 + v_2 = v, \tag{5}$$

$$\mathbf{J} = m\mathbf{r} \times \mathbf{v}, \quad E = \frac{1}{2}mv^2 - \frac{GMm}{r}. \tag{6}$$

The shape of the orbit is determined by $dr/d\theta = v_r/(v_\theta/r)$, with $\mathbf{v}_\theta = (\hat{\mathbf{r}} \times \mathbf{v}) \times \hat{\mathbf{r}} = \mathbf{J} \times \hat{\mathbf{r}}/mr$, $v_\theta = J/mr$ and $\mathbf{v}_r = (\hat{\mathbf{r}}.\mathbf{v})\hat{\mathbf{r}}$, giving

$$\left(\frac{dr}{rd\theta}\right)^2 = \left(\frac{v_r}{v_\theta}\right)^2 = \frac{v^2 - v_\theta^2}{v_\theta^2} = \frac{2E/m + 2GM/r - (J/mr)^2}{(J/mr)^2}. \tag{7}$$

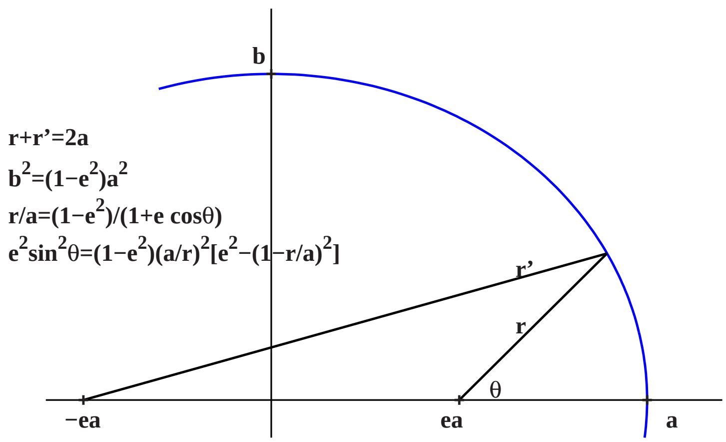
For motion along an ellipse we have

$$\left(\frac{dr}{rd\theta}\right)^2 = \frac{e^2 - (1 - r/a)^2}{(1 - e^2)} = \frac{-1 + 2a/r - (1 - e^2)(a/r)^2}{(1 - e^2)(a/r)^2}. \tag{8}$$

Comparing eqs. (8) and (7) we find that the orbit is an ellipse with

$$a = -\frac{GMm}{2E}, \quad b = a\sqrt{1 - e^2} = \frac{J}{\sqrt{2m|E|}}. \tag{9}$$

Suppose that the orbit is observed at a direction making an angle i with the direction perpendicular to the ellipse plane, and that its projection on the ellipse plane makes an angle ϕ with the major axis of the ellipse. The binary is completely determined by $\{m, M, E, J\}$ or by $\{m, M, a, e\}$, and the observed properties by the additional $\{i, \phi\}$. 2 constraints are provided



by the (observed) velocity ratio and period. The period $T = A/\dot{A}$, where $A = \pi ab$ and $\dot{A} = rv_\theta/2 = J/2m$ (Kepler's 2nd), implying

$$T^2 = \frac{4\pi^2 a^3}{GM}, \quad \frac{v_{1\,\mathrm{obs}}}{v_{2\,\mathrm{obs}}} = \frac{m_2}{m_1}. \tag{10}$$

The observed, line of sight, velocity is given by (the 2nd equality requires some algebra and note $v_{i\,\mathrm{obs}} = (m/m_i)v_{\mathrm{obs}}$)

$$v_{\mathrm{obs}} = [v_r \cos(\phi - \theta) + v_\theta \sin(\phi - \theta)] \sin i = -\left[v_r \cos\phi + \left(\frac{r}{a} - 1\right) v_\theta \sin\phi\right] \frac{\sin i}{e}. \tag{11}$$

Solving the differential eq. $\dot{r} = (dr/d\theta)\dot{\theta} = (dr/d\theta)J/mr^2$ we find

$$\frac{2\pi t}{T} = -\sqrt{e^2 - (x-1)^2} + \arctan\left[\frac{x-1}{e^2 - (x-1)^2}\right], \quad x = r/a. \tag{12}$$

Thus, $r/a = f(t/T, e)$ and $Tv_r/a = f'(t/T, e)$. Thus, the functional dependence of v_{obs} on t determines $\{e, \phi\}$, but does not determine the multiplicative constant $\sin i$. The amplitude of the velocity determines $\tilde{a} = a \sin i$, so that $M = (4\pi^2\tilde{a}^3/GT^2)/\sin^3 i$. For circular orbits $2\pi\tilde{a} = Tv_{\mathrm{obs,max}} = T(v_{1\mathrm{obs,max}} + v_{2\mathrm{obs,max}})$ and $M \sin^3 i = v^3_{\mathrm{obs,max}} T/2\pi G$.

- For eclipsing binaries $\pi/2 - i \simeq R_*/d \ll 1$. Allows to determine M (and also R_* from the eclipse photometry).

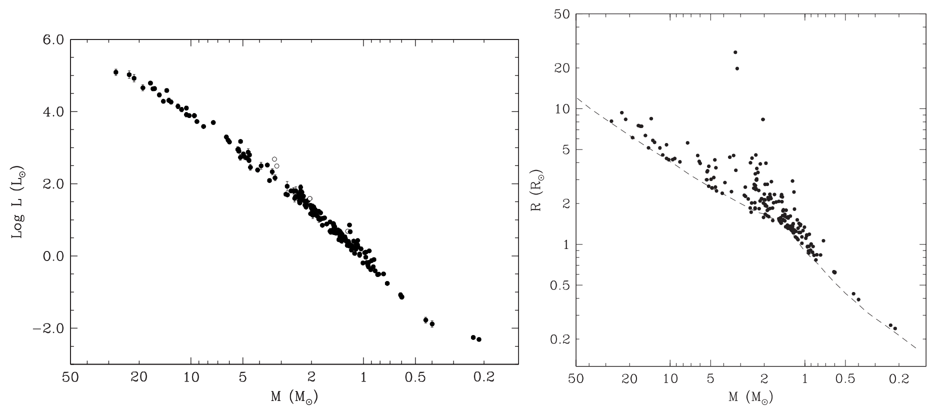


Figure 1: 94 detached non-interacting eclipsing binaries, from Torres, Andersen & Gimenez (2010 A&ARv 18, 67).

- If only v_1 determined, $M \sin^3 i/(1 + m_1/m_2)^3 = (m_2 \sin i)^3/M^2$ is determined (but not m_1/m_2) since $v = v_1 + v_2 = v_1(1 + v_2/v_1) = v_1(1 + m_1/m_2)$. For circular orbits $(m_2 \sin i)^3/M^2 = v_{1\mathrm{obs,max}}^3 T/2\pi G$.

3.2 Stellar structure: Main sequence

3.2.1 Structure eqs. [2hr]

Stellar structure eqs.: Derive hydrostatic and L (rad diffusion eq- simplified get $lc/4$), introduce eos and κ, mention composition-nuclear burning.

$$\frac{1}{\rho}\frac{dp}{dr} = \frac{GM}{r^2}, \tag{13}$$

$$\frac{dL}{dr} = 4\pi r^2 q, \quad L = 4\pi r^2 j, \quad \mathbf{j} = \frac{lc}{3}\nabla U_r. \tag{14}$$

Mention metalicity (Solar by mass: 75% H, 24% He, 1.6% Z, 1% CNO, 0.1% Fe) and it's evolution.

Ideal gas, $e = p/(\gamma - 1)$, $(\partial \ln p/\partial \ln \rho)_S = \gamma$, R/NR example, $\gamma = 4/3, 5/3$. Integrate hydrostat to get virial theorem

$$E_G = -3(\gamma - 1)E_i, \quad E_G + E_i = -3(\gamma - 4/3)E_i = \frac{\gamma - 4/3}{\gamma - 1}E_G. \tag{15}$$

Contraction leads to larger internal energy (heating). From hyd/virial we have

$$T \simeq \frac{GM\mu}{R} = 1\frac{(M/M_\odot)(2\mu/m_p)}{R/R_\odot}\,\text{keV} \tag{16}$$

when pressure is plasma dominated, $p = \rho T/\mu$.

Stability: adiabatic compression $p/R\rho \propto \rho^{\gamma-1}/R \propto R^{-3(\gamma-1)-1}$, $(p/R\rho)/(GM/R^2) \propto R^{-3(\gamma-1)+1}$, stable for $-3(\gamma-1)+1 < 0$ i.e. $\gamma > 4/3$. For $\gamma > 4/3$ contraction leads to larger binding energy. Ionization example: $\Delta p \sim \Delta n_I T - n\Delta T \sim \Delta n_I(T - I) < 0$,

$$\frac{n_I}{n_0} = \frac{g_I}{g_0}e^{-I/T} \sim \frac{(2mT/\hbar^2)^{3/2}}{n_e}e^{-I/T}. \tag{17}$$

Luminosity. Derive

$$L \sim 4\pi R^2(lc/3)U_r/R \sim E_r/(R^2/lc), \tag{18}$$

explain diffusion time. Simple l: at high enough T, opacity dominated by Thomson scattering, $\sigma_T = (8\pi/3)(e^2/m_ec^2)^2 = (2/3\pi)\alpha^2(h/m_ec)^2 = 0.66 \times 10^{-24}\text{cm}^2$. For fully ionized Thomson dominated plasma with X (mass) fraction of He, $n_{\rm He}/n_{\rm H} = X/4(1 - X)$, $n_e/\rho = (1 - X/2)/m_p$, $\kappa = n_e\sigma_T/\rho = (1 - X/2)(\sigma_T/m_p) = 0.4(1 - X/2)\text{cm}^2/\text{g}$: κ independent of ρ, T.

Radiation energy density is $U_r = (\pi^2/15)(1/\hbar c)^3 T^4$. For matter dominated pressure we have $L \sim (4\pi/3)^2(\rho l c)(M/\hbar c)^3(G\mu)^4$, and writing $l = 1/\kappa\rho$,

$$L \sim (c/\kappa)(M/\hbar c)^3(G\mu)^4 = 20\frac{(\mu/0.5m_p)^4}{\kappa/1\text{cm}^2/\text{g}}\left(\frac{M}{M_\odot}\right)^3 L_\odot. \quad (19)$$

Detailed gives $10^4 L_\odot$ at $10M_\odot$. Thus, for κ independent of ρ, T- nuclear energy production rate set by M. Since reaction rate depends strongly on T, T is close to threshold, implying $R \propto M$.

The ratio of radiation to plasma pressure is

$$\frac{p_{\text{rad}}}{p_{\text{plasma}}} \simeq \frac{T^4/5(\hbar c)^3}{nT} = \frac{(T/\hbar c)^3\mu}{5\rho} \simeq 0.05\frac{T_{\text{keV}}^3}{\rho/1\text{g cm}^{-3}}. \quad (20)$$

For the Sun, $\rho \sim (M/R^3) \sim 10\text{g/cm}^3$. For larger M, $R \propto M$ gives $\rho \propto M^{-2}$. When radiation pressure dominates, virial gives

$$\frac{T^4}{(\hbar c)^3} \simeq \frac{GM^2}{R^4}, \quad (21)$$

and $L \sim (GM^2/R)(lc/R^2)$,

$$L \sim \frac{GMc}{\kappa} = 10^4\frac{M/10M_\odot}{\kappa/1\text{cm}^2/\text{g}} L_\odot. \quad (22)$$

Eddington luminosity. The force on an electron $\sigma_T(L/4\pi R^2)/c$ balanced by $GM(\rho/n_e)/R^2 = GMm_p/(1-X/2)R^2$ gives

$$L_{\text{Edd.}} = 4\pi\frac{GMm_pc}{(1-X/2)\sigma_T} = 4\pi\frac{GMc}{\kappa} = 1.3\times10^{38}\frac{M/M_\odot}{1-X/2} = 3\times10^4\frac{M/M_\odot}{1-X/2} L_\odot. \quad (23)$$

Note- 1 e^- per 2 nucleons for all Z higher than H. Very massive stars have $L \sim L_{\text{Edd.}}/3$.

3.2.2 $L(M)$ scaling at low M: Kramers (ff/bf) opacity [2hr]

Lower temp, ff & bf opacity. Kirchoff $\alpha_\nu^{-1} j_\nu = B_\nu$, $4\pi B_\nu = ch\nu n_\nu = 8\pi(h\nu^3/c^2)(e^{h\nu/T}-1)^{-1}$,

$$\alpha_\nu^{-1}(4\pi j_\nu) = 8\pi(h\nu^3/c^2)(e^{h\nu/T}-1)^{-1}. \quad (24)$$

Comment on derivation using Einstein coefficients ($n_2A_{21} = n_1B_{12}n_\nu - n_2B_{21}n_\nu$, $A_{21}/B_{21} = 8\pi\nu^2/c^3$, $n_1B_{12}/n_2B_{21} = e^{h\nu/T}$, $4\pi j_\nu = n_2A_{21}h\nu$,

$c\alpha_\nu n_\nu = (n_1 B_{12} - n_2 B_{21})n_\nu)$. Bremsstrahlung: dipole $P = (2/3)|\ddot{\mathbf{d}}|^2/c^3 = (2/3)(e^2/c^3)|\mathbf{a}|^2 = (2/3)(e^2/c^3)(Ze^2/b^2 m_e)^2 = (2/3)(Z^2e^6/b^4m_e^2c^3)$ over $T = 2b/v$ for $h\nu < m_e v^2/2$, $E_r = PT = (4/3)(Z^2e^6/b^3m_e^2c^3v)$, $\nu = 1/T = v/2b$, $d\nu = vdb/2b^2$, $4\pi j_\nu d\nu = n_Z n_e v(2\pi b db)PT$, $4\pi j_\nu = n_Z(16\pi/3)(n_e/v)(Z^2e^6/m_e^2c^3)$. For thermal e, $n_e/v \simeq \sqrt{m_e/2T}e^{-m_e v^2/2}$ (in square brackets- the factor missing for an exact result),

$$4\pi j_{\nu,\mathrm{Brem.}} \simeq \frac{16\pi}{3}\left[g_{\mathrm{ff}}\sqrt{\frac{16\pi}{3}}\right] n_e n_Z \sqrt{\frac{m_e}{2T}}\frac{Z^2e^6}{m_e^2c^3}e^{-h\nu/T}, \tag{25}$$

$$\alpha_{\nu,\mathrm{ff}} \simeq \frac{2}{3}\left[g_{\mathrm{ff}}\sqrt{\frac{16\pi}{3}}\right] n_e n_Z \sqrt{\frac{m_e}{2T}}\frac{Z^2e^6}{m_e^2ch\nu^3}(1-e^{-h\nu/T}), \tag{26}$$

$$l^{-1}_{\nu,\mathrm{ff}} \simeq \frac{2}{3}\left[g_{\mathrm{ff}}\sqrt{\frac{16\pi}{3}}\right] n_e n_Z \sqrt{\frac{m_e}{2T}}\frac{Z^2e^6}{m_e^2ch\nu^3}, \tag{27}$$

$$\kappa_{\mathrm{ff}} \simeq \frac{1}{\rho l_{\nu,\mathrm{ff}}(h\nu = T)} \simeq \frac{[g_{\mathrm{ff}}\sqrt{16\pi/3}]X_Z\rho}{3\sqrt{2}Am_p^2}\frac{Z^2h^2e^6}{cm_e^{3/2}T^{7/2}} = 1\frac{X_Z Z^2}{A}\frac{\rho/1\mathrm{g\,cm^{-3}}}{T_{\mathrm{keV}}^{7/2}}\,\mathrm{cm^2/g}. \tag{28}$$

For bf, a factor correction.

For $\kappa \propto \rho/T^{7/2}$ we have

$$L \propto M^{5.5}R^{-0.5}. \tag{29}$$

Assuming const T from nuclear threshold, $R \propto M$ and $L \propto M^5$.

3.2.3 Characteristic times

$$t_{\mathrm{dyn}} = \frac{1}{\sqrt{G\rho}} = 1(\rho/1\mathrm{g\,cm^{-3}})^{-1/2}\mathrm{hr}, \quad t_{\mathrm{therm}} = \frac{R^2}{lc} = \frac{\kappa\rho R^2}{c} \sim 10^4\frac{\kappa\rho}{1\mathrm{cm^{-1}}}(R/R_\odot)^2\,\mathrm{yr}. \tag{30}$$

3.2.4 Convection

Relate adiabatic to t_{dyn} and t_{therm}. $(\partial p/\partial\rho)_s\delta\rho_{\mathrm{ad.}} = \delta p = (\partial p/\partial\rho)_s\delta\rho + (\partial p/\partial s)_\rho\delta s$, $(\partial p/\partial\rho)_s(\delta\rho_{\mathrm{ad.}} - \delta\rho) = (\partial p/\partial s)_\rho\delta s$. Since $(\partial p/\partial\rho)_s = c_s^2 > 0$ and $(\partial p/\partial s)_\rho > 0$, stability requires $ds/dr > 0$.

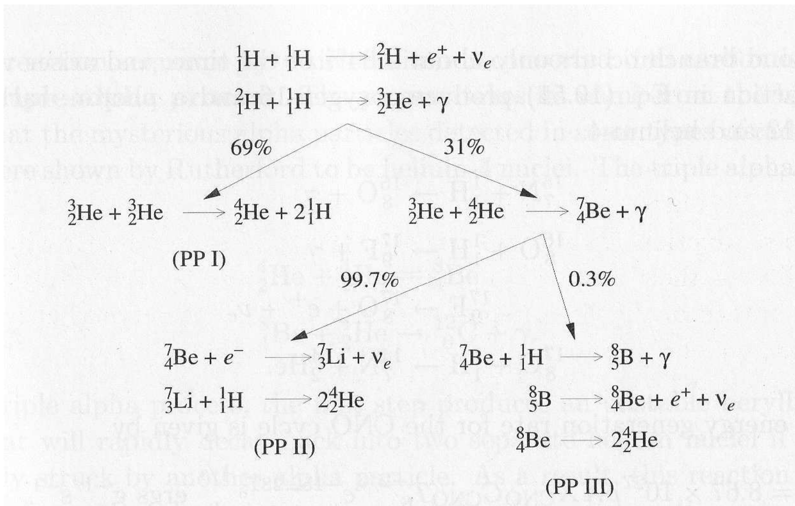


Figure 2: The pp chain (from Carroll & Ostlie)

3.3 Nuclear energy production 2×[2hr]

3.3.1 Nuclear reactions

Binding energy $\sim$ 1 MeV. Rough nuclei binding energy per nucleon plot: D$\sim$ 1 MeV, ^{3}He$\sim$ 2.5 MeV, ^{6}Li$\sim$ 5.5 MeV, ^{4}He$\sim$ 7 MeV, ^{12}C$\sim$ 7.5 MeV, ^{16}O$\sim$ 8 MeV, ^{56}Fe$\sim$ 8.5 MeV, U$\sim$ 7.5 MeV. Life-time estimate

$$t_{\text{nuc,H}} \sim \frac{7\,\text{MeV}}{1\,\text{GeV}}\frac{Mc^2}{L} = 10^{10}\frac{M/0.1M_\odot}{L/L_\odot}\,\text{yr}. \tag{31}$$

Conversion of 4 p to ^{4}He produces 26.73 MeV, in the main pp branch (fig. 5) the 2 neutrinos carry 0.52 MeV. CNO: ^{12}C+$p \to$ ^{13}N+γ, ^{13}N$\to$^{13}C+e^++ν_e, ^{13}C+p $\to$ ^{14}N+γ, ^{14}N+p $\to$ ^{15}O+γ, ^{15}O$\to$^{15}N+$e^+ + \nu_e$, ^{15}N+p $\to$ ^{12}C+^{4}He. At $\sim$ 1%, the last step is replaced with ^{15}N+p $\to$ ^{16}O+γ, ^{16}O+$p \to$ ^{17}F+γ, ^{17}F$\to$^{17}O+$e^+ + \nu_e$, ^{17}O+$p \to$ ^{14}N+^{4}He.

Potential energy as function of separation plot, $r_b = e^2/E = 10^{-13}E_{\text{MeV}}^{-1}$ cm, $\lambda_{dB} = \hbar/p = \hbar/\sqrt{2mE} = 10^{-11}E_{\text{keV}}^{-1/2}$ cm, $\pi\lambda_{dB}^2 = 10^{-21}E_{\text{keV}}^{-1}$cm^2.

$H\Psi = E\Psi$, $\hbar k = \sqrt{2m(E-V)}$ (m reduced mass and $\Psi(\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2)$). Suppression $kr \sim \sqrt{2mV(r)}r/\hbar$, since $V \propto 1/r$ take the value at largest r, $V(r) = E$, $r = Z_A Z_B e^2/E$ and approximate $kr \sim \sqrt{2m/E}Z_A Z_B e^2/\hbar$.

Probability e^{-2kr},

$$P_{\rm tun.} = e^{-\sqrt{E_G/E}}, \quad E_G = 8[\pi^2/4](\alpha Z_A Z_B)^2 mc^2 = 0.5(Z_A Z_B)^2 \frac{2mc^2}{1\,{\rm GeV}}{\rm MeV}, \tag{32}$$

$$\sigma(E) = \frac{S(E)}{E} e^{-\sqrt{E_G/E}}. \tag{33}$$

For thermal E distribution,

$$\begin{aligned} q &= Q n_A n_B \langle \sigma v \rangle = Q n_A n_B \frac{4}{\sqrt{\pi}} \left(\frac{m}{2T}\right)^{3/2} \int dv v^2 v \sigma(E) e^{-E/T} \\ &= Q n_A n_B \left(\frac{2m}{T}\right)^{3/2} \frac{1}{\sqrt{\pi} m^2} \int dE S(E) e^{-\sqrt{E_G/E} - E/T} \end{aligned} \tag{34}$$

max of $f = \exp(-E/T - \sqrt{E_G/E})$ at $E_m/T = (E_G/4T)^{1/3}$, with width $((f_m/f''_m)^{1/2})$ of $\Delta E/T = (E_G/4T)^{1/6}$. We therefore approximate

$$\begin{aligned} q &\approx Q n_A n_B \sqrt{\frac{8}{\pi m T}} S(E_m) \frac{\Delta E}{T} e^{-3E_m/T} \\ &= Q n_A n_B \sqrt{\frac{8}{\pi m T}} S(E_m) \left(\frac{E_G}{4T}\right)^{1/6} e^{-3(E_G/4T)^{1/3}}. \end{aligned} \tag{35}$$

For pp the slowest part is pp to D involving weak interaction, for which $S \sim 10^{-44}{\rm cm}^2{\rm keV}$. For H plasma and $Q = \epsilon m_p c^2$ we may write

$$\begin{aligned} L = \frac{qM}{\rho} &\approx 2X_H^2 \epsilon \frac{\rho M c^2}{m_p} \sqrt{\frac{16}{\pi m_p T}} S x^{1/6} e^{-3x^{1/3}} \\ &= 10^{41} \frac{\rho_1 M/M_\odot}{T_{\rm keV}^{1/2}} S_{-44} \frac{x^{1/6}}{2} e^{-3x^{1/3}}\ {\rm erg/s}, \end{aligned} \tag{36}$$

where $\rho = 10\rho_1 {\rm g/cm}^3$, $S = 10^{-44} S_{-44} {\rm cm}^2{\rm keV}$, $x = E_G/4T$. We finally obtain

$$\frac{E_G}{T} \approx 700 \left[1 + 0.06 \ln\left(\frac{\rho_1 M/M_\odot}{T_{\rm keV}^{1/2} L/L_\odot} S_{-44}\right)\right]^3, \tag{37}$$

which gives $T \simeq 1$ keV. Since L grows faster then linear with M, T is somewhat larger for larger M. For p+C, $E_G = 36$ MeV,

$$L \approx 10^{60} \frac{X_{{\rm C},-2} \rho_1 M/M_\odot}{A_1 T_{\rm keV}^{1/2}} S_{-22} \frac{x^{1/6}}{5} e^{-3x^{1/3}}\ {\rm erg/s}, \tag{38}$$

and $e^{-3x^{1/3}}$ is smaller at $T = 1$ keV by 20.5 orders of mag compared to pp. For power-law approx., $q \propto T^\alpha$, $\alpha = d\ln q/d\ln T = x^{1/3} - \frac{2}{3}$, which gives $\alpha = 4, 20$ for pp, CNO at 1 keV. The steeper dependence on T for the larger E_G of CNO implies that for somewhat higher T CNO takes over, which implies that for stars more massive then the Sun CNO takes over.

3.3.2 Solar neutrinos

The pp ν_e (< 0.4 MeV) flux is $\simeq 2L_\odot/26.2\text{MeV}/4\pi d^2 \simeq 7\times 10^{10}\text{cm}^{-2}\text{s}^{-1}$, ^{8}B decay ($< 15$ MeV) flux is $6\times 10^6\text{cm}^{-2}\text{s}^{-1}$, ^{7}Be e$^-$ capture (0.9 MeV) flux is $5\times 10^9\text{cm}^{-2}\text{s}^{-1}$.

- Absorption cross section, $\sigma \simeq G_F^2 p_e E_e$, for ^{8}B ν_e on $^{37}_{17}$Cl ($^{71}_{31}$Ga) is 10^{-42}cm^2 ($\times 2.4$), giving $6\times 10^{-36}\text{s}^{-1}$ per target atom = 6 SNU or 10^{-4}s^{-1} per kiloton. Davis: 615 ton of C_2Cl_4, 2×10^{30} ^{37}Cl atoms,

$$\nu_e(> 0.814\,\text{MeV}) +^{37}\text{Cl} \rightarrow e^- +^{37}\text{Ar}(35.0\text{d half life}). \tag{39}$$

  Gallex/SAGE:

$$\nu_e(> 0.233\,\text{MeV}) +^{71}\text{Ga} \rightarrow e^- +^{71}\text{Ge}(11.4\text{d half life}). \tag{40}$$

- Absorption cross section for ^{7}Be e$^-$ capture ν_e on ^{37}Cl (^{71}Ga) is $2.5\times 10^{-46}\text{cm}^2$ ($\times 30$), giving $1\times 10^{-36}\text{s}^{-1}$ per target atom =1 SNU or $2\times 10^{-5}\text{s}^{-1}$ per kiloton ($\times 15$).

- Absorption cross section for pp ν_e on ^{71}Ga is 10^{-45}cm^2, giving $7\times 10^{-35}\text{s}^{-1}$ per target atom = 70 SNU or $6\times 10^{-4}\text{s}^{-1}$ per kiloton.

- Cross section for e^- scattering, $\sigma \simeq G_F^2 m_e E$, at 10 MeV $\sim 10^{-43}\text{cm}^2$ (times 0.1 for ν_μ) [Kamoikande, Super-K, SNO], for D at 10 MeV $\simeq 10^{-42}\text{cm}^2$ [SNO].

$$\nu+e \rightarrow \nu'+e', \quad \nu_e(> 1.4\,\text{MeV})+d \rightarrow 2p+e^-, \quad \nu(> 2.2\,\text{MeV})+d \rightarrow \nu+p+n. \tag{41}$$

  SNO- 1kt D_2O, measured ^{8}B all flavor neutrino flux 0.9$\pm$0.1 predicted.

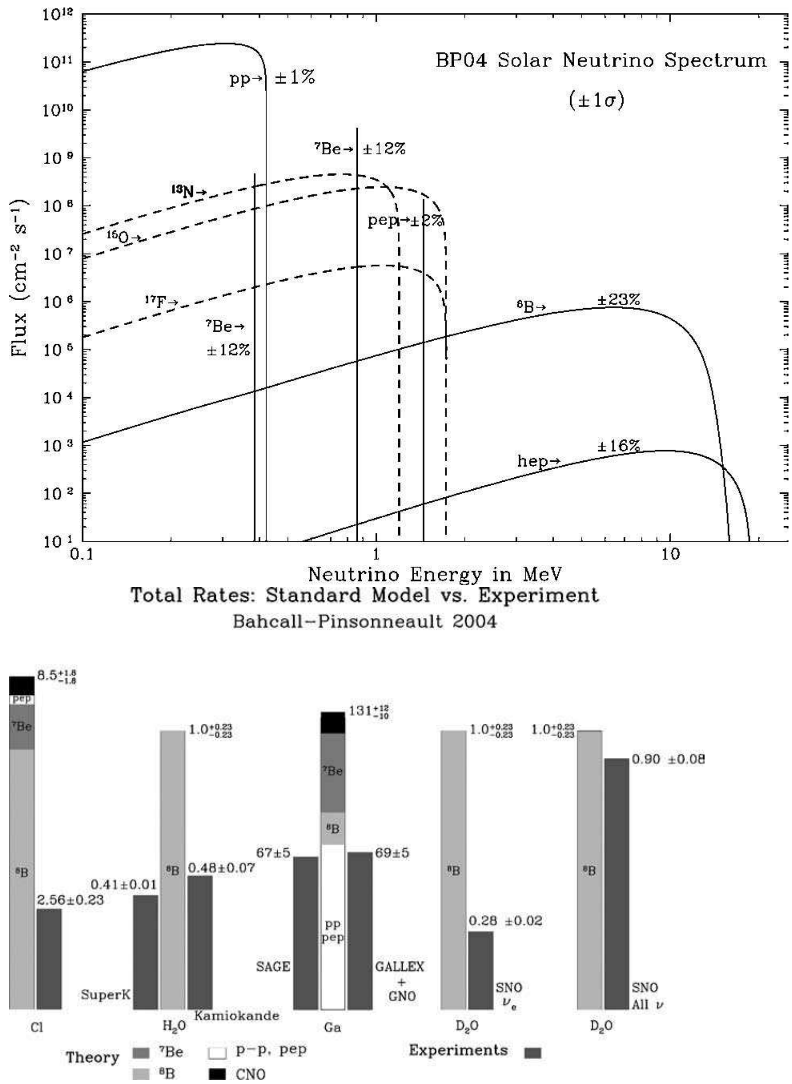


Figure 3: From Bahcall's Nobel Symp. Lecture; For continuum the flux is per MeV.

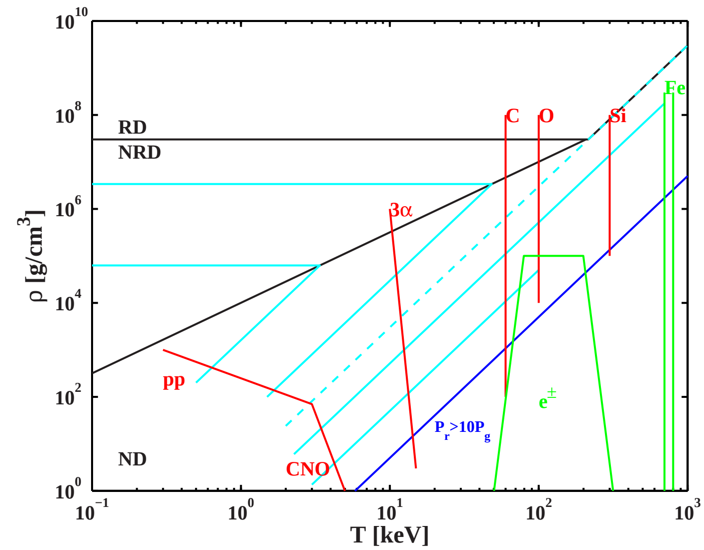


Figure 4: Schematic core evolution (solid cyan lines) for $\{0.1, 1, 10, 100\}M_\odot$ stars. Dashed- Chandrasekhar. Min mass for H ignition: 0.08 $M_\odot$. Stars with $8 > M/M_\odot$ develop cores with $M_{\rm core} < M_{\rm ch}$, end their lives as WDs. Stars with $8 \le M/M_\odot \le 70$ develop cores with $M_{\rm core} > M_{\rm ch}$ and do not enter the pair-instability region, move to Fe dissociation and core collapse (Type II, Ib/c SN). More massive stars enter the pair-instability region (pair instability SN). Type Ia- probably accretion induced collapse of WD (mass accretion from companion drives the mass above $M_{\rm ch}$).

3.4 Post main sequence evolution 2×[2hr]

3.4.1 Post main sequence evolution: Cores

Cold ($T = 0$) degenerate e^-: $\Delta p = h/L$, $2(4\pi/3)(p_f/h)^3 = n_e$,

$$p_f = (3\pi^2)^{1/3}\hbar n_e^{1/3}, \quad P = \frac{1}{3}n_e < pv >= \frac{1}{3}\int dp\frac{dn_e}{dp}pv, \tag{42}$$

$$< pv >= \frac{\int dpp^3 v}{\int dpp^2} = \begin{cases} \frac{3}{4}cp_f, & v = c \\ \frac{3}{5}p_f^2/m, & v = p/m \end{cases} \tag{43}$$

$$P_{\rm NRD} = \frac{(3\pi^2)^{2/3}}{5}\frac{\hbar^2}{m_e}n_e^{5/3}, \quad P_{\rm RD} = \frac{(3\pi^2)^{1/3}}{4}c\hbar n_e^{4/3}. \tag{44}$$

NRD=RD: $n_e = (1/3\pi^2)(5m_e c/4\hbar)^3$, NRD=NRND: $n_e = (1/3\pi^2)(5m_e T/\hbar^2)^{3/2}$, RD=RND: $n_e = (1/3\pi^2)(4T/c\hbar)^3$.

$$\begin{aligned}
\text{Radiation P}: &\quad \rho \ll 0.05 T_{\rm keV}^3 \text{g/cm}^3, \\
\text{RD}: &\quad \rho \gg \max\left[5\times 10^6, 1T_{\rm keV}^3\right] (A/2Z)\text{g/cm}^3, \\
\text{NRD}: &\quad 2\times 10^3 T_{\rm keV}^{3/2} \ll \rho/[(A/2Z)\text{g/cm}^3] \ll 5\times 10^6. \quad (45)
\end{aligned}$$

NRD/NRND lies on $\rho \propto T^{3/2}$ (adiabat), NRND cores (with plasma pressure domination) have $T \propto M/R \propto M^{2/3}\rho^{1/3}$ i.e. $\rho \propto T^3/M^2$ and may propagate to cross into NRD.

WD. Crossing into NRD, increase in internal energy gives increase in ρ, not T. Cooling leaves internal energy, and ρ fixed. $R \sim GM\rho/P_{NRD}$,

$$R_{\rm WD} = 4.3\frac{\hbar^2}{Gm_e m_p^{5/3}}\left(\frac{Z}{A}\right)^{5/3} M^{-1/3} = 2.3\times 10^9 \left(\frac{Z}{A}\right)^{5/3}\left(\frac{M}{M_\odot}\right)^{-1/3} \text{cm.} \quad (46)$$

For RD,$R \sim GM\rho/P_{RD}$, Chandrasekhar mass

$$M_{\rm Ch.} = 3.5\left(\frac{Z}{A}\right)^2\left(\frac{\hbar c}{Gm_p^2}\right)^{3/2} m_p = 1.4\left(\frac{2Z}{A}\right)^2 M_\odot. \quad (47)$$

He Burning (10 keV). No stable $A = 5, 8$. $^4\text{He} +^4\text{He} \rightarrow^8 \text{Be} + \gamma$, $\tau(^8\text{Be}) = 2.6\times 10^{-16}$s, ^{8}Be+^{4}He$\rightarrow$^{12}C+$\gamma$. Releases 0.61 MeV per nucleon, accompanied by $^{12}\text{C} +^4 \text{He} \rightarrow^{16} \text{O} + \gamma$. Hoyle predicted the existence of excited energy level of ^{12}C corresponding to ^{8}Be+^{4}He based on observed He:C:O ratio.

C & O Burning (60,100 keV). ^{12}C+^{12}C$\rightarrow$^{24}Mg+γ,^{23}Mg+n,^{23}Na+p,^{20}Ne+α,^{16}O+2α with 0.54 MeV per nucleon. ^{16}O+^{16}O$\rightarrow$^{32}S+γ,^{31}S+n,^{31}P+p,^{28}Si+α,^{24}Mg+2α with 0.5 MeV per nucleon. Released p, n absorbed fast by nuclei (lower E_G) to produce large variety of isotopes. From O, most abundant is Si.

Si burning (0.3 MeV), NSE. At $T > 0.1MeV$, approaching the nuclei binding energy, photons begin to cause photodisintegration. At 0.3 MeV light nuclei produced by Si disintegration lead to a network of nuclear reactions, near Nuclear Statistical Equilibrium with "leakage" to Iron group, $_{26}$Fe/$_{27}$Co/$_{28}$Ni (giving 0.2 MeV per nucleon), for which photodisintegration sets in at 0.7 MeV.

Trajectories in $\{\rho_c, T_c\}$***.***

3.4.2 Post main sequence evolution: Envelopes

More on the "story telling side".

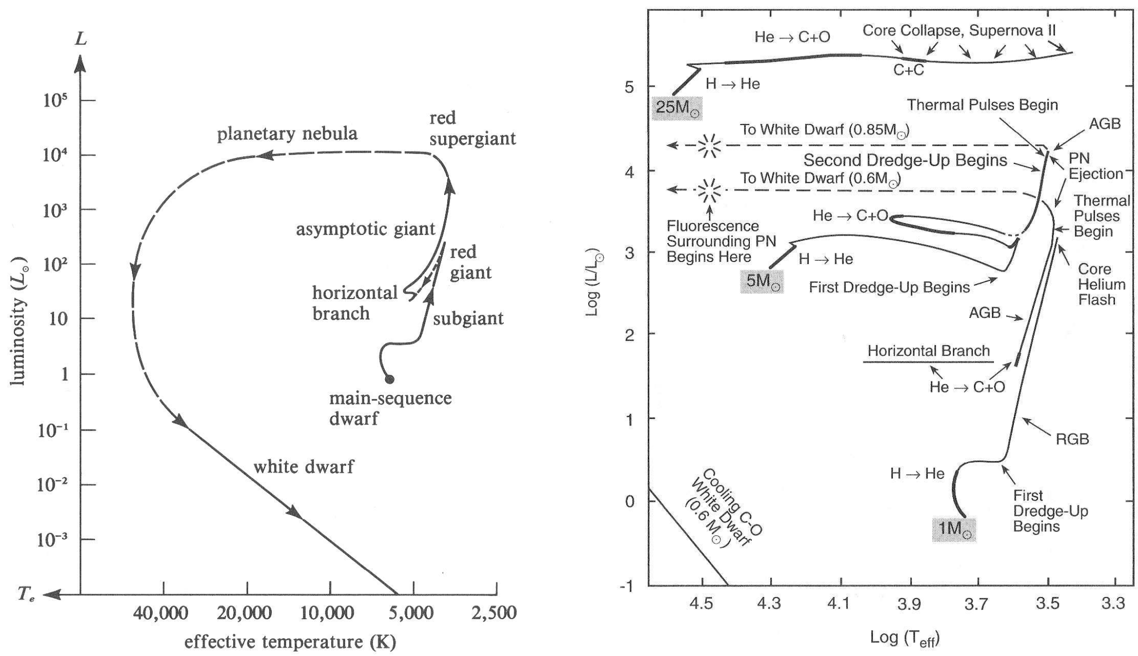


Figure 5: Low (left, from Shu) and general (right, from Iben 1985) mass evolution.

Low mass stars. H exhaustion in the core, core contraction and heating, ignition of H shell. Now- p, ρ of burning shell determined by core, not by balancing nuclear/L. Increased core gravity and T (initially checked by shell expansion, const L lower $T_{\rm ef.}$ but then) leads to increased nuclear production, L exceeds radiative $L(M)$, convection and expansion: "Ascending the giant branch" to red giant, $> 100L_\odot$, $R \sim 50R_\odot$. The strong dependence of κ on T for low T forces evolution at nearly fixed $T_{\rm ef.}$ (Hayashi): $L \propto R^2 T_{\rm ef.}^4$,$p = \rho T/\mu = \rho g l$, $l = T/g\mu$, strong dependence of l on T implies nearly fixed T for wide range of R (g). (Low mass $< 2.25M_\odot$ leads to He ignition with degenerate core, "He flash") Followed by He "main sequence". L on He MS fixed by M, note $L \propto \mu^5$ for Thomson (μ^7 from Kramers). Increased L and line opacity/convection lead to (ill understood) mass loss. $T_{\rm ef.}$ depends on mass loss (higher for higher mass loss), gives "horizontal branch".

He exhaustion leads to a similar story: inert CO core, He and H burning shells, "Ascending the asymptotic giant branch" to red super giant, $10^4 L_\odot$, $R \sim 300R_\odot$ with convective envelope. Extensive mass loss give Planetary Nebula, star ends its life as a WD.

High mass stars. $> 8M_\odot$ do not reach degeneracy, hit the Fe disintegration or pair production instability zones. Luminosity close to $L_{\rm Ed.}$ on MS, evolution with nearly fixed L and accompanied by strong mass loss at all stages. Wolf-Rayet stars: "exposed" He with little H (C/O with little else), extensive mass loss. Pre-core-collapse-SN: type II- Inert Fe core, Si/O/Ne/C/He/H burning shells $< 10^{-2}R_\odot$, H envelope extending to tens-thousand $R_\odot$ (BSG-RSG) depending on H loss; type Ib/c- exposed WR.

Seismology, Pulsation. Sound waves, modes, seismology. Helioseismology success- stellar structure. Pulsation: Cepheids, fundamental radial mode, opacity increasing with density possible in partial ionization region.

3.5 Compact objects: NS/BH

- Will be discussed in "spectacular explosions".