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Mathematical self-schema: a framework for analyzing adults' retrospection on high school mathematics

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Abstract

This paper reports on a qualitative study, which explored the nature of affective recollections elicited by adults in regard to their experience as high school mathematics students. The research subjects were 24 highly educated men and women, between the ages of 30 and 45. During individual interviews, subjects were requested to reflect upon themselves as learners of mathematics in the past and also as consumers of mathematics in the present. The interview transcripts were analyzed within a framework designed during the research, of which the central part was termed 'mathematical self-schema.' The rationale and definition of this concept are explained, and its components specified. Results are presented as a review of each component across all subjects, categorized by the diverse reactions obtained. The summary of categories in regard to all components was used to group subjects into four main mathematical self-schemas. Possible connections between the mathematical self-schema and other characteristics of subjects are discussed, as well as prospects for future use of this framework.

Keywords: Adults; Affective characteristics; Self-schema; Qualitative methods

1. Introduction

As a mathematics educator, I never cease to wonder how loaded the word "mathematics" is among adults. It seems, from my own experience and that of colleagues, that when people meet a mathematics educator in a social circumstance, they tend to develop a type of "shrink session" conversation, in which the word "math" often sounds as if it was a psychiatric phenomenon. Frequently you are bound to hear how much they have suffered from math, how it still haunts them, and how they still can't figure out why they

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had to go through this obstacle course. Conversely (but less often), others will share with you how much they enjoyed learning math and how important they think it is for developing young minds. Either way (and of course there are many more variations of these conversations), the cumulative impression is that mathematics studies are an experience which leave a significant affective imprint on most learners, perhaps more than other school subjects. This impression was the initial motivation for the study reported herein. On the one hand, I was curious to find out what kinds of different "imprints" can be found. On the other hand, it has long been my intention to explore another important issue in regard to adults' recollections of their high school mathematics: To what extent can adults recall basic mathematical concepts and procedures learned many years ago in high school? I therefore decided to combine the affective and the cognitive goals, and conduct a research in which adults' recollections about their high school mathematics studies will be scrutinized from both perspectives. This article reports on the affective investigation carried out within the research. In the following, after a short description of the study, I will present the framework developed for the affective analysis of the subjects' responses, called "the mathematical self-schema." The definition and implementation of this term constitute the lion's share of this report. I will also refer to connections between the mathematical self-schema and other characteristics of the research subjects. Finally, prospects for future use of this framework will be examined.

2. The study: an overview of design and methodology

As mentioned above, the research conducted had both affective and cognitive objectives. In an earlier paper (Karsenty, 2002), I reported results concerning the cognitive goal of the study (i.e., findings about the diversity of ways in which adults recall mathematical material learned in high school). Within that report I presented a detailed description of the study's characteristics and methods, including target population and case selection. In the present report, concentrating on affective analysis of the data, I will repeat only details that are essential for understanding the research design in general, and add details that specifically concern the affective goal of the study.

The research was a qualitative one, of the type known as "collective case study" (Stake, 1994, 1995). It was designed to focus on the higher-educated sector of the society in Israel. Subjects were 24 Israeli high school graduates (12 men and 12 women), aged 30–45, half of whom took mathematics in the low-level track and half in the intermediate or high level tracks.[1] All subjects had post-secondary education, mostly from colleges or universities, and were engaged in professional careers in areas such as law, medicine, psychology, art, business, high school teaching, and other. The main research method was an extensive, semi-structured, individual interview. The interview included two parts, of which the first was dedicated to affective aspects. Several questions, prepared beforehand, were used as a basis for this part of the interview. The aim was to trigger subjects to express, as comprehensively and openly as possible, their feelings, dispositions, and attitudes in regard to the experience of learning and using mathematics. To

[1] In the Israeli educational system, high school mathematics can be studied in three tracks: high level, intermediate level and low level. Substantial differences exist between the lower track and the other two, in terms of curriculum and number of hours studied. The difference between the intermediate and the high tracks is smaller. Students in these tracks study a similar curriculum in regard to most topics, with more complicated exercises given in the high level track. The annual report of the Israeli Central Bureau of Statistics usually refers to the intermediate level and the high level together, in distinction from the low level.

put it in a slightly different way, I was interested in how subjects perceive themselves as learners of mathematics in the past and as consumers of mathematics in the present. Subjects were requested, for example, to describe a typical high school mathematics lesson as they remembered it, and recollect their feelings at the time; to assess the impact of studying mathematics on their lives in comparison to other high school topics; to estimate their ability to recall or re-learn high school mathematics if needed again; to describe the nature of their involvement in the mathematical studies of their children; to suggest a future state policy in regard to mathematics education; to elicit free associations to the words “professional mathematician,” and more. In spite of these prepared questions, the course of conversation was, to a certain degree, determined also by the interviewee’s responses. This means that the order of questions and the time devoted to each question were flexible, and that further spontaneous questions were posed if an interesting direction emerged or if clarifications were needed. In all cases, the conversations seemed to be flowing in a natural way, as subjects tended to willingly speak about their mathematical experience, regardless of its nature (this phenomenon was coherent with my previous informal experience, as described in the opening of this paper).

In the second part of the interview, aimed at investigating cognitive aspects of recall, subjects were asked to solve mathematical tasks involving basic concepts and procedures (see Karsenty, 2002, for some results in regard to this objective of the research). The interviews were recorded and transcribed. Transcripts of the first part of the interviews were then analyzed within a framework developed during the research, which I termed ‘The Personal Mathematical Profile.’ The method by which this analysis was carried out is known as *categorical-content analysis* (Lieblich, Tuval-Mashiach, & Zilber, 1998), a procedure that will be further specified in Section 3 below.

Each profile consisted of two sections. The section called ‘general opinions towards mathematics,’ of which I will not report here, relates to attitudes and ideas in regard to mathematics education and the place of mathematics in public priority (see Karsenty & Vinner, 2000). The section named ‘the mathematical self-schema’ is the central issue of the present report, and will now be explained and illustrated in detail.

3. Mathematical self-schema: rationale, definition, and constituting elements

The term *mathematical self-schema* was generated from the wider notion of *self-schema,* a notion which itself derives from the more general and well-known idea of *schema*. Thorndyke (1984) defined a schema as a cluster of generic knowledge about a concept (or object, event, procedure, etc.), a mental structure used for assimilating and processing new information. A schema is constructed through a process of induction from several experiences concerning the concept, refined with every additional experience. New information is assimilated into an active schema in accordance with prior expectations stemming from the schema’s characteristics. Thus, schemas influence the interpretation, storage and retrieval of information (Neisser, 1976; Silver, 1982). In the past 30 years, schema theory was used extensively by researchers in various branches of cognitive psychology, as an explanatory framework for human behavior phenomena (see Brewer & Nakamura, 1984, for a review). Two examples of topics in which the influence of schemas was investigated, are memory for written texts (see reviews by Cortazzi, 1993; Thorndyke, 1984), and problem solving in mathematics (Hinsley, Hayes, & Simon, 1977; Mayer, 1982; Schoenfeld, 1982; Silver, 1982). Yet another central issue that gained the attention of researchers concerned the application of schema theory in regard to one of the most basic notions in psychology, the notion of ‘the self’ (Epstein, 1973). This application yielded the term “self-schema.”

Brewer (1986, p. 27) defined a self-schema as "the cognitive structure that contains generic knowledge about the self". This knowledge is organized by subconscious mental constructs that respond to information encountered by a person about him or herself. Some of this information is private and accessible only to its owner, while other information is public and accessible to outside observers. The self-schema, Brewer says, is probably one of the richest knowledge structures in the long-term memory of every individual. When a person repeatedly undergoes experiences that relate to a particular aspect of the self, a self-schema is created. Similar to schemas in other knowledge domains, a self-schema influences the receiving, interpreting, and retrieving of new information, and thus effects many actions carried out by the individual.

In her illuminating discussion about self-schemas, Markus (1980) describes the development of these schemas from childhood to adulthood. A child's self-schemas are characterized mainly by concrete, perceptible attributes such as appearance, place of residence, hobbies and so forth. During adolescence, more complex self-schemas are formed which include abstract traits; for instance, sociability, political affiliation, persistence (see also Montemayor & Eisen, 1977). Based on several studies, Markus suggests that an individual's self-schemas will evolve around those aspects that lend opportunities for constant comparisons of him or herself with surrounding others, while emphasizing one's distinctive and particular traits. For example, for a redheaded child, the hair color is an important component of a self-schema. According to Markus, information about the repeated experiences that created a self-schema might be forgotten over time, leaving some kind of 'summaries' in memory, such as "I am a person who usually enjoys parties." These personally-created compact pieces of knowledge will often gain priority in our memory over other kinds of knowledge. Thus, an emotionally loaded experience might create a self-schema that will push aside other knowledge schemas relevant to the same issue. Moreover, self-schemas frequently cause people to recollect their own history in a biased way (Greenwald, 1980).

For every one of us there exists, according to self-schema theory, many self-schemas relating to different domains in which we acquire and preserve knowledge about ourselves. Hence, in order to analyze adults' recollections of their mathematical experience, I focused on what I called the *mathematical self-schema* of a person. The idea of reducing a general theoretical construct to create an analysis tool for mathematics specifically is quite a common practice in the research arena of the psychology of mathematics education. A well-known example is the study by Betz and Hackett (1983), who applied Bandura's (1977) theory of *self-efficacy expectations* to produce the term *mathematics self-efficacy expectations* (indeed, Bandura's theory lends itself to a wide spectrum of context-specific applications, and has been adapted for diverse research areas such as decision making, health, academic attainment, teacher beliefs and more. However, a construct need not be so broadly defined in order to allow for its reduction to a more specific domain). A more recent example is the use that Boaler and Greeno (2000) have made of Belenky, Clinchy, Goldberger, and Tarule's (1986) typology, termed *ways of knowing*, to consider aspects of mathematics knowing and learning specifically.

Following Brewer's (1986) definition, I therefore defined a mathematical self-schema as *a specific cognitive structure containing the knowledge a person has of oneself as a learner and a user of mathematics*. As said earlier, self-schemas can be viewed as powerful psychological mechanisms; a self-schema associated with a certain domain is assumed to have major effects on receiving, interpreting, and retrieving of information relevant to that domain, and consequently on the actions of the individual in regard to related issues. There is, therefore, reason to believe that a mathematical self-schema of a person will influence his or her functioning in regard to mathematics-related issues. This might include cognitive functioning, i.e., how does that person *acquire and retrieve* mathematical information; affective functioning, for instance, how that person *reacts* (*in the emotional sense*) when encountered with mathematical information; and practical functioning, concerning the ways in which the person *uses* mathematics (or perhaps avoids using it) in the course of his or her life. These are all core issues that are centralized in mathematics education research, though not often with adults as the focus of inquiry (unless they are college students or teachers). Thus, I suggest that characterizing and investigating mathematical self-schemas may lead to better understandings of topics that concern mathematics at the stage of adulthood. According to this view, my goal was to characterize the mathematical self-schemas of the adults participating in the study, and distinguish between different types of those schemas.

The above stated rationale and goal call for some key questions to be asked and addressed. Four central questions are:

- How were the mathematical self-schemas characterized?
- Are the mathematical self-schemas defined in such a way that allows for their applicability in other research?
- How is the framework of mathematical self-schemas useful for other studies?
- In what ways might this framework be further explored?

The first question is addressed in the following paragraph. In order to refer to the other questions, it is necessary that the reader will first be presented with the process of applying the mathematical self-schema framework to the subjects of the present study. Through the description of this process, appearing in Section 4 below, I attempt to establish the grounds for discussing possible applications and further exploration of the framework. These are considered in Section 5.

Characterizing the mathematical self-schemas of the research subjects. The analysis of the interview transcripts was carried out, as noted earlier, through a process known as *categorical-content analysis* (Lieblich et al., 1998). When using this method, the researcher begins with examining the original interview text and creating a subtext from it, on which he or she wishes to focus, by collecting all sentences referring to a certain issue. In this case, sentences were collected out of each interview that related to mathematics from a personal affective perspective. Due to the course of interviews as described in Section 2 above, most sentences were taken from the first part of each interview. However, if during the second part of the interview (while attempting to solve mathematical tasks) the subject had made an affective comment, such as “must I do it?” or “I like this question,” these sentences were also included in the personal affective subtext. The next stage in a categorical-content analysis is a meticulous reading of the subtext, yielding a preliminary outline of content classification. What follows then is a circular process of reading, eliciting components for the categorization of subtexts, initial classification by the suggested components while rethinking, suggesting additional components and/or refining the existing ones, re-reading and so on. In this case of mathematical self-schema, the process was fairly simple. Of the seven components that were eventually used (see below), four have manifested themselves as a direct result of questions asked (Components 1 and 4–6). The other three components (2, 3 and 7) were not the result of direct questions, but were formed during the circular process described above. Following is the list of seven components that served as indicators of the mathematical self-schema:

1. The nature of feelings expressed by the subject when recollecting the experience of studying math in high school.
2. Characteristics of mathematics regarded by the subject as attractive or repellent.
3. Self-evaluation of mathematical ability.

4. Self-evaluation of the possibility to recall mathematical material today.
5. The relevancy ascribed to mathematics by the subject, in relation to his or her professional and everyday life.
6. The subject's involvement in the mathematical studies of his or her children.
7. The nature of feelings that were elicited while the subject was dealing with mathematical tasks during the second part of the interview.

In the next section I will present a detailed description of the categories obtained for each of these schema components, as they emerged while scrutinizing the transcripts.

4. Results: the mathematical self-schema framework applied to the study subjects

As already mentioned, the study was defined as a collective case study. In this type of study, the aim is first to characterize each case individually, and then find similarities and diversities between cases in order to gain a more general view. According to this approach, each interview transcript was first analyzed as a separate unit in an in-depth manner, based on the seven components described above. This first stage resulted in 24 individual mathematical self-schemas. Only then did the cross-subject exploration begin. In the second stage of analysis, categories were formed for each of the seven components. In the third stage, possibilities of grouping were considered, using those categories. Due to space limitations, the first stage of analysis (i.e., the 24 schemas) will not be presented in this article, and I will turn directly to results from the second and third stages. However, the extensive citations reveal parts of some personal stories.

4.1. Categories for the mathematical self-schema components

4.1.1. Component 1: The nature of feelings expressed by the subject when recollecting the experience of studying math in high school

I found that references to this component in the 24 sub-texts could be grouped into the following five categories.

1a. Describing high school math lessons as an exciting experience, an intellectual challenge that invigorated a constructive competition between students. Subjects in this category said that doing math in high school was fun, and that they were able to see beauty in mathematical ideas. Most of them mentioned that together with peers, who viewed math in the same way, they felt like members of a selected, and even superior, group. An illustration of this category can be seen in the following citation[2] from the interview with Ruth, a 33-year-old physician who studied math in the high level track:

Ruth: It was a marvelous experience for me. Apparently I had very good teachers that gave me the flavor of enjoying math, enjoying the challenge. [. . .] It became a challenge to come to math classes, and not a punishment. It was fun. [. . .] I would find myself at 3:00 A.M. after 6 hours of exercising from the books, without noticing, like. . . just for the fun of it, because it's, like, tempting, to show them in the morning that I managed to solve even this problem, that they couldn't do. [. . .] There was this atmosphere, it was such a constructive competition.

[2] All citations from subjects' interviews are translated from Hebrew.

1b. Describing high school math as an important but difficult subject, that was studied with motivation and effort, sometimes even with some interest and enjoyment, but without enthusiasm. The following two citations illustrate this category. The first one is taken from the interview with Shaul (male), 45, a sound technician who studied math in the high level track. The second is taken from the interview with Judy, 35, a lawyer who studied in the intermediate level track:

Shaul: I remember that in this subject I had to work hard in order to achieve. I remember that it was difficult, and I dealt with it through self-learning, help from friends and so on. I had to study and concentrate in class, and memorize [. . .]. We wanted to know math, we had to, in order to study electronics. In all the calculations of currents there are complicated mathematical calculations.

Judy: I decided to take the intermediate track and it was hard for me, we were a small group and I think I was the weakest. But I liked it, I liked the game, the exercises. [. . .] He [eleventh grade math teacher] said we have to exercise a lot, and this is what we did. All the time we sat and exercised. All the others were such geniuses, that the material was easy for them. I had to invest and work to manage the material.

1c. Describing feeling of indifference towards math learnt in high school, resulting from the low priority that was ascribed to this subject in comparison to other school subjects and out-of-school occupations. The following citation from Eli's interview illustrates this category. Eli (male), 40, currently a principal of a post-secondary institution, had quit math lessons during his senior year in high school. After his army service he passed the low-level matriculation exam in math with a final grade of 90:

Eli: I didn't have a problem, I just didn't think it was important enough. In my senior year I regarded humanities as more important. I was busy with the youth movement activities, and really, who had time to deal with math. It didn't seem important. I'll do it when I come to the conclusion that it's important. So then I gave up beforehand. . . I decided not to take the exam and leave it until after the army. [. . .] Up to mid senior year I did study, and the grades were average. It was a low-level class. There was this excellent teacher, which afterwards helped me to take the test in private lessons. An amazing teacher. But well, I felt quite bored during class. These were lessons that I didn't think of as especially important. . . There was something oppressive about it. It wasn't a subject that I liked.

1d. Describing lack of interest in math, as part of a general lack of interest in high school altogether. This category is illustrated below with citations from the interviews with Tamar (female), 42, an art history high school teacher, and Arie (male), 38, an insurance representative. Both of them studied math in the low-level track:

Tamar: As for feelings [towards mathematics], I didn't feel any abysmal frustration, and not a great delight either. No subject was particularly a celebration in high school. . . I knew how to manage, how to survive there.

Arie: I remember that in my time, for most of the class, we came to school feeling like 'going to work,' finishing our workday and that's that. It was not a first priority calling. I didn't study, didn't put any effort in school, really, ah. . . [. . .] For me to survive was passing the exams. In mathematics I was comfortable with the low-level track, it really didn't demand any grand effort.

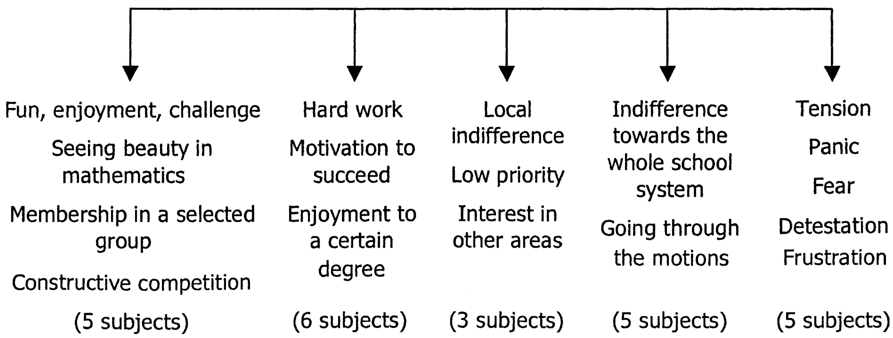
1e. Describing high school math lessons as a stressful, and even terrifying, experience. Subjects in this category recalled feelings of panic, fear, detestation and frustration during mathematics classes. These feelings were in most cases ascribed to the behavior of the teacher (see first and second citations below), except in one case, in which the origin of unpleasant feelings seemed to be internal (see third citation). The illustrative citations below are of the following interviewees: Ilan, 40, a businessman who studied in the high-level track; Ahinoam (female), 30, an interior and jewelry designer, and Tmira (female), 36, a producer, both of whom studied in the low-level track:

Ilan: He was an authoritarian teacher. He would make you flounder... [During the lesson] he would begin to develop formulas, asking who understands and who doesn't, of course no one dared to raise a hand, including those who did understand, because then he'll ask them, and those who didn't understand, because he'll ask them even more. [...] There was a kind of tension, a feeling of tension.

Ahinoam: Until ninth grade I had 100 in math [...], I had very good teachers. Then, at tenth grade came this terrible, horrible monster and managed to scare and create stress, and since then I had 60 and a great lack of confidence. It was a mechanical atmosphere, no, not even that. It was more like reciting. This is theorem so and so, here you take this angle... It wasn't like riddles, wasn't like the fun of discovering, and with all the threats about final grades and blah blah blah, and if you make no effort you won't pass, it was a dictatorial scene.

Tmira: I suppose I had good math teachers, I just couldn't figure out what they were talking about. Truly. I used to sit in class really terrified, wishing no one would ask me nothing... I sat at the far end of the class, just so I won't be asked... The fear, what will happen if I fail the matriculation exam. The matriculation was, if I don't pass, I can't get education... I remember it as panic.

Summary of Component 1. The recollections of feelings elicited in regard to high school mathematics lessons can be arranged as a sequence of key words, as shown below.



4.1.2. Component 2: Characteristics of mathematics regarded by the subject as attractive or repellent

Subjects' responses in regard to this component were divided into three categories: attractive characteristics of mathematics, repellent characteristics of mathematics, and mixed feelings about the subject matter of mathematics. These categories are specified below, with illustrative citations following each point.

2a. Attractive characteristics of mathematics.

- Doing mathematics gives one opportunity to exercise the brain:

Isaac: I think it's some kind of mental food for the brain, something like that. My brain demands of me to practice, like, to do all sorts of games, arithmetic games.

(Isaac, 41, a lawyer, studied math in the high-level track)

- Mathematics is based on pure logic, and working logically is an appealing pursuit:

Judy: Anything that is logical thinking attracts me very much [. . .]. In law school, too, whatever had to do with logic, like accounts, which is a very logical issue — I was very good at it. Mathematics is totally pure. In law the logic is similar, but you also consider all sorts of external factors.

(Judy, 35, a lawyer, studied math in the intermediate level track)

- Mathematics requires understanding rather than memorizing:

Uri: I, personally, am not gifted with extraordinary memory. So I really liked that you don't need memory, but that once you got the principle, you can solve from it many things.

(Uri (male), 36, gynecologist, studied math in the high-level track)

- The precision and complete correctness of mathematical outcomes creates a feeling of confidence:

Isaac: I'm speaking [. . .] about the confidence that this subject has given me, about knowing that in this thing no one can argue with me, because I know in math what the result should be. While in other things you can argue with me and say that I expressed myself incorrectly and I'm not being accurate, in math no one could dispute me.

(Isaac, 41, a lawyer, studied math in the high-level track)

2b. Repellent characteristics of mathematics.

- Mathematics is boring:

Dov: I do not recall that during all those years, let's say from fifth grade to senior year, that I said 'wait a minute, math is actually nice.' I don't recall anything of the kind. [. . .] I saw it as a duty, very unappealing, not interesting, very boring. So I imagine that it can be made more appealing, more refreshing, even if you're not attracted to it, that at least one positive memory would remain. The fact that I have no positive memory, I suppose it can't be only me, that nothing interested me. Apparently there was also nothing to attract me.

(Dov (male), 37, a government official, studied mathematics in the low-level track)

Ahinoam: A train leaves Jerusalem at such and such hours, a train leaves Tel-Aviv, when will they meet, when will driver A wave at driver B, as if I care. There is nothing exiting about it, there is nothing about it at all.

(Ahinoam, 30, an interior and jewelry designer, studied mathematics in the low-level track)

- Mathematics is dogmatic:

Amira: Once the study is dogmatic, that is, I teach something and you should look, understand and exercise, just because — so for me it didn't work.

(Amira (female), 31, a museum director, studied math in the intermediate level track)

2c. Mixed feelings about the subject matter of mathematics.

- Fondness of mathematical general principles, yet impatience towards exercises:

Gadi: At the time of Algebra, then I really liked it. I liked the way it was built, that here and there there's this trick you've got to know and notice. Once I knew the principle, to exercise it a thousand times seemed to me a waste of time, that if I exercise it, it won't be math any more. [. . .] I mean, I had to do it in order to prepare, but this I didn't like. Because. . . because of all sorts of follies of forgetting the comma or adding instead of subtracting, I arrived at the wrong answers, and then it made me feel uneasy and unconfident.

(Gadi (male), 41, an architect, studied math in the intermediate level track)

Yafa: I liked the games of thinking. [. . .] The main difficulty was really the degree of precision, of working accurately and patiently. This is not a thing I was ever particularly fond of.

(Yafa (female), 38, a humanities high school teacher, studied math in the high-level track)

- Satisfaction from the efficiency and concreteness of calculations, yet impatience towards theories and generalizations:

Isaac: The teacher, for instance, could have come and say to me 'listen, let's talk about it from the theoretical side,' and I would tell him, 'I'm not interested in the theoretical side. Please give me an exercise. You want a truck leaving here, a car leaving there, when do they meet — this interests me, I'd be glad to give you an answer about this thing.' What is *a*, and what is *x* and what is *y*, this did not interest me. I want to give answers to concrete specific problems. I don't care about creating a formula that will be good for all generations.

(Isaac, 41, a lawyer, studied math in the high-level track)

4.1.3. Component 3: Self-evaluation of mathematical ability

Reactions relating to this component were grouped into five categories, described and illustrated below.

3a. Viewing oneself as a very competent person in mathematics, someone who demonstrates a high mathematical ability and a fast understanding of mathematical concepts:

Yosi: My mathematical perception was much higher than that of the rest of the class. [. . .] Nothing was difficult for me, I didn't work at home at all, I didn't study before tests [. . .] my understanding was immediate, absolutely without studying.

(Yosi (male), 31, an accountant, studied math in the low-level track)

3b. Regarding oneself as having a good mathematical ability, which was not fully fulfilled due to a mistreatment or a mismatch between school factors and personal factors:

Tehila: This teacher threatened us [. . .] He created a lot of tension, which drove me crazy. At a certain stage I dropped to the lower track just to eliminate all this stress [. . .] I think that in other circumstances the whole subject would have been painted in different colors.

(Tehila (female), 38, teacher in adult school, studied math in the low-level track)

3c. Regarding oneself as having a reasonable mathematical ability, just enough to successfully handle mathematics in the level chosen:

Eli: I knew I could do it [. . .] A month before the [final low-level] test, I learned very intensively [. . .] and that was it. I took the test and I got the grade I got.

(Eli, 40, a principal of a post-secondary institution, studied math in the low-level track)

3d. Description of oneself as a person with average-to-low capability of doing mathematics, who "came to terms" with this state of affairs:

Tali: Some things I don't know and I will never know, and it's ok. A sort of acceptance [. . .] I can't sew either [. . .] It's not low self-esteem, it's a fact.

(Tali (female), 42, a midwife, studied math in the low-level track)

Tuvia: Like they say about fat people that there comes a stage when they are fed up with hiding their paunch, and then this is it, we're fat. [. . .] I too will be very happy to know mathematics, but I'm fed up.

(Tuvia (male), 35, a musician, studied math in the low-level track)

3e. Viewing oneself as having very low, or almost lacking, mathematical ability, a description which often hints to feelings of inferiority.

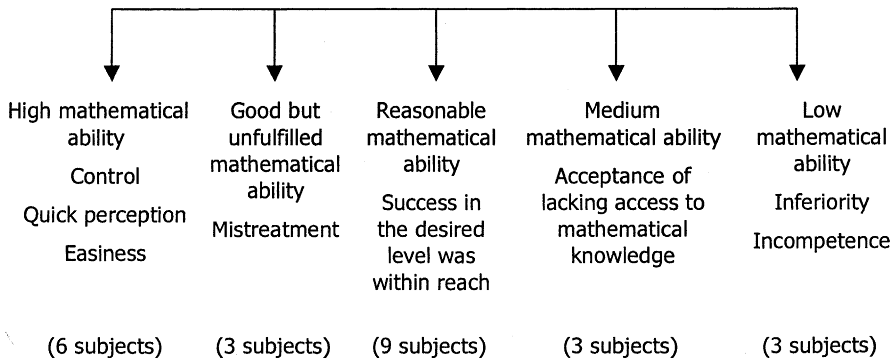
Dafna: It's not a terrible disaster that I don't know arithmetic, mathematics. It's not terrible, I'm ok, I'm a fine person and nothing happened. It's a shame, though, because it seems to me that there are a lot of interesting things in it. The fact is, that the beginning [of a new mathematical topic] is always very interesting for me, until I'm blocked by some sort of a wall and it's complicated for me and this is it.

(Dafna (female), 33, a nurse, studied math in the low-level track)

Tmira: I simply didn't have the brains for it. Simply, look, I was not talented for it.

(Tmira (female), 36, a producer, studied math in the low-level track)

Summary of Component 3. Similarly to Component 1, the variety of descriptions regarding this component could be viewed as creating a sequence, moving between low and high self-evaluation of mathematical ability. Each stage is characterized by key words. This sequence is described below.



4.1.4. Component 4: Self-evaluation of the possibility to recall mathematical material today

Three main types of responses were obtained in regard to this component, as described below.

4a. The subject considers the mathematical knowledge learnt in the past to be available, even if it cannot be immediately recalled. Re-exposure to the material, through self-reading, is expected to yield satisfactory results within a short period. This type of reaction was obtained from eight subjects. Ruth, for example, said:

Ruth: I'll open a book and I'll remember what it's all about [. . .] It's only the starter that's missing.

(Ruth, 33, a physician, studied math in the high-level track)

4b. The subject conjectures that the mathematical knowledge is not completely forgotten, yet cannot be retrieved without external help, such as formal lessons. The re-study is expected to be easier than the original study of mathematics during high school. This conjecture was elicited by four subjects, for example:

Tehila: Even if I'd open a book I wouldn't know how to read it. I didn't remember that this is how you write π. I forgot, everything. So what would it help, it would have been Greek to me. [. . .] Look, it's not a total vacuum [. . .], it's somewhere in the brain, and I think if I go today, if I enter a lesson, then I'll come back. I'll come back to myself, I just forgot. It's not a vacuum, I'm not a Tabulla Rasa. I've got it inside me somewhere.

(Tehila (female), 38, teacher in adult school, studied math in the low-level track)

4c. The subject considers his or her mathematical knowledge to be at a state of "total loss." In case of needing to learn mathematics again, no prior knowledge could be assumed, and studies will have to start from zero. This type of response was given by 10 subjects. Most of them ascribed this state of complete forgetting to the original manner of study; the mathematical material was only learned for the purpose of passing the final exam, and it was mostly memorized during short periods of time before tests. Arie, for example, said:

Arie: In my opinion I'll have to start all over again. That's what I think. Because I think we learned the topic, its technique, and if you would ask me about that topic two weeks later, I wouldn't remember and wouldn't know anything. I could get 10 before the test, and a week and a half later get zero if there was an unexpected exam.

(Arie (male), 38, an insurance representative, studied math in the low-level track)

The reader may have noticed that of the 24 subjects, only 22 were categorized within the three categories of Component 4. Two subjects were not categorized: Gadi (an architect, studied math in the intermediate level track) did not fit any of the formed categories, saying that he thinks he could easily retrieve formal geometry, which he enjoyed learning, but as for other topics he expects great difficulties. Dov (a government official, studied mathematics in the low-level track) claimed he could not answer this question, since he has no idea whatsoever what the process of preparing for a high school math test involves (apparently, Dov added, he never had such an idea, and his passing of the final test — with the grade of 60 out of 100 — was the result of using a little common sense together with "bits and pieces" he managed to gather).

4.1.5. *Component 5: The relevancy ascribed to mathematics by the subject, in relation to his or her professional and everyday life*

Reactions regarding this component divided subjects into three groups, as follows.

5a. Subjects who referred to certain mathematical topics as relevant to their lives, and could exemplify some uses of mathematical knowledge. This group included seven subjects, one of whom was Ruth. Her reaction is briefly summed below, as an illustration of this category.

Ruth, a physician currently specializing in gynecology, spoke both about the mathematical knowledge that is relevant to her work, and about the impact that mathematics and science studies have on thought processes needed in a physician's work. In her opinion, the relevant knowledge includes first and foremost the issue of statistics, which is essential but nevertheless, as she criticizes, underrepresented in high school curriculum and in medical school as well. Ruth also exemplified the use of other mathematical issues: blood flow calculations carried out by integrals, estimating the fetus' weight and calculating the area of the placenta (understanding these calculations is highly regarded by Ruth, although they are performed by computer), exponential processes of growth (e.g., pregnancy hormone, the quantity of which doubles every two days), calculation of the bladder's volume after a gynecological surgery on the basis of its radius as seen in the ultrasound appliance, reading data presented by means of graph, and more. As for thought processes, Ruth claimed that handling mathematical and scientific materials contributes to the development of a certain logical perception that is vital to a physician's practice:

Ruth: The perception that there are basic data, and then there are data that builds upon it [. . .] I think this is part of science teaching. [. . .] In professions that are tangent to the exact sciences, like the medical professions, you use quite a lot of this thing of trying to deduce. The ability to deduce from certain data to more general, broad things. That is, the ability to deduce is something that you learn in basic math and you use it as a physician every day.

The other subjects in this category mentioned different professional actions, as well as everyday actions, that demand the use of mathematical issues. For example: calculating the percentage of the increase of shares, interpreting statistical results of child development tests, economical shopping, economical building using extreme values of area in regard to different possibilities of shape, using geometry to solve space design problems, and more.

5b. Subjects who claimed that although in their daily actions they do not use extensive mathematical knowledge, nevertheless studying mathematics has educated them to think in an ordered and logical manner (five subjects). For example:

Judy: I see many times that people start to talk about something, and suddenly — wait a minute, what were we talking about? They're lost on the way. But when you have a method, when you work on a lesson, or a lecture, where do you begin, where are you heading, what is your summary — in my opinion these are all things connected to mathematical thinking, not necessarily with numbers.

(Judy, 35, a lawyer, studied math in the intermediate level track)

5c. Subjects who claimed that apart from few basic operations, the mathematics they have learned is irrelevant to their lives today. Twelve subjects were assigned to this category, of which two are cited below:

Tali: I think I could have studied in three months what is useful for me today. Addition, subtraction, areas [. . .], ah, interest percentages, ah. . . bank account, what else do I deal with? Addition, subtraction, multiplication, percentages.

(Tali (female), 42, a midwife, studied math in the low-level track)

Tmira: Multiplication table, playing a little with percentages, these are things that assist me today. Addition, subtraction, the simple things. I can't remember when was the last time I used a square root, or searched for *x* and *y*. And what will I do with an equilateral triangle? Do you do something with it? Really, I don't know. I'm trying to think where. . . We have a business, we deal with money, and I can't see where I use all these things. Except for basic operations, I can't see.

(Tmira (female), 36, a producer, studied math in the low-level track)

4.1.6. Component 6: The subject's involvement in the mathematical studies of his or her children

The term 'involvement' here refers to the following questions: Does the subject assist his or her children when working on homework assignments in mathematics? How does the subject feel about that? What is the importance ascribed by the subject to his or her children's achievements in mathematics? In spite of many different nuances, it was feasible to divide subjects into the three main groups described below.

6a. Subjects who frequently assist their children in mathematics, actively follow children's progress, and put much emphasis on the aim of acquiring mathematical knowledge and understanding per se, not only because mathematics is a compulsory school subject (nine subjects). For example:

Nurit: I'm terribly sensitive to them not understanding something in math, or missing something. I'll see it right away. And then I sit with them and they get cross with that. That is, they want the answer to the given homework, and I don't care about the answer. I want them to understand the process. And I don't care if they'll sit for two hours and no homework will be finished, but they'll get the process. It hurts me to see something being done automatically, without understanding, I can't stand it. [. . .] There was for example the multiplication table with my daughter, so I remember that I sat with her and didn't agree that she won't know it [. . .]. I remember us sitting on this for four days, it was a lot, and each day we played with rows of the multiplication table, or something like that - apparently more than four days, and that was it, until she sang it. I remember that when I did it I enjoyed it, and thought it was terribly important. And in fact she passed that, and I'm not sure that hadn't I been there, she could have carried some sort of a gap, and this might have caused a deterioration.

(Nurit (female), 38, a psychologist, studied math in the high-level track)

6b. Subjects who are willing to assist their children in preparing their homework in mathematics, motivated by a sense of parental concern to see that children stand the system's requirements (eight subjects). Some of these subjects remarked, however, that novel teaching methods make it difficult for them to provide assistance, since they can only explain in the way they were taught, but the children reject such explanations. Most of the subjects in this group said that if mathematics will cease to be a compulsory matriculation subject, they would not press their children to take it beyond the required minimum. For example:

Eli: My daughter is in sixth grade [. . .] and she has a hard time with math. [. . .] Arithmetic demands of her a great effort, and it causes her frustration. [. . .] I'll be very glad if it won't be a compulsory [matriculation subject], from the sheer family interest perspective. For example, now she studied fractions, so in class for instance they studied one way of calculating fractions, addition and subtraction, and I remembered a different method. . . My wife preferred that we don't confuse her, that we go in the way they learn. My method seemed more simple, by the way.

(Eli, 40, a principal of a post-secondary institution, studied math in the low-level track)

6c. Subjects who refrain from active attendance to their children's needs in regard to mathematics, and leave it in the hands of their spouse or a professional (seven subjects). Stated reasons for this situation were either unwillingness to deal with mathematics again, or a sense of lack of capability to do so. Some of these subjects mentioned their aspiration for their children's success in mathematics, mainly since it is a compulsory matriculation subject; however they claimed that they could nevertheless identify with failure, due to their own experiences. For example:

Dafna: I didn't, and I won't, say anything to my daughter that relates to mathematics, so that she will not be prejudiced, and she'll have a positive attitude. My husband is very patient, and can explain in a way that you can understand even Chinese. So if she'll have any questions or problems, he will be the one to turn to. [. . .] If she won't understand, then she won't. We'll consult the teacher [. . .]. I'm supposed to be her mother, not her teacher. If my daughter will dislike it, then she'll just have, in my opinion, to do the minimum required to get to university, not more.

(Dafna (female), 33, a nurse, studied math in the low-level track)

4.1.7. Component 7: The nature of feelings that were elicited while the subject was dealing with mathematical tasks during the second part of the interview

Reactions regarding this component can be described, as done for Components 1 and 3, as a sequence on which the five following main responses can be distinguished.

7a. Attempts to handle the tasks were performed with motivation, while showing interest and even some pleasure. Yafa, for example, said:

Yafa: Such fun, do you know how long it's been since I was engaged in such thoughts?

(Yafa (female), 38, a humanities high school teacher, studied math in the high-level track)

7b. Attitude towards the tasks could be regarded as attentive and cooperative, yet no emotions, either positive or negative, were expressed.

7c. Strong willingness to answer the questions was evident, but the main motif elicited was disappointment. Subjects expressed regret about what they felt was an almost complete loss of mathematical material mastered in the past. For example:

Judy: It's like, you know, you had a time when you took quite a high-leveled exam, you could use those things and suddenly being in a state when you can't even recall concepts, it's embarrassing. [. . .] Not because I'm supposed to remember, but because I can remember a stage when I did know all that.

(Judy, 35, a lawyer, studied math in the intermediate level track)

7d. Attempts to recall the needed material are accompanied with expressions of discomfort, lack of confidence or even inferiority. Yet, this does not cause the subject to try and avoid the tasks. Examples:

Eli: It seems to me so simple and I can't do it.

Tuvia: There, I've exposed myself, I don't know anything.

Nir: It must look idiotic that someone will try such a simple thing and will think it over, ha?

(Eli and Tuvia, cited earlier, are graduates of the low-level track. Nir (male) is a 43-year-old photographer and carpenter, a graduate of the intermediate level track.)

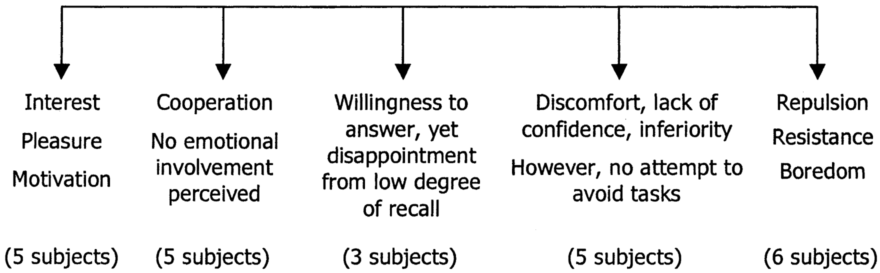
7e. While trying to recall mathematical concepts, the main feelings expressed are repulsion and resistance, either because the questions elicit unpleasant memories, or because the content is perceived as boring or irrelevant. Examples:

Dafna: Must we go on? [. . .] This makes me sweat.

Dov: What can be interesting in that, for heavens' sake? [. . .] Gross.

(Dafna and Dov, cited earlier, are both graduates of the low-level track.)

Summary of Component 7. The sequence of descriptions regarding this component is described below, each stage characterized by key words.



4.2. *Grouping subjects into four main mathematical self-schemas*

In the previous section, I described the seven components of the mathematical self-schema in detail. For each component, several categories or groups were formed, in order to encompass the variety of subjects' reactions. In this section I will show how these categories were used for constructing four main types of mathematical self-schema.

Fig. 1 presents an integration of all the different categories or groups formed for the seven components, into one summarizing table.

For each subject there now exists a certain "path" through Fig. 1, according to the categorization of his or her reactions in regard to the various components. Since, in spite of a few difficulties in classification, each response was eventually assigned to one category only,[3] this path is unique. Furthermore, due to an apparent connection between the different components, the paths are not random (e.g., it is not expected to receive a positive recollection of mathematics lessons together with a description of repulsion towards mathematics). Consequently, it was feasible to divide the 24 paths obtained, into four main clusters, each such cluster containing paths, which do not differ greatly from one another. This process was performed as shown in Fig. 2.

[3] This is true for all components except for the second one; few subjects stated both attractive and mixed characteristics of mathematics, or expressed repulsion towards a certain topic in mathematics, but mixed feelings towards other topics. For these cases the path indeed splits. However, this did not have much affect on the determination of the four main schemas.

1. Key words characterizing the mathematical experience in high school:	Fun, enjoyment, challenge Seeing beauty in mathematics Membership in a selected group Constructive competition	Hard work Motivation to succeed Enjoyment to a certain degree	Local indifference Low priority Interest in other areas	Indifference towards the whole school system Going through the motions	Tension Panic Fear Detestation Frustration
2. Attractive / repellent characteristics of mathematics:	Attractive characteristics: Opportunity to exercise the brain, pure logic is appealing, understanding rather than memorizing is required, precision and correctness creates a feeling of confidence.		Mixed feelings: Fondness of mathematical principles, yet impatience towards exercises. Satisfaction from efficiency and concreteness of calculations, yet impatience towards generalizations.		Repellent characteristics: Mathematics is boring and dogmatic.
3. Self-evaluation of mathematical ability:	High mathematical ability Control Quick perception Easiness	Good but unfulfilled mathematical ability Mistreatment	Reasonable mathematical ability Success in the desired level was within reach	Medium mathematical ability Acceptance of lacking access to mathematical knowledge	Low mathematical ability Inferiority Incompetence
4. Self-evaluation of the possibility to recall mathematical material today:	Knowledge is available, even if it cannot be immediately recalled. Re-exposure to the material, through self-reading, will yield satisfactory results within a short period.		Knowledge is not completely forgotten, yet cannot be retrieved without formal lessons. The re-study is expected to be easier than the original study.		Knowledge is at a state of "total loss". Re-studies will have to start from zero.
5. Relevancy ascribed to mathematics:	Certain mathematical topics were mentioned as relevant today, and their uses were exemplified.		The mathematical knowledge is not used, but the logical aspect of mathematics is relevant.		Apart from few basic operations, mathematics is irrelevant.
6. Involvement in the mathematical studies of children:	Frequent assistance in children's homework, active interest in their progress, emphasis on acquiring mathematical knowledge and understanding per se.		Assistance motivated by parental concern to see that children stand the system's requirements, but not beyond the required minimum.		Refrain from active attendance to children's needs in mathematics. This issue is left in the hands of the spouse or a professional.
7. Feelings elicited during the interview:	Interest Pleasure Motivation	Cooperation No emotional involvement perceived	Willingness to answer, yet disappointment from low degree of recall	Discomfort, lack of confidence, inferiority However, no attempt to avoid tasks	Repulsion Resistance Boredom

Fig. 1. Summary of the mathematical self-schema components.

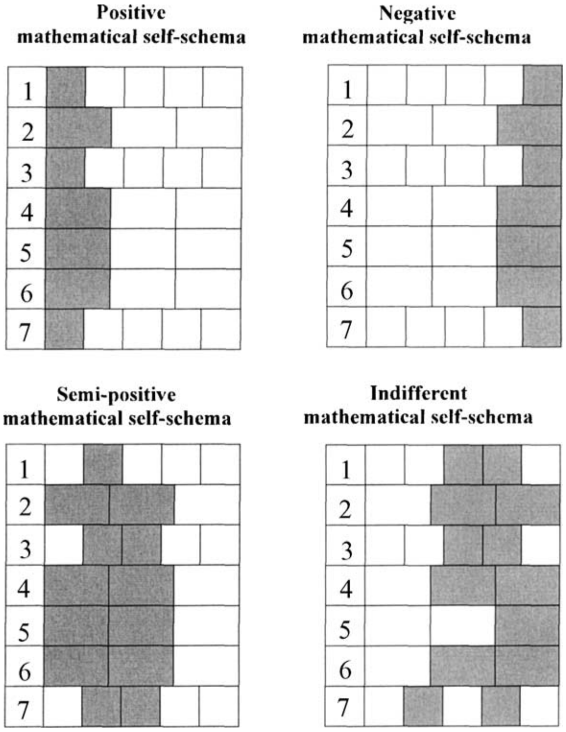


Fig. 2. Primary paths for four mathematical self-schemas.

The two extreme clusters were defined as “positive” and “negative” mathematical self-schemas. Each of these schemas relates to an endmost prime path, as described in the top half of Fig. 2. Subjects were classified to one of these schemas if their personal path was similar to the prime path, i.e., differed from it in no more than two places. Thus, for instance, I characterized five subjects as having a positive schema: Uri and Issac, whose paths were identical to the prime path, Ruth and Nurit, whose paths diverged from the prime path in one component, and Irit, whose path was different from the prime one in two components.

In addition, two other clusters were determined. These clusters, called the “semi-positive” and the “indifferent” schemas, were non-extreme clusters, and were thus each assembled from several prime paths, as shown in the lower half of Fig. 2. Subjects were assigned one of these schemas, if their personal paths diverged from the schema’s prime paths on one component for the most.

This procedure resulted in the classification of each of the subjects, excluding three, to a certain mathematical self-schema. For three subjects, the decision was problematic; according to the above definitions, two of them could fit close enough to the paths of two schemas, and one did not fit any schema. This itself is an interesting outcome, portraying once again the diversity of human behavior, with which it seems that one always comes across cases that do not lend themselves to any placement. After some consideration, however, I decided to classify these three subjects according to the nature of their recollections from high school mathematics lessons (i.e., Component 1), as this was the most dominant component in every schema.

The main themes of the four mathematical self-schemas are presented below (see also Karsenty & Vinner, 2000).

4.2.1. The positive schema

Among the features of this schema are feelings of pleasure, challenge and superiority associated with studying mathematics in high school, along with high self-confidence in regard to mathematical ability. Mathematics is perceived as an appealing and intellectually gratifying pursuit. Subjects demonstrating this schema appeared to have a mathematical cast of mind.[4] They willfully use numbers in their professional lives as well as in their everyday surroundings. They are actively involved in their children's mathematical education. Most of them felt they could regain mastery of high school mathematics within a short period of self-reading. They usually enjoyed dealing with the mathematical tasks presented during the interview.

4.2.2. The semi-positive schema

This type of schema is basically characterized by feelings of respect towards mathematics. High school mathematics is remembered as an important topic, a top priority goal that could not be easily achieved. Subjects demonstrating this schema described themselves as highly motivated former mathematics students. They had worked hard, not always with pleasure, in order to succeed in mathematics. Most of them were attracted by the need to reason logically, but lacked the patience to accurately follow necessary procedures. Today they perceive themselves as rational people, able to handle daily quantitative data with reasonable success. Most of them doubt their ability to re-study mathematics without formal assistance, but provided such help, "the second time around" is expected to be easier. They regard their children's mathematics studies as very important, but do not always take an active part in it. They value their mathematics education, mostly because it contributed to their reasoning skills. In this sense, they find the mathematical knowledge per se to be of secondary importance. When handling mathematical tasks during the interview, they tended to express mainly feelings of disappointment or embarrassment, but were often quite eager to reach the correct answer.

4.2.3. The indifferent schema

This schema is essentially defined by its title. Subjects exhibiting this schema described themselves as indifferent mathematics students in high school. They were not enthusiastic, nor frustrated by it. They simply went through the motions of this requirement. This sort of recollection is not associated with low self-confidence in regard to mathematical competence; on the contrary, most of these subjects claimed to have a reasonable mathematical ability. It was motivation they lacked. Efforts in mathematics were given low priority, and this subject was considered boring and technical, with rare gleams of appealing ideas. Today, these subjects are still indifferent towards mathematics, though some of them said they assume it has attractive sides, only unknown to them. They all use calculators regularly in the course of their workday, but refer to the mathematics that is relevant to their lives as minimal and trivial. They sometimes assist their children with homework in math, but only as part of their parental duties; most of them said that if math became optional in high school, they would not insist that their children study it. Attempts to recall the requested mathematical material elicited mainly feelings of discomfort and some expressions of inferiority.

4.2.4. The negative schema

Fear, resentment, stress and frustration are the leading motifs in the portrayal of this schema. Subjects who manifested this schema described mathematics lessons in high school as the worst part of their

[4] A term used by Krutetskii (1976) for the inclination to view the world mathematically.

studies. Capable students at other areas, they found mathematics to be difficult and unattractive, a tedious list of exercises to be carried out, usually not very successfully. Today, most of these subjects avoid any unnecessary contact with mathematics. They minimize their involvement with the mathematical studies of their children and if possible leave it entirely in the hands of their spouse. They assume that all the mathematics they had memorized in high school is now forgotten. During the interview, when mathematical tasks were posed, they tended to look repelled or bored, and used expressions of anxiety.

The distribution of subjects within the four types of mathematical self-schema is presented in Table 1. Data includes the subjects' professions, gender and level of mathematics taken in high school. Two findings are apparent from this table. The first finding concerns the connection between the mathematical self-schema and the level of high school mathematics: The positive schema was ascribed to subjects who, except one, were all graduates of the high-level track in mathematics. The semi-positive schema was characteristic of subjects of which half were graduates of the high-level track and half were graduates of the intermediate-level track. In other words, the positive and semi-positive schemas were not identified among low-level track graduates. Conversely, almost all subjects demonstrating the indifferent and negative self-schemas were graduates of the low-level track (except for one intermediate-level graduate). This finding hints to a possible interdependent connection between mathematical self-schemas, and choices made by adolescents regarding the extent of their mathematical studies: It is reasonable to assume that such choices are guided a priori by existing — though maybe fragile and yet unstable — mathematical self-schemas, initially formed before entering high school. The choice of a mathematical track is then, in turn, liable to effect the adolescent's future habits as a user of mathematics, or, to put it in another way, crystallize the developing schema. Such a premise gives rise to many intriguing questions, several of which I will mention in Section 5 below.

The second finding revealed in Table 1 concerns the issue of gender. Although this article is not intended to focus on gender-related topics, nevertheless a brief observation can hardly be avoided. It can be seen that the positive and the semi-positive schemas were ascribed almost equally to men and women (six and five, respectively). Yet, the situation with the other two schemas is quite different. The indifferent self-schema was exhibited mostly by men (six of eight), while the negative schema was demonstrated by women only. This finding is consistent with the known literature, which reports on the inclination of girls/women to (a) view themselves as less competent in mathematics, (b) develop a low self-image in regard to mathematics, (c) experience the study of mathematics as an unpleasant element of the education process, and (d) perceive mathematics as irrelevant to their lives (see, for example, Eccles et al., 1985; Fullerton, 1995; Isaacson, 1990; Meyer & Schatz-Koehler, 1990). The result reported above, i.e., that among the "non-positive" subjects, men tend to demonstrate the indifferent schema while women tend to develop a negative schema, is therefore not surprising. However, I consider thought-provoking the fact that this phenomenon was found among a highly educated group of people who belong to a social sector that takes equity for granted.

5. Prospects for future use of the mathematical self-schema framework

After describing in detail the framework designed in this study, namely the mathematical self-schema, I now turn to address questions I raised earlier in this paper, in regard to possible future use and exploration of this framework. First, the issue of feasibility should be considered, i.e., the question of whether a certain mathematical self-schema can be ascribed to any adult individual other than the research partic-

Table 1
Distribution of subjects within the four types of mathematical self-schema ($N = 24$)

Type of mathematical self-schema	Positive		Semi-positive		Indifferent		Negative	
Attributes of subjects who demonstrated this schema: profession, gender[a] and level of math taken in high school[b]	Lawyer	M, H	Businessman	M, H	Principal of a post-secondary institution	M, L	Museum director	F, I
	Gynecologist	M, H	Sound technician	M, H	Businessman	M, L	Teacher in adult school	F, L
	Psychologist	F, H	History teacher	F, H	Musician	M, L	Interior designer	F, L
	Physician	F, H	Lawyer	F, I	Insurance representative	M, L	Producer	F, L
	Banker	F, I	Architect	M, I	Accountant	M, L	Registered nurse	F, L
			Photographer	M, I	Government official	M, L		
					Art teacher	F, L		
					Midwife	F, L		
Number of subjects	Five subjects		Six subjects		Eight subjects		Five subjects	

[a] M, male; F, female.
[b] H, high level; I, intermediate level; L, low level.

ipants. I propose a positive answer to this question. Given the first six of the seven components specified above, a researcher can now pose direct questions to subjects,[5] that will tackle issues "covered" by those components (regardless of whether the component was originally a result of a direct question, or was formed as part of a categorical-content process). For instance, the second component ('characteristics of mathematics regarded by the subject as attractive or repellent'), can be attended to by a question such as "Are there things about mathematics that you find particularly attractive or unattractive?," etc. Answers can then be classified according to the categories listed within each component (except perhaps for a minority of unclassifiable cases), and thus lead to a "path" which can be grouped according to the process described in Section 4.2.

As for the last component, i.e., the nature of feelings that are elicited while the subject actually deals with mathematical tasks, I acknowledge that its implementation might be less simple, due to the fact that the researcher will need to add some mathematical tasks to the interview. Unless there is a particular interest in recalling or problem solving processes, as was the case in this study, this procedure might be viewed as an overburden to both the subjects and the researcher.[6] Indeed, this component came as a "byproduct" of the cognitive objective of this study, and although it had enriched the analysis, perhaps a replication could be comprehensive enough without it. I leave this consideration to the erudite reader. Furthermore, I suggest that the first six components of the framework can be adapted into a Likert-type questionnaire that will enable a more quantitative investigation. This may be done by collecting various responses representing different categories, using them as a basis for creating closed items, and designing a corresponding analysis method. However, describing such a process in detail is beyond the scope of this article.

The second prominent issue that should be addressed when reflecting on a possible contribution of the mathematical self-schema framework, is the issue of the framework's potential part in advancing future investigations on mathematics learning. I suggest that introducing this framework may open two directions for prospective research. First, as a research tool, its implementation might serve to control research groups' characteristics. Second, as a construct, it may be worthwhile to study whether mathematical self-schemas can promote our understanding of certain phenomena, such as the diversity found in regard to preservation of mathematical knowledge (Bahrick & Hall, 1991; Karsenty, 2002). In the following, I describe each of these two directions in more detail.

5.1. Control of research groups' characteristics

Studying mathematical behavior, in the broad sense of this pursuit, frequently calls for selecting a group of learners, in order to portray certain phenomena that characterize this group, or assess the impact of an intervention. Selections might be performed according to various criteria, such as achievements, sociological background, etc. Often, affective variables are considered. Regardless of whether the focus of study is cognitive, instructional, socio-cultural, or other, it is conceivable that a research design should aim to control for affective variables, if these might have a certain influence on results. The term 'control'

[5] Provided that subjects meet the basic requirement of being high school math graduates. Also, it is preferable that subjects will be parents, so that answers regarding Component 6 will be grounded in real life and not just given on the basis of presuppositions.

[6] One could also claim that in order to achieve replication, the mathematical tasks posed to subjects should resemble — in terms of extent and level — the ones given in this study. Perhaps it should be noted, therefore, that a detailed account on these tasks may by obtained from the author via personal communication.

can mean either restricting the research group to subjects who satisfy pre-specified affective conditions, or distinguishing, in the analysis stage, between different sub-groups on the basis of affective parameters, in order to find connections between these parameters and variables which are in the center of investigation. I suggest that the mathematical self-schema framework can be helpful in such control attempts, as will be illustrated by the following two examples.

5.1.1. Example I

Consider a research whose aim is to explore the nature of mathematical thinking processes at adulthood. It is possible that a research of this kind will want to focus on "average" people with respect to mathematics, that is, people who are neither particularly drawn, nor resentful, towards mathematics. The selection of subjects may therefore benefit from using the mathematical self-schema framework, in order to rule out subjects demonstrating the two extreme schemas (positive and negative). Another option might be to investigate adults of all four schemas, and examine possible connections between schema type and characteristics of mathematical thinking processes.

5.1.2. Example II

In some countries, primary school teachers are general teachers, meaning that they teach all school subjects, including mathematics. One of the several problems that this situation creates is that many primary teachers tend to view the teaching of mathematics as an unavoidable duty and not as a challenging pursuit (Nisbet & Warren, 2000). Preparing future general teachers, who will view mathematics (and the practice of teaching it) in a positive way, can be an extremely difficult task (see, for example, Breen, 2001). Under these circumstances, suppose that a teachers college presents a new pre-service program that is expected to increase primary teachers' motivation to teach mathematics (a step in this direction is described, for instance, in Grootenboer, 2003). Evaluation of the program should probably take into account the starting point of the participating teachers, in terms of their affective characteristics. Using the mathematical self-schema framework (perhaps adapted to a Likert-type questionnaire form, as mentioned above) can serve as an indicator of the pre-program state.

5.2. Subsequent investigations of the construct of mathematical self-schema

The mathematical self-schema of an individual, as defined in this study, includes the knowledge this individual has of oneself as a learner and a user of mathematics. If this self-schema is an influential mechanism, as all self-schemas are assumed to be, it is interesting, in my opinion, to examine its effect in regard to various mathematical behaviors of people. One of the questions that can be suggested is the following: Do mathematical self-schemas play any role in regard to the ability to preserve mathematical material once it has been acquired? Studies about factors that could account for the variance in the ability to maintain knowledge learned in school are relatively few (see review by Semb, Ellis, & Araujo, 1993), and even less so when it comes to mathematics specifically (Bahrick & Hall's work (1991) is a unique example). Affective factors are scarcely considered within this research area, which may therefore be enriched by investigations that will consider different mathematical self-schemas as a potential factor.

In order to examine this issue, and other issues in which the construct of mathematical self-schema is possibly involved, it may be helpful to explore this construct in a more extensive manner, especially in regard to its formation and stability. Earlier I discussed the connection found in Table 1, between subjects' mathematical self-schema and the high school mathematics track in which they studied. I suggested that the decision made by an adolescent on the verge of high school (within school constraints), in regard to the level of mathematics in which he or she intends to enroll, might be influenced by a mathematical self-schema that is still in formation. In turn, the outcome of this decision (i.e., the mathematical track chosen) plays a role in the crystallization of the mathematical self-schema. What follows from this conjecture is that major themes in an individual's mathematical self-schema are not likely to alter from high school to adulthood. This raises several questions such as: What are the influential conditions in the process of forming a mathematical self-schema at earlier stages? What is the role of teachers and parents in this process? Is it possible to consciously change a student's mathematical self-schema on the verge of high school? Are certain schemas more resistant to change than others (e.g., would the indifferent schema be harder to modify than the negative schema, considering that the former seems to be more emotionally equilibrated than the latter)? These questions remain open for further investigations.

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