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Video-based peer discussions as sources for knowledge growth of secondary teachers

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This paper reports on a study conducted as part of a larger project, named VIDEO-LM, which centres on video-based professional development for secondary mathematics teachers. The project aims to facilitate the reflective skills and the Mathematical Knowledge for Teaching (MKT) of secondary school teachers, in particular those teaching advanced mathematics courses. At the core of the project is a 6-component framework developed for analysing videotaped lessons in collaborative discussions with teachers. We describe the rationale and novelty of the project and the framework. Then, we focus on a study which examines the MKT growth of a group of teachers who participated in VIDEO-LM peer discussions, and present some preliminary findings.

Keywords: Video-based professional development, peer discussions, mathematical knowledge for teaching (MKT), secondary mathematics teachers.

INTRODUCTION

The power of videotaped teaching episodes as a vehicle for stimulating discussion and reflection among mathematics teachers has been discussed in recent years from various angles (Borko et al., 2011; Coles, 2013; Sherin & van Es, 2009). Although video has been used as a tool for teachers' professional development for the past 50 years (Sherin, 2004), the rapid advancements of digital video documentation has allowed for significant amplification in this field, which manifests in a host of professional development programs in various countries that include video as a major resource (e.g., KIRA and Mathe sicher können in Germany; the Problem-Solving Cycle and the Learning and Teaching Geometry programs in the US; WMCS in South Africa). Online video resources are now largely available to educators (MET in the US and Teachers Media in UK are prominent examples) and at least two international symposia were dedicated

recently to the use of video in mathematics teacher education (see <http://www.weizmann.ac.il/conferences/video-lm2014>).

The following citations distil the main feature of video that can explain why it is regarded as a valuable tool for teacher development. Sherin and van Es (2009, p. 21) claim that "Teachers benefit from opportunities to reflect on teaching with authentic representations of practice"; Brophy (2004, p. 287) argues that video can introduce "the complexity and subtlety of classroom teaching as it occurs in real time"; and Nemirovsky and Galvis (2004, p. 68) suggest that "because of the unique power of video to convey the complexity and atmosphere of human interactions, video case studies provide powerful opportunities for deep reflection". All these scholars emphasize the role of video as a window to the authentic practice of teaching, which allows teachers to focus on complex issues that may be unpacked through observing, re-observing and reflecting on specific occurrences.

Generally speaking, there are three main trends in using videotaped episodes from mathematics lessons as resources for teacher development. First, video is utilized for introducing new curricula, activities, pedagogical strategies, etc. This target is mainly implemented through supplying teachers with video cases that model how teaching the new curricula, or using the pedagogical strategies, may be enacted (e.g., Seago et al., 2010). A second trend, becoming more and more prevalent (particularly in the US), is using videotaped lessons as a source for feedback and evaluation. Teachers watch videos from their own classrooms and discuss them with colleagues or instructors, often with the use of a pre-constructed standard-based rubric such as the one developed by Hill and colleagues (2008). The third trend is using videotaped episodes to enhance teachers' proficiency to notice, understand and discuss students' mathemat-

ical thinking (Sherin et al., 2011), usually in the form of “video clubs” (Sherin & van Es, 2009).

This paper brings forward a different direction for using video as a major resource for professional development of mathematics teachers, as emerges from a new project in Israel, named VIDEO-LM (Karsenty & Arcavi, 2014). In what follows, we describe the project and introduce the framework of analysis which lies at its core. We then report on a study conducted as part of the project, investigating the development of new Mathematical Knowledge for Teaching (MKT) in a group of teachers who participated in VIDEO-LM sessions.

THE VIDEO-LM PROJECT

VIDEO-LM (Viewing, Investigating and Discussing Environments of Learning Mathematics), is a project launched on 2012 at the Weizmann Institute of Science. Its over-arching goal is to improve mathematics teaching, with particular emphasis on the advanced tracks in secondary schools, through enhancing the reflective skills and the mathematical knowledge of teachers. The means to achieve this goal is by creating a pool of videotaped mathematics lessons, which serve as a basis for guided peer discussions with teachers. We use the lessons as “vicarious experiences” for teachers, centering on how the filmed teacher displays multifaceted elements of practice. The videos we use do not necessarily display ‘exemplary teaching’; rather, we pick lessons that can potentially trigger fruitful conversations. As opposed to the first trend noted above, teachers are not presented with demonstrative videos focused around new materials or strategies, nor do they engage in evaluative discourses, as in the second trend. They may relate to students’ thinking, the aspect centralized in the third trend, but only as part of the whole “teaching picture” revealed on the screen. In other words, the discussions are intentionally *teacher-centered*, and not *student-centered*.

Rationale

It is well known that teaching can be a lonely profession. Despite participation in professional communities, online forums and other forms of communication and collaboration with other teachers, the reality is that the vast majority of teachers are the “solo adult actors” in their classrooms, where they spend the lion’s share of their professional life. In many countries teachers seldom get the chance to watch their peers

in action once the pre-service period is over. This is not merely a social deficit, but also a barrier to certain processes of professional evolutions embedded in peer learning *in situ*. For instance, watching peers may expose teachers to alternative instructional strategies, which makes it possible to change routine thinking and actions (Santagata et al., 2005; Sherin, 2004). Thus, the VIDEO-LM project aims at creating opportunities for teachers to watch whole lessons given by others. Moreover, we seek to enhance the potential gains from these opportunities, by directing teachers to collectively analyse these vicarious experiences through a systematic use of a 6-component framework.

The framework for analysing videotaped lessons in teachers’ discussions

We have developed a unique framework for analyzing videotaped lessons, inspired by the work of Schoenfeld (1998) and Arcavi and Schoenfeld (2008). Schoenfeld’s theoretical model of “teaching in context” describes and predicts how teachers’ goals, knowledge and beliefs affect their in-the-moment decision-making during lessons. Arcavi and Schoenfeld have taken this model as a basis for creating analytical tools with which mathematics teachers can reflect upon their own practice while watching videotaped lessons of other teachers. In the VIDEO-LM project, we have modified and extended these analytical tools to include six components, which are the building blocks of the framework we use. In the following, we briefly describe these six components.

- 1) *Mathematical and meta-mathematical ideas.* Given the topic of the lesson, there is a range of relevant concepts, procedures and ideas that may be associated with this topic. For instance, the topic of the square root function may involve the following ideas: the non-negativity of the function’s domain; its monotonously increasing graph; continuity and derivability of the function; its relation to the function $y=x^2$; and so forth. Topics may also evoke meta-mathematical ideas, such as what makes a proof legitimate, why is one counter example sufficient to refute a conjecture, the arbitrariness of certain mathematical definitions, etc. Before watching a videotaped lesson, teachers are requested to elicit ideas, in an attempt to gauge the boundaries of this range. Then, once the tape is screened, they can refer to questions such as: Which of these ideas, or oth-

ers, did the teacher bring forward in the lesson? Which ideas were left out? How can this decision be explained? Which meta-mathematical notions were evident in the lesson?

- 2) *Explicit and implicit goals.* The rich span of mathematical ideas around a given topic enables choices of the goals teachers wish to pursue within a lesson. One of the reasons that lessons of different teachers on the same topic do not resemble one another is that teachers derive different goals from the range of relevant mathematical ideas. While watching a video, teachers try to identify the goals they think the filmed teacher was attempting to achieve, whether explicitly or implicitly. In other words, they ascribe goals to the teacher, just as one would ascribe meaning to a poem or some other piece of art. In this context, our aim is not to scientifically verify any “true situation” (i.e., what were the teacher’s “real” intentions); Rather, we encourage the mental exercise of ascribing goals, targeted at (a) promoting the skill of articulating goals; and (b) enhancing awareness to the fact that alternative (sometimes even competing) goals to teaching a certain mathematical subject may exist.
- 3) *Tasks and activities selected by the teacher.* The tasks, problems and activities presented by the teacher during the lesson are the means by which the teacher’s goals are fulfilled, hence reflecting the mathematical ideas chosen by the teacher. The video enables teachers to watch a “task in action”; how it is implemented, the nuances in introducing it and how the teacher addresses students’ reactions. This enables quite a different exploration than the one teachers may preform when presented with the task in its written form (i.e., as it appears in textbooks or other written resources). We refer to such an exploration as an *a posteriori task analysis*, which can potentially enrich the discussion, giving an additional angle to that of the *a priori task analysis*.
- 4) *Interactions with students.* The implementation of the tasks and activities selected by the teacher is carried out through classroom interactions. This component includes generic elements such as positive and negative feedbacks given by the teacher, listening to students, wait time, etc., but also considerations that are more related to the subject matter, for instance how the teacher navigates the students’ responses during the mathematical activity and poses subsequent questions. Following Clarke (2014), questions of equity, authority and knowledge construction are also valuable as triggers of productive conversations: Who gets permission to speak? Who is responsible for the flow of ideas? Is the mandate to produce new knowledge distributed or centralized?
- 5) *Dilemmas and decision-making.* The mathematics education community has learned much about teachers’ decision making processes, from the work of Schoenfeld (1998; 2008) and others. However, for many teachers the “diving” into another teacher’s decisions is a novel experience. The exercise offered to teachers participating in the discussion is to focus on the filmed teacher’s dilemmas as they may be uncovered in the lesson, the decisions taken in order to resolve these dilemmas, and their consequent tradeoffs. The risk is drifting into criticism and judgmental talk, a problem pointed out in the literature on video sessions (e.g., Coles, 2013; Jaworski, 1990). To avoid this, teachers are guided to consider the choices made by the teacher under the assumption that she acts in the best interest of her students (Arcavi & Schoenfeld, 2008). Taking this as a starting point, the constraints and affordances of the teacher’s choices can then be examined, and alternative paths can be elicited and explored.
- 6) *Beliefs about mathematics teaching.* The issue of how teachers’ beliefs shape their practice has been widely studied (e.g., Li & Moschkovich, 2013; Schoenfeld, 1998). In fact, all the components (1) through (5) above are likely to be guided by the set of beliefs the teacher brings into the classroom. Facilitating discussion about beliefs is a highly complicated and delicate matter. However, we suggest that such a conversation can be valuable. Teachers are not always aware of messages they convey during mathematics lessons, through direct or latent communications, nor of their considerable influence on how students perceive the domain of mathematics and how they function during the lesson. Thus there is a potential gain in the exposure to explicit and implicit attitudes reflected in another teacher’s actions. The discussion focuses on questions such as: What

may be the filmed teacher's views about the nature of mathematics as a discipline? How does the teacher perceive her role? What may be her ideas about what "good mathematics teaching" is? What does she think about the students' role as learners?

THE STUDY

The framework of analysis described above was implemented by the VIDEO-LM team in several courses with in-service teachers. As described earlier, and in line with Sherin's review (2004), different video-based programs set different learning goals for discussions with teachers. Ours was a two-folded goal, strongly linked to two agendas that we found valuable. One is the need to promote teachers mathematical knowledge for teaching (MKT), as defined by Ball and colleagues (2008). The other is the important move from a judgmental or evaluative discourse about the mathematics teaching profession towards a reflective and more constructive discourse, as advocated for instance by Jaworski (1990). In this paper, we limit our focus to the first goal only. We report on an exploration conducted on a group of teachers, who experienced peer discussions using the VIDEO-LM framework. The research question was defined as follows:

- What may be the gains of video-based peer discussions around the VIDEO-LM framework, in terms of the teachers' MKT?

Design, data collection and data analysis

During 2013–2014, a group of teachers participated in VIDEO-LM workshops, as a pilot for a professional development program conducted later on with other groups. The group met once a month throughout the 2013 academic year, and continued to meet monthly, with some change in members, during 2014. Each session lasted about 4–5 hours, a total of 60 workshop hours. For every session we used one videotaped lesson (45 minutes on average) with various modes of watching implemented (e.g., watching together or in small groups, focusing on different components of the framework, watching the whole lesson uninterrupted vs. breaking it to selected episodes). About half the lessons used were filmed in high track high school classes (grades 10–12), and the others were junior high school lessons (grades 7–9). Data collection means included field notes taken during workshops,

video-documentations of all discussions, documentation of e-mail correspondences initiated by the teachers after some of the sessions, and questionnaires administered at the end of the course, focusing on the participants' views about the 6-component framework used in workshops. The content analysis performed on the collected data included (a) tracing all of the participants' utterances associated with MKT (i.e., unpacking mathematical concepts or relating to teaching these concepts); (b) grouping utterances into units of analysis that share similar ideas; (c) using the units to form "utterances maps" that convey the development of mathematical, meta-mathematical and pedagogical ideas throughout different parts of the sessions. This process is still ongoing; therefore the results reported herein are preliminary.

Subjects

The 2013 group comprised of 10 teachers, of which 7 continued to the second year in 2014, with 5 new members joining in. All participants were secondary school mathematics teachers with a teaching record of over ten years, and were well acquainted with the mathematics curriculum of grades 7–12. Nine of them were lead teachers, i.e., holding additional positions such as heads of mathematics departments in their schools, instructors, or principals. The group was diverse in terms of gender and sector (i.e., included religious and secular Jews, Israeli and Palestinian Arabs). None of the subjects had a prior significant experience with watching videotaped lessons.

Preliminary results

The data analysis revealed that discussions focused around the six components of the framework were rich in examples, insights and suggestions brought up by participants. The deep mathematical conversations during sessions and in subsequent e-mail correspondences, although not fully analyzed yet, point to the joint development of new mathematical knowledge for teaching, triggered by the videotaped lesson. We chose to present here two detailed examples, demonstrating processes of collective knowledge growth.

Example I: How do we define an inflection point? The videotaped lesson in this case was given in an 11th grade high track calculus class. The teacher explored with her students the concept of concavity of functions, leading to the definition of inflection points as points where the graph changes from concavity

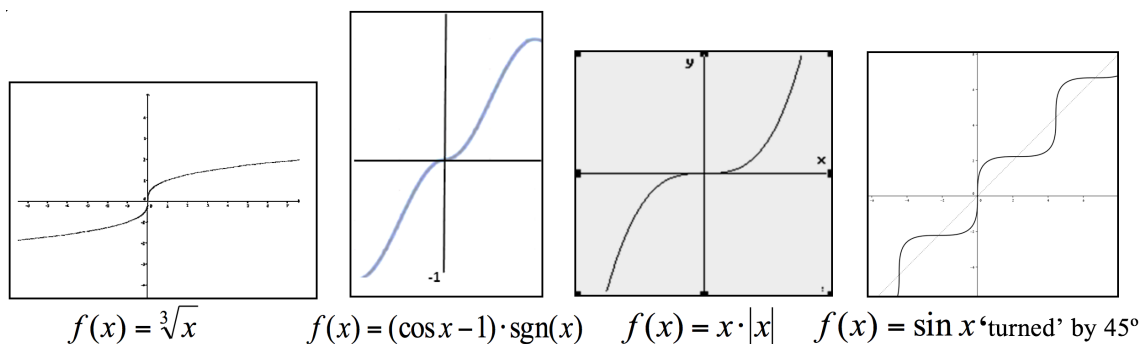


Figure 1: Teachers' generated examples

upwards to concavity downwards, or vice versa. This was then translated into a “working tool”, associating inflection points of $f(x)$ with the extreme points of $f'(x)$, or the zeros of $f''(x)$. Discussing the mathematical ideas introduced in this video, participants raised the following question: What about an inflection point where the first or the second derivatives do not exist? The group became motivated to find counterexamples where $f(x)$ has an inflection point in x_0 but $f'(x_0)$ or $f''(x_0)$ do not exist, and found a graphic example but not an algebraic representation of such a function. Following the session, in an intense and rich e-mail conversation, teachers found and shared different counter-examples, as described in Figure 1.

In all these examples, $f(x)$ has an inflection point in 0, but $f'(0)$ and/or $f''(0)$ do not exist. Furthermore, one teacher generalized that the product of $\text{sgn}(x)$ and any even function in which $f'(0)=0$ and $f''(0) \neq 0$ would be a suitable counterexample (e.g., $g(x) = (\cos x - 1) \times \text{sgn}(x)$).

As a result, the group reached a consensus about the accuracy of definitions of inflection points that are customarily presented in advanced calculus classrooms. This new collectively generated MKT was explicitly articulated by one of the teachers, as follows: “I think that everything we have seen so far shows that the correct definition of an inflection point is a point where the second derivative changes its sign, that is, there is an opposite sign in the neighborhoods before and after the point. The ‘usual’ definitions are incorrect – (1) a point where the second derivative is zero, and (2) a point where the first derivative has an extremum”.

The process of knowledge development also included valuable pedagogical ideas offered by participants, such as the idea to have students find on their own counter-examples to the “rule” that identifies inflection points with $f''(x)=0$. Another component of the

process evolved during the session, when the goals of the videotaped teacher were discussed. Participants attempted to justify the choice of the teacher to present an inaccurate working definition, by ascribing to her two major considerations: firstly, students may not be ready to grasp the correct definition, which requires advanced thinking, and secondly, left/right derivatives and functions such as $x \cdot |x|$ are not included in the curriculum and in the final exams. This part of the discussion opened a debate on a more general question, i.e., when is it legitimate to “sacrifice” mathematical accuracy for the sake of our students’ best interests?

Example II: Are the commutative and associative properties interdependent? In this case, the teachers watched an episode from a lesson on the commutative and associative laws, given in a 7th grade heterogeneous class. Prior to watching the video, they were asked to elicit any mathematical ideas that may be associated with this topic. They suggested a fairly wide range of ideas, from the simple fact that addition and multiplication satisfy both laws, while subtraction and division do not, through various models that demonstrate the laws, to efficient solutions of multi-term exercises using the laws. It appeared that most teachers perceived the topic as natural and intuitive for students, at least in the numerical level. Thus, the lion’s share of the discussion was dedicated to considering the general algebraic forms of these properties (e.g., $a+b=b+a$), and suggesting why and how they should be taught. Some teachers viewed the teaching of the algebraic generalizations as necessary for consolidating students’ intuitive knowledge, while others perceived it as a difficult goal to achieve in 7th grade.

In the video episode screened, the teacher asked the class whether operations that satisfy the commutative law necessarily satisfy the associative law as well, and vice versa. The students’ spontaneous collective

Verbal description:	Operation #1: The operation on a given pair of numbers returns the <i>first</i> number in the pair.	Operation #2: The operation on a given pair of numbers returns the <i>larger</i> number in the pair.	Operation #3: None																																									
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Associative law	✓	✓	✗																																									

Figure 2: The operations discussed in the 'commutative and associative laws' episode

answer was “yes”. The teacher then introduced three examples of mathematical operations (see Figure 2), and led a discussion about which property exists in each example, resulting in the conclusion that the properties are not interdependent.

Each pair of teachers was requested to focus, while watching the episode, on one of the components of the analysis framework. Then, in the plenary, findings were described by the teams and discussed by all participants. According to our goal, we focus herein on the mathematical ideas which were elicited and discussed. It should be noted, however, that these ideas were triggered not only by the first component ('mathematical and meta-mathematical ideas'), but also when discussing the videotaped teacher's goals, choice of tasks, interactions with students and beliefs.

On the whole, teachers were surprised by the episode, since the main mathematical idea they have noticed was not included in the span of ideas constructed by the group earlier: they described it as “undermining the perception that an operation can either satisfy both the associative and commutative laws, or none of them”. The teachers used representations from set

theory to express this idea (see Figure 3), noting that addition and multiplication are in the intersection of the commutative operations and the associative operations sets, while subtraction and division are in the complement of the union of these sets. While students might hold the misconception that the other possible two sets are empty, the lesson demonstrates that operations exist in all possible sets.

A major discussion evolved around the use of operation tables to exemplify operations that satisfy only one of the two properties. Some teachers asserted that operations on small finite groups are not equivalent, both mathematically and pedagogically, to operations defined on the real numbers. The discussion facilitator asked the teachers to consider the advantages and disadvantages of the teacher's choice of operations. The advantages offered by participants were that (a) these examples clearly serve the teacher's apparent goal – challenging what students erroneously perceive as obvious; (b) the process of considering these operations and checking which properties they hold may contribute to the development of critical thinking, which is another goal that can be ascribed to the teacher; (c) using such examples conveys that operations



Figure 3: A teacher presenting the mathematical idea of the episode using set theory

can appear in different contexts, for example, real numbers, finite groups, or height of students (which was one way the teacher used to illustrate operation #2), and furthermore, demonstrate that operations can be defined for one's needs. On the other hand, the disadvantages mentioned concerned the same fact that the operations were "made up", i.e., somewhat artificial, and in 2 of the 3 examples were defined only on 3–4 objects. This was viewed by some teachers as limiting, irrelevant to the students' prior or subsequent knowledge and therefore unconvincing; they opined it was problematic to generalize from these examples that the laws are not interdependent. Thus, the teachers were challenged to find an operation, defined on the real numbers and relevant to students' school learning, for which only one of the properties holds. They have eventually found such examples: $a \square b = (a + b)^n$, $a \square b = \sin(a + b)$ and $a \square b = |a + b|$. In all these cases the operation satisfies the commutative law but not the associative law.

CONCLUDING WORDS

The instructional practice for using video with teachers is still underdeveloped (Ball, 2014). Despite a notable progress in this field, essential questions such as how to design and facilitate effective discussions need further exploration (Coles, 2013). This paper reports on a pilot work that may contribute to the development of such practice, by using a unique framework of analysis in video-based teacher discussions. The framework is deeply rooted in the subject matter of mathematics, thus discussions are perceived, alongside their role as promoters of reflective skills, as opportunities to deepen mathematical knowledge. The collaborative discussions support teachers' attempts to unpack the practice observed on the screen, through the implementation of mechanisms such as ascribing goals and weighing alternatives. The two examples presented clearly demonstrate the potential contribution of such video-based discussions to the evolution of a rich and multifaceted mathematical knowledge for teaching. We hope that the continuation of our studies, exploring teachers' use of this framework, will amplify the understanding about the nature and impact of this process.

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