Cosmology: Introduction

CONTENTS

I. OVERVIEW: THE CONTENT OF THE OBSERVABLE UNIVERSE

Much of § I A & § I B is covered in the excellent into chapter of *Galactic Dynamics* of Binney & Tremaine. Much of § II can be found in the excellent book *Gravitation & Cosmology* of Weinberg.

A. Content: In the Galaxy

- What is a star? $M > M_J (= 2\times10^{30}\text{g})$, T at solar core ~ 1 keV, fusion $E \sim 10^{51}$ erg and $t \sim 10^{10}$ yr.

- The Sun: solar constant $f = 1.36\times10^6\text{erg/cm}^2\text{s}$, $d = 1\text{a.u.} = 1.50\times10^{13}$ cm ($e = 1.7\%$), $L_\odot = 3.90\times10^{33}\text{erg/s}$, $R_\odot = 6.96\times10^{10}$ cm, $M_\odot = dv^2/G = 1.99\times10^{33}$ g, $T_{\text{eff.}} = 5800°$K. $t_{\text{KH}} \approx 30$ Myr, $t_{\text{rad}} \approx 5$ Gyr, $t_{\text{H}}(\text{7MeV He binding per nuc.}, 0.1M_\odot) \approx 10^{10}$ yr.

- $\{U,B,V,R\} = \{0.37, 0.44, 0.55, 0.7\}\mu$, $\Delta\lambda/\lambda \approx 0.2$; $m = -2.5\log f + const.$, mag at 10 pc: $M = m - 5\log(d/10\text{pc})$, $m - M$ distance modulus; $M_{\odot B} = 5.48$, $M_{\odot V} = 4.83$; closest- Proxima Centauri, 1.3 pc, $m_V = 11.05$; brightest- Sirius, 2.7 pc, $m_V = -1.45$; eye: $m_V \le 6$.

- Basic distances: a.u., parallax- 1 pc= 1a.u./1" $= 3.09\times10^{18}$ cm, Hipparcos [1989-93] 0.002"- distances to $\sim 10^5$ stars, local density of stars $\approx 1\text{pc}^{-3}$. Gaia [2013-]- 24 μas for 15 mag (= Sun at 1 kpc), ~ 40 times better than Hipparcos– $\sim 1\%$ distance accuracy at 1 kpc, 1B stars.

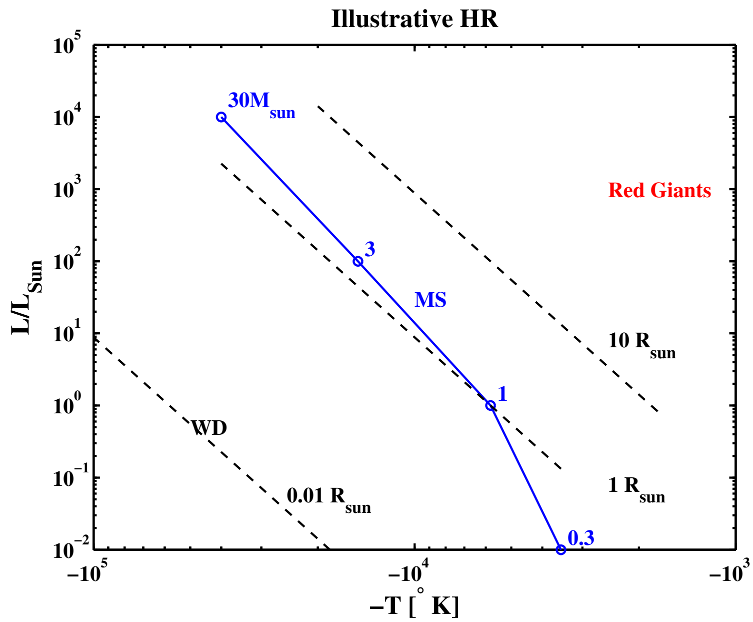
- **Slide: Hipparcos HR of local stars (100k at 1 mas).**

- The HR diagram interpretation (R, T), $L \propto M^4$, $\tau_\odot \sim 10^{10}$ yr, $t_\odot \simeq 0.5\times10^{10}$ yr.

- **Slide: Globular clusters' HR**, age: $t_* = 14\pm2$ Gyr.

- Distances: Main sequence fitting- kpc, Cepheids $L = 10^3(T/1\,\text{d})L_\odot$ for $T = 1-100$ d (two sequences, Z dependent)-10 Mpc. Hipparcos 26 Galactic Cepheids calibration of zero point $->$ distance modulus to LMC, M31 accurate to 0.1–0.2 mag (d to $\sim 5\%$).

- **Slide: Milky way**: l, b; disk $L_V \simeq 10^{10}L_{\odot V}$, $R \simeq 10$ kpc, $h \simeq 1$ kpc; spheroid $L_V \simeq 2\times10^9 L_{\odot V}$, $R \simeq 2$ kpc; $M_G \simeq 10^{11}M_\odot$; $h_{\text{gas}} \sim 0.1$ kpc, $M_{\text{gas}} \simeq 10^{10}M_\odot$, 75% H, 24% He, "$Z$".



- The Sun at $r_0 = 8.1 \pm 0.1$ kpc (Sgr-S2), $I(r) \propto \exp(-r/3.5 \pm 0.5\text{kpc})$, $v_c = 220 \pm 15$km/s ($2\pi r_0/v_c \sim 10^8$ yr), Sun moves at 16.5km/s in $l = 53°, b = 25°$ with respect to local standard of rest (local stars' motion) (massive star radio masers- radio interferometry parallaxes).

- Stellar clusters: 10^5 open in disk, 10^{2-3} stars, $\sim 10^9$ yr "Pop I"; 10^2 globular in halo, 10^{4-6} stars, $> 10^{10}$ yr "Pop II".

B. Content: Out of the Galaxy

- (Spirals &) Elliptical galaxies: Hubble-Reynolds $I \propto r_H^2/(r+r_H)^2$, $\sigma_v \simeq 220(L/L_*)^{1/4}$km/s.

- **Slide: Ia's Hubble's Law**, $1+z = \sqrt{(1+\beta)/(1-\beta)} = 1+\beta+O(\beta^2)$, equivalent observers, $H_0 = 70 \pm 3\,\text{km/s}\,\text{Mpc}$.

- Local galaxy density $n_* \simeq 10^{-2}\text{Mpc}^{-3}$, Schechter's law $dn/dL = \Phi(L) = (n_*/L_*)(L/L_*)^\alpha \exp(-L/L_*)$ with $L_{*V} = 10^{10} L_{\odot V}$, $\alpha = -1.25$.

- LSS. The cosmological principle. **slide: 2dF**. $\delta n/n \sim 1$ at ~ 10 Mpc.

- Radiaton: **slide: CMB**. $T = 2.7$ K, $\delta T/T \sim 10^{-5}$.

- DM: Zwizky Coma (10^3km/s, $R \sim 1$ Mpc) $M/L \sim 100(\propto H_0)$ due to H_0 error; **slide: Rotation curves**.

II. NEWTONIAN COSMOLOGY

Consider first the evolution of a cold, pressureless, plasma.

Birkhoff & Newtonian cosmology.

$$\ddot{r} = -\frac{GM}{r^2} = -\frac{4\pi}{3}G\rho r\,, r(r_0,t) = a(t)r_0 \Rightarrow\ \ddot{a} = -\frac{4\pi}{3}G\rho a. \tag{1}$$

$\dot{a}_0 = H_0$.

$$\rho = \rho_0 a^{-3}\ \Rightarrow\ \dot{a}^2 = \frac{8\pi}{3}G\rho_0 a^{-1} + \left(H_0^2 - \frac{8\pi}{3}G\rho_0\right). \tag{2}$$

Critical density $\rho_c = 3H_0^2/8\pi G = 10^{-29}h_{75}^2\mathrm{g/cm}^3$, $a_0 = 1$, with $H_0 = 75\,h_{75}\mathrm{km/s/Mpc}$,

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\frac{\rho_0}{\rho_c}a^{-3} + \left(1 - \frac{\rho_0}{\rho_c}\right)a^{-2}\right]. \tag{3}$$

Age, fate- $tH = 1$ for $\rho_0 \ll \rho_c$, $tH = 2/3$ and $a \propto t^{2/3}$ for $\rho_0 = \rho_c$. $\rho_{b0}/\rho_c \sim 10^{-2}$. Recall DM, so that ρ_0 may be $\gg \rho_{b0}$.

Modification: λ. λ as additional length scale in GR- we'll be discussed later. Static:

$$\ddot{a} = -\frac{4\pi}{3}G\rho a + \frac{1}{3}\lambda c^2 a, \tag{4}$$

with $\lambda = 4\pi G\rho_0/c^2 = (2/3)(\rho_0/\rho_c)(H_0/c)^2$. Modified Friedmann

$$\dot{a}^2 = \frac{8\pi}{3}G\rho_0 a^{-1} + \frac{1}{3}\lambda c^2 a^2 + \left(H_0^2 - \frac{8\pi}{3}G\rho_0 - \frac{1}{3}\lambda c^2\right), \tag{5}$$

or

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega a^{-3} + (1-\Omega-\Lambda)\,a^{-2} + \Lambda\right], \tag{6}$$

with

$$\Omega = \frac{\rho_0}{\rho_c}, \quad \Lambda = \frac{\rho_\Lambda}{\rho_c}, \quad \rho_c = \frac{3H_0^2}{8\pi G} = 1.05\times 10^{-29}h_{75}^2\mathrm{g/cm}^3, \quad \rho_\Lambda = \frac{\lambda c^2}{8\pi G}. \tag{7}$$

For $a \ll 1$ we have $\dot{a} = (8\pi G\rho_0/3)^{1/2}a^{-1/2}$, giving $a \propto t^{2/3}$ and $tH = 2/3$ as for an $\{\Omega = 1, \Lambda = 0\}$ universe. For $a \gg 1$ we have $a \propto \exp\left(H_0\Lambda^{1/2}t\right)$. Revised discussion of age, fate.

A. Radiation evolution

Recall ***CMB:*** Planck spec., $T = 2.73°\mathrm{K}$, Dipole, $\delta T/T \sim 10^{-5}$. Radiation density:

$$n_\gamma = \frac{2\zeta(3)}{\pi^2}\left(\frac{T}{\hbar c}\right)^3 = 0.244\left(\frac{T}{\hbar c}\right)^3 = 420\mathrm{cm}^{-3}, \quad u_\gamma = \frac{\pi^2}{15}\left(\frac{T}{\hbar c}\right)^3 T = 0.26\mathrm{eV/cm}^3. \tag{8}$$

Denoting the baryon density by n_b,

$$\eta \equiv \frac{n_\gamma}{n_b} = 0.7 \times 10^8 \Omega_b^{-1} h_{75}^{-2}. \tag{9}$$

Assuming photons decoupled from matter. Photon redshift:

$$\begin{aligned} \lambda(t+dt) = & \ (1+\beta)\lambda(t) = (1 + Hcdt/c)\lambda(t) \\ & \Rightarrow \tfrac{\dot\lambda}{\lambda} = H = \tfrac{\dot a}{a} \Rightarrow \lambda \propto a, \end{aligned} \tag{10}$$

and

$$1 + z \equiv \frac{\lambda_{\rm obs.}}{\lambda_{\rm emt.}} = \frac{1}{a}. \tag{11}$$

Conservation of Planck spec., $T \propto a^{-1}$. Comment on adiabatic expansion ($\hat\gamma = 4/3$).

$u_\gamma \propto a^{-4}$, $\rho \propto a^{-3}$, Matter-Radiation equality:

$$z_{\rm eq.} = \frac{\rho_0 c^2}{u_\gamma} = \frac{\Omega \rho_c c^2}{u_\gamma} = 2.3 \times 10^3 (\Omega h_{75}^2/0.1). \tag{12}$$

For $z > z_{\rm eq.}$ energy density dominated by radiation, GR must be used to derive dynamics. $z > z_{\rm eq.}$: Radiation domination, $z < z_{\rm eq.}$: Matter domination.

Assume that at high T, for which the plasma is fully ionized, radiation and matter are strongly coupled and in thermal equilibrium. For $n_\gamma \gg n_b$, the internal energy and the pressure are dominated by radiation (assuming $T \ll m_e c^2$). The eos is then $e/n_b = m_p c^2 + 3p/n_b$, and $Td(S/N) = d(e/n_b) + pd(1/n_b)$ (assuming conserved number of baryons) gives $p \propto n_b^{4/3}$ for adiabatic processes, implying $T \propto n_b^{1/3}$ and $n_\gamma \propto n_b$. Thus, for thermal equilibrium we obtain $\eta = Const.$, i.e. $\eta \gg 1$ and given by its value today, and $T \propto 1/a$ (since $a^3 n_b = Const.$) as for free expansion.

The ionized fraction is approximately given by

$$\begin{aligned} \frac{n_I}{n_0} &= \frac{g_I}{g_0} e^{-I/T} \cong \frac{(2mT/\hbar^2)^{3/2}}{n_e} e^{-I/T} \\ &= \frac{1}{0.244} \eta \left(\frac{2m_e c^2}{T}\right)^{3/2} e^{-I/T} \approx 10^{19} T_{\rm eV}^{-3/2} e^{-I/T}. \end{aligned} \tag{13}$$

The plasma becomes neutral ("recombination" occurs) therefore at $T_{\rm rec.} \approx 13.6{\rm eV}/\ln(10^{19}) \approx 0.3$ eV, $z_{\rm rec.} = T_{\rm rec.}/T_0 \approx 10^3$. Exact calculation gives

$$z_{\rm dec.} = z_{\rm rec.} = 1.1 \times 10^3. \tag{14}$$

Note that

$$\frac{z_{\rm eq.}}{z_{\rm dec.}} = 2.1 (\Omega h_{75}^2/0.1). \tag{15}$$

$z_{\rm rec.}$ is also the redshift at which radiation decouples from matter. Comparing the photon scattering rate, $n_e\sigma c$, to the expansion rate, $\dot{a}/a = H = H_0\Omega^{1/2}a^{-3/2}$ (for $a_{\rm eq.} \ll a \ll 1$), we have

$$\frac{n_e\sigma c}{H} = \frac{n_{e0}\sigma c}{\Omega^{1/2}H_0a^{3/2}} \approx 2\times 10^3 \frac{\Omega_b h_{75}}{\Omega^{1/2}}\frac{\sigma}{\sigma_T}\left(\frac{a}{10^{-3}}\right)^{-3/2}. \tag{16}$$

For $z > z_{\rm rec.}$ the electrons are free, $\sigma = \sigma_T$ radiation and matter are coupled. For $z < z_{\rm rec.}$ the atoms are neutral, $\sigma \ll \sigma_T$, and the radiation is "decoupled". While $T_{\rm rad.} \propto 1/a$ also for $z > z_{\rm rec.}$, $T_{\rm mat.} \propto 1/a^2$.

III. RELATIVISTIC EVOLUTION, $z > z_{\rm eq.}$

At early time, $a_{\rm eq.} \ll a \ll 1$, the a^{-3} term dominates and eq. (6) may be approximated as

$$H^2 = \frac{8\pi G}{3c^2}e, \tag{17}$$

where e is the energy density, dominated by rest mass. We will show later, in the brief discussion of GR principles, that this equation is exact also for the relativistic phase. When radiation dominates we therefore have

$$H = \left[\frac{8\pi^3 G}{45c^2}\left(\frac{T}{\hbar c}\right)^3 T\right]^{1/2} = 0.9\times 10^{-5}T_{\rm eV}^2{\rm yr}^{-1}. \tag{18}$$

Since $T \propto 1/a$ we have $\dot{a} \propto 1/a$, i.e. $a \propto t^{1/2}$ and $tH = 1/2$, i.e.

$$t = t_H/2 = 1/2H = 0.5\times 10^5 T_{\rm eV}^{-2}{\rm yr} = 2T_{\rm MeV}^{-2}{\rm s}. \tag{19}$$

Mention entropy "injection" to photons at particle annihilation ($e^{\pm}$).

If eq. (17) is valid at all temperatures, $\dot{a}/a \propto a^{-2}$ and $\dot{a} \propto t^{1/2}$. The age is proportional therefore to t_H, $t = \int^a da/\dot{a} = t_H/2$, and the the size of causally connected regions is proportional to the Hubble distance, $ct = ct_H/2 = d_H/2$. The current size of regions which were causally connected at the time the temperature was T is

$$a^{-1}d_H(T)/2 = 70T_{\rm eV}^{-1}{\rm Mpc}, \tag{20}$$

corresponding to angular size

$$\theta \approx \frac{a^{-1}d_H(T)/2}{c/H_0} \approx 1^\circ h_{75}T_{\rm eV}^{-1}. \tag{21}$$

Discuss causality problem, new physics at $T > 1$ TeV?