The Hubble tension

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1. Constraints from CMB

We have seen that from measurements of $C_{TT,l}$ we are able to accurately determine $\Omega_b h^2$, $\Omega_m h^2$, and $\theta \equiv 1/l_H = d_H/d_A$, where the acoustic horizon distance and the angular diameter distance are given at the time of last scattering. The Planck 2018 results are $\Omega_b h^2 = 0.02212 \pm 0.00022$, $\Omega_m h^2 = 0.1434 \pm 0.0020$ and $100\theta = 1.04077 \pm 0.00047$. The determination of $\Omega_b h^2$ and $\Omega_m h^2$ sets the redshift of last scattering, z_L, and d_H, so we can accurately determine d_A. However, d_A is also a function of Ω_Λ and h, so it is impossible in this way to separately determine both h and Ω_Λ.

We can proceed in a few ways:

- Assume a flat Universe, $\Omega_m + \Omega_\Lambda = 1$. In this case, one can determine d_A to $\approx 0.5\%$ accuracy, which translates to $\approx 4\%$ determination of Ω_m (0.321 ± 0.013, see Figures 1 and 2), which in turn translates to $\approx 1.5\%$ determination of h (0.6688 ± 0.0092).
- Allowing Ω_k to vary, but then only weak constraints can be placed on Ω_m ($\approx 0.45 \pm 0.15$) and h ($\approx 0.55 \pm 0.1$).
- Adding the results of BAO constrains $\Omega_k = 0.0007 \pm 0.0019$. Together with the full polarization maps and lensing of high l, one can constrain $h = 0.6766 \pm 0.0042$ ($\approx 0.6\%$ error).

2. Constrains from local measurements

There are a few independent measurements of h in the local Universe ($z \lesssim 1$):

- Using gravitational wave signals from known galaxies. So far, only one NS-NS event was located to its host galaxy, so there is a large uncertainty $h = 0.7 \pm 0.1$.
- Using lensed quasars, which requires a model for the galaxy lens mass profile. So far, this method provides too large uncertainties. For example, the TDCOSMO sample yields $h = 0.74 \pm 0.06$ and the SLACS sample yields $h = 0.67 \pm 0.04$.
- Using Type Ia supernovae distances to $z \approx 0.1$, calibrated by the tip of the red giant branch (TRGB). This method provides small uncertainties, although the role of systematics in the determination of the TRGB from different RGB distributions is not clear. The reported value is $h = 0.698 \pm 0.006(stat) \pm 0.016(sys)$, consistent with the Planck value.

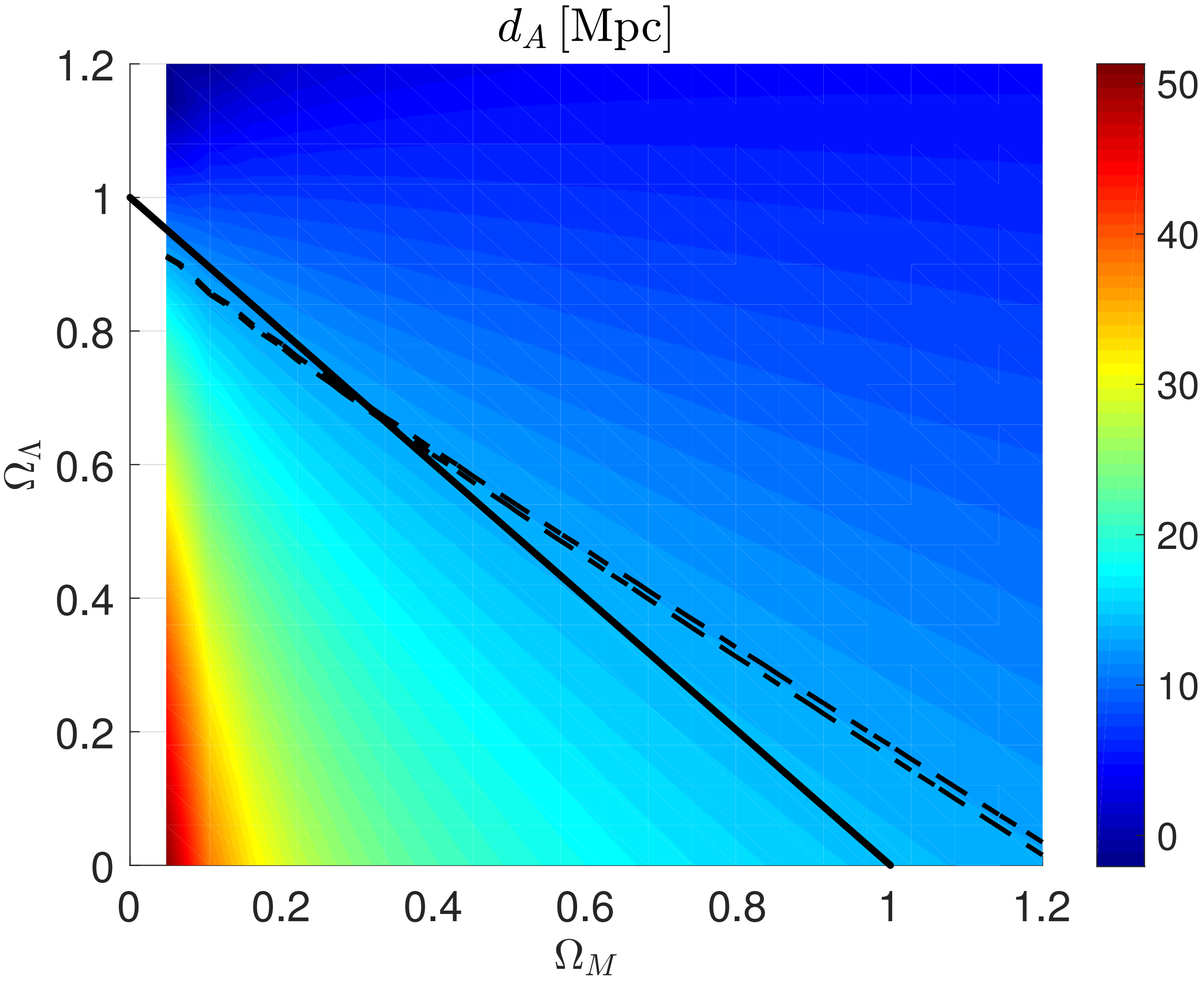

Fig. 1.— d_A as a function of Ω_m and Ω_Λ, for $\Omega_m h^2 = 0.143$ and $z_L = 1080$. The solid line is $\Omega_m + \Omega_\Lambda = 1$. The dashed lines are 0.5% error around $d_A = 12.94$ Mpc.

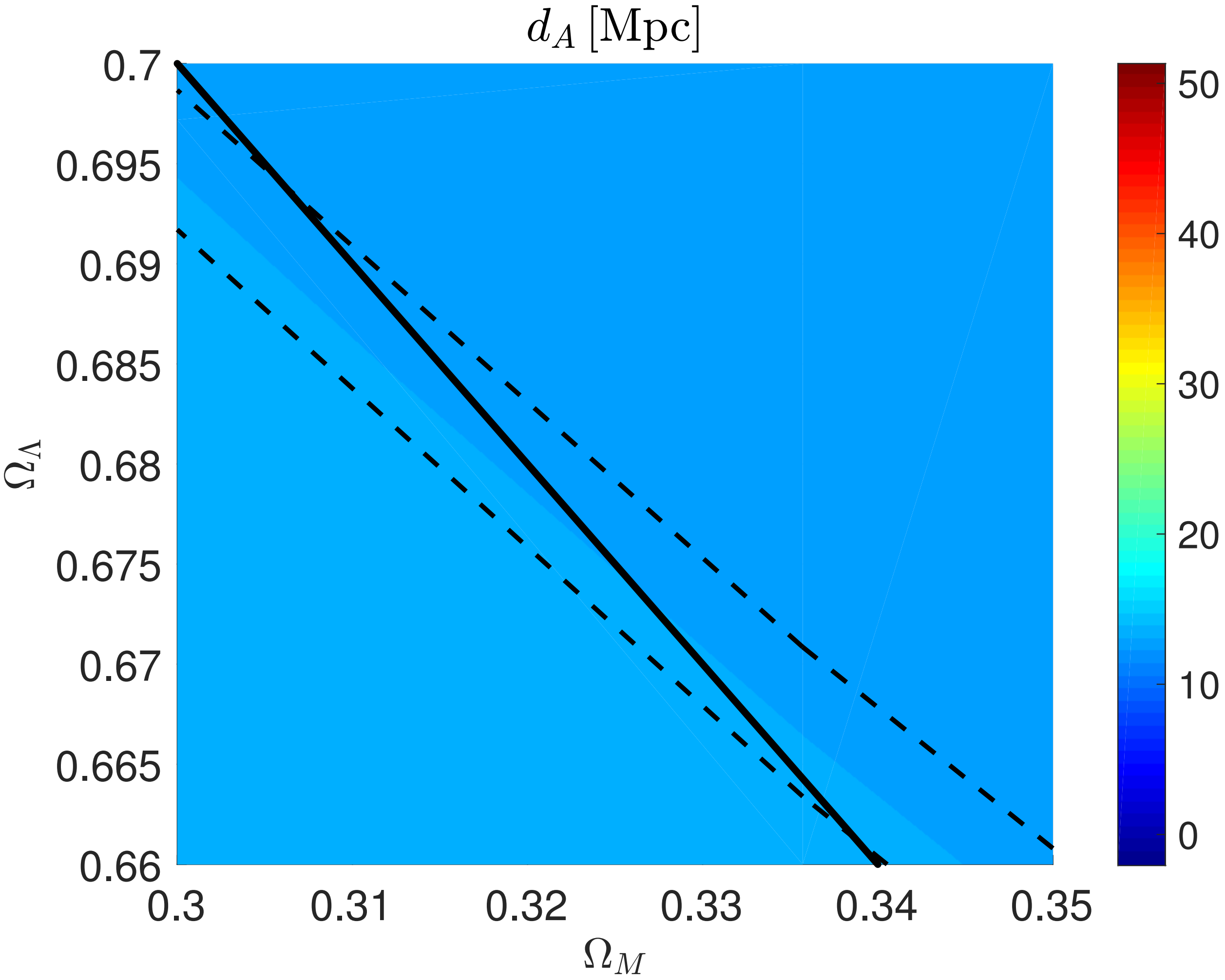

Fig. 2.— A zoomed version of Figure 1.

- Using Type Ia supernovae distances to $z \approx 0.1$, calibrated by Cepheids (SH0ES team). This method provides small uncertainties, 0.732±0.013, in a definite tension with the Planck value. Some details are given below.

3. The SH0ES measurement

A standard candle with absolute magnitude M at a redshift z will have an apparent magnitude

$$m = M + 25 + 5\log_{10}(d_L(z)) = -5a + 5\log_{10}(c\hat{d}_L(z)), \tag{1}$$

with the luminosity distance, $d_L(z)$, measured in Mpc, and a is the intercept of the magnitude-redshift relation, $5a = -(M + 25 - 5\log_{10} H_0)$ and $\hat{d}_L(z) = H_0 d_L(z)/c$. In practice, the second order expansion

$$\hat{d}_L(z) = z\left[1 + (1-q_0)\frac{z}{2} - \frac{1}{6}(1 - q_0 - 3q_0^2 + j_0)z^2 + O(z^3)\right] \tag{2}$$

is used with $q_0 = -0.55$ and $j_0 = 1$ to determine $a = 0.71273 \pm 0.00176$. The absolute magnitude of Type Ia SNe is determined with Cepheids, where the observed magnitude of the j-th Cepheid in the i-th host is expressed as

$$m_{i,j} = \mu_i + M_{i,j} + b\log_{10} P_{i,j} + Z\Delta\log_{10}(\mathrm{O/H})_{\mathrm{i,j}}, \tag{3}$$

with $\mu = m - M$, $P_{i,j}$ is the period of the Cepheid in days, $\Delta\log_{10}(\mathrm{O/H})_{\mathrm{i,j}}$ is the metallicity assigned to the Cepheid relative to Solar metallicity, and b and Z are fitted parameters. The magnitudes here are the H-band Weisenheit magnitudes, defined by $H - R(V - I)$, where the color term prefactor is chosen as $R = A_H/(A_V - A_I)$ to make the Weisenheit magnitude extinction-free. Combining the Cepheid data with the geometrical distance estimates of the maser galaxy NGC 4258, detached eclipsing binaries in the LMC and GAIA parallax measurements of MW Cepheids, they find $M = -19.2141 \pm 0.037$, which sets the derived value of h.

4. Possible solutions for the tension

So far, a clear solution to the tension was not proposed. A few possibilities are:

- Unidentified systematic in the SH0ES measurements. One concrete suggestion is variation in the extinction law used to derive the Weisenheit magnitudes.
- Radical departure from conventional cosmology, including departures from GR.
- Changes to the physics of the early Universe (e.g., additional relativistic species or neutrino interactions).

- New physics at matter-radiation equality or recombination, that alters the value of the sound horizon.
- Changes to the expansion history at late times. This is constrained to $z \lesssim 0.05$, since the BAO detemination of $H(z)$ at $0.5 \lesssim z \lesssim 2.5$ and the Type Ia supernovae magnitude-redshift relation at $0.023 < z < 0.15$ are fully consistent with base ΛCDM.