The interplay of factors involved in shaping students' opportunities to engage in argumentative activity in the mathematics classroom

Ruhama Even
Weizmann Institute of Science
ISRAEL

Improving students' opportunities to learn mathematics

"Conventional" approach:

- New curriculum
- Professional development for teachers

Interactions/interplay

- curriculum
- teachers
- classrooms

= "noise".

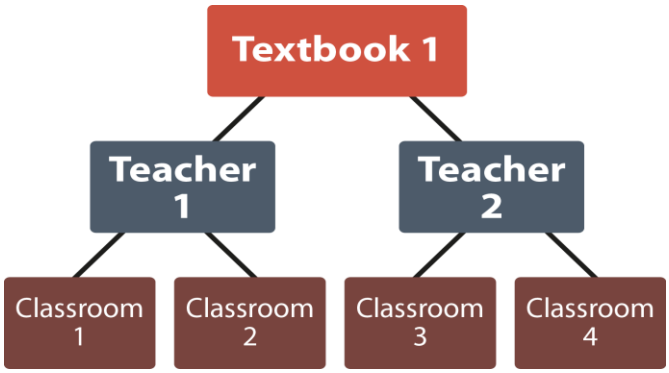
Research program:
Same Teacher – Different Classes

- Examination of how students' opportunities to engage in mathematics are shaped by the interplay of characteristics of the textbook, the teacher, and the classmates.

Even (2008, 2014), Even & Kvatinsky (2009, 2010), Eisenmann & Even (2009, 2011), Ayalon & Even (2015, in press)

Research design

- Case studies



Main findings

Differences in the mathematics addressed in class, and in students' opportunities to engage in mathematics:

- **Same** textbook, **Different** teachers, **Different** classes.
- **Same** textbook, **Same** teacher, **Different** classes.

Factors shaping students' opportunities to engage in argumentative activity

Ayalon & Even (in press)

Argumentation

- Justifying claims, generating conjectures and their justifications, evaluating arguments…
- Central to doing mathematics.

Argumentative activities

- encourage students to explore, confront, and evaluate alternative positions, voice support or objections, and justify different ideas and hypotheses.
- have the potential to promote the learning of mathematics.

(e.g., Douek, 1999; Schwarz, 2009; Yackel & Hanna, 2003)

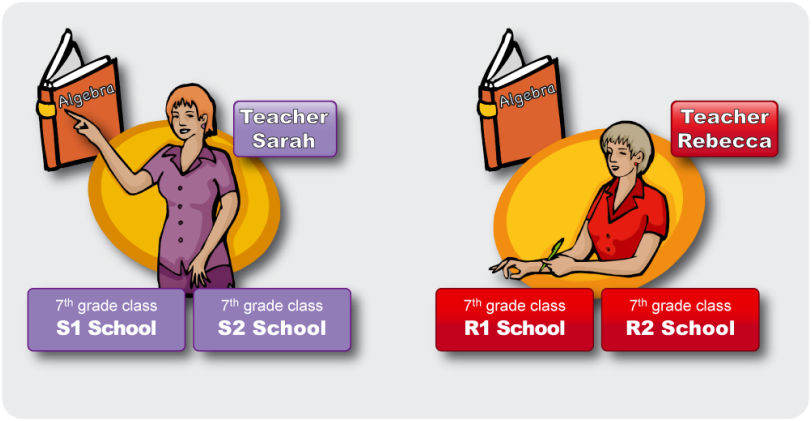
Research goal

To examine how classroom argumentative activities are shaped by the textbook, the teacher and the class.

Research question

How do classroom argumentative activities vary between two classes taught by the same teacher using the same textbook?

Research design



The teachers: Similar resources

- The same textbook.
- Both teachers followed closely the lesson plans suggested in the textbook, and mainly used textbook tasks and activities.

The teachers: Different teaching approaches

Sarah – direct teaching:

- Provided explicit explanations of central mathematical ideas and solutions of key tasks.
- Did not let students play a significant role in the development and evaluation of mathematical ideas.

Rebecca – inquiry-based no-telling teaching

- Rarely provided explicit explanations of mathematical ideas or solutions of tasks.
- Encouraged students to propose their own ideas.
- Followed-up on students' suggestions with questions.
- Seldom provided explicit appraisal of students' work.

The classes: S_1, S_2, R_1, R_2

Sarah's classes

S_1: Active participation of students.

S_2: Lack of student participation and frequent disciplinary problems.

Rebecca's classes

R_1: Cooperative, highly motivated students.

R_2: Active participation of students; some difficulties with the mathematics.

Data sources*

- Video and audio tapes of the teaching of two textbook units on *equivalence of algebraic expressions* (2-3 lessons in each class).

And also -

- Interviews with the teachers.
- Field notes.

*Collected as part of Eisenmann and Even's (2009, 2011) study.

Focus of the two textbook units

- Identifying, generating, and justifying equivalence or non-equivalence of expressions.

Proof-related ideas

- Substituting numerical values into expressions as a means to prove non-equivalence.
- Substituting numerical values into expressions as an inadequate means to prove equivalence.
- Expanding and simplifying expressions as a means to maintain/prove equivalence.

Examples of tasks

1. Find among the following pairs of expressions a pair in which the expressions are not equivalent:

   a) $2 \cdot m$, $m \cdot 2$ b) $1 \cdot m$, m

   c) $m-4$, $4-m$ d) $m+4$, $4+m$

   For each of the remaining pairs, find a property that shows that the expressions are equivalent.

2. Write equivalent algebraic expressions for the following expressions:

   a) $5 \cdot k$ b) $12 \cdot k + 8$ c) $10 - 4 \cdot k$

Data analysis

Unit of analysis: whole class work on a task.

S_1: 13 tasks (32 min).
S_2: 11 tasks (33 min).

R_1: 33 tasks (43 min).
R_2: 30 tasks (60 min).

- Almost all tasks were common to both classes of a specific teacher.
- Nearly all tasks worked on in Sarah's classes were also worked on in Rebecca's classes.

Data analysis: **argumentative moves**

Coding the teacher's and students' utterances according to their argumentative function within the whole class work on one task, differentiating between contributors (teacher or students):

- Claim
- Request for a claim
- Justification
- Request for justification
- Elaboration of justification
- Challenge for evaluating arguments
- Agreement
- Opposition
- Repetition
- Concession
- Approving

(e.g., Asterhan & Schwarz, 2009)

Example of a **sequence of argumentative moves** within work on one task in S_2

Contributor	Utterance	Arg. move
Sarah	[writing on the board $2 \cdot m$, $m \cdot 2$] Are they equivalent?	Request for a claim
Student	They are equivalent.	Claim
Sarah	Because of the commutative law. We have here multiplication, so we can change the order of the addends, and we will always get the same result. So if we switch the 2 and *m*, it doesn't matter at all.	Justification

Data analysis: **types of justification**

Coding the types of justifications raised in class:

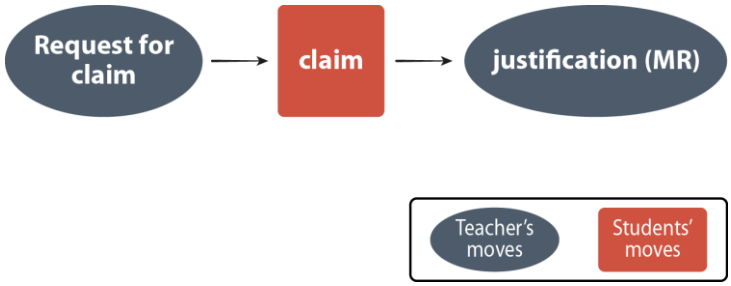
- MR – Justifications based on general mathematical rules (e.g., using the commutative law to show equivalence of two expressions).
- NE – Justifications based on numerical examples (e.g., substituting 3 in two expressions to check equivalence).

(e.g., Bieda, 2010)

Data analysis: graphical representation of argumentative **sequences**

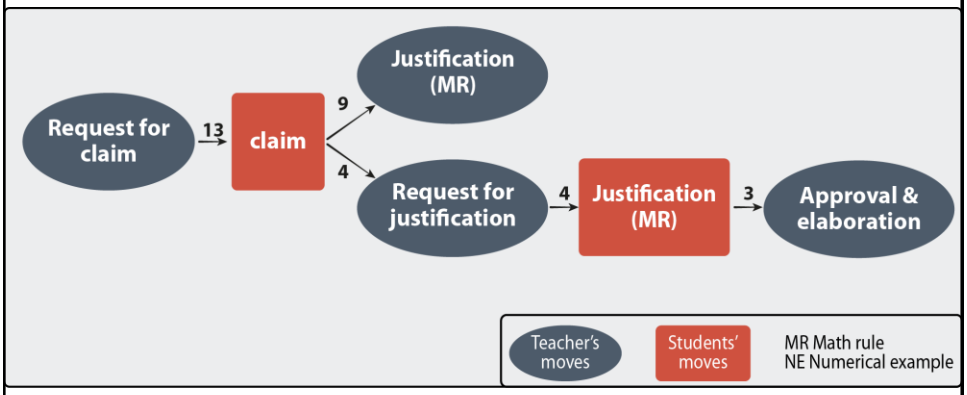
Displaying the sequence of argumentative moves, the contributors, and the types of justifications within whole class work on one task.

For example (S_2):



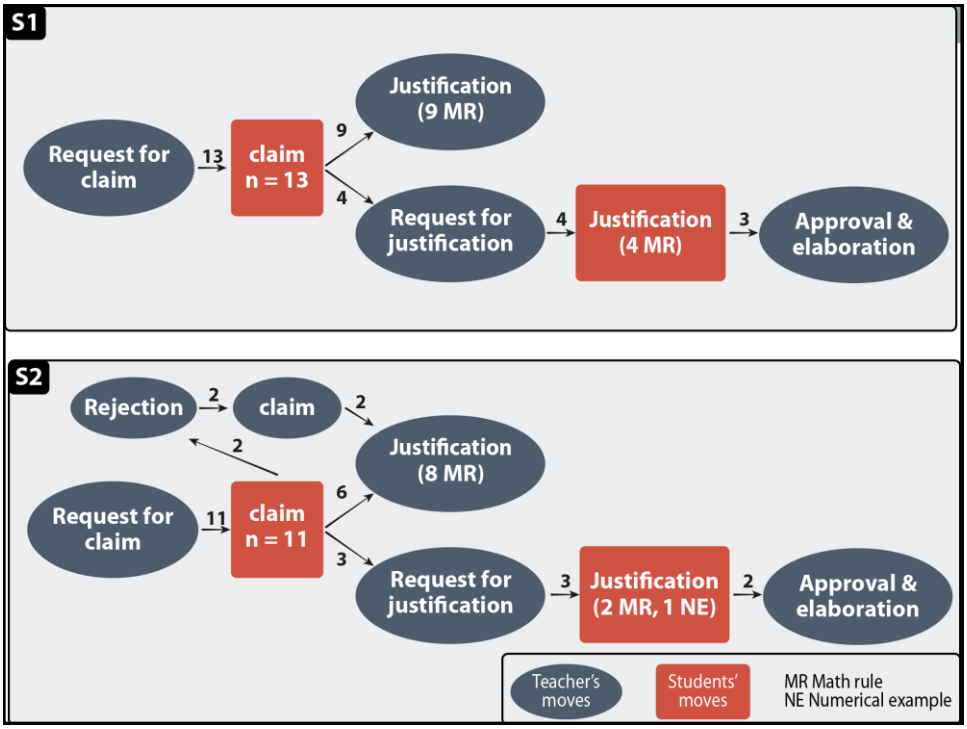
Data analysis: graphical representation of argumentative **schemas**

- Combining the graphical representations of the sequences of argumentative moves, for each class. For example (S_1),



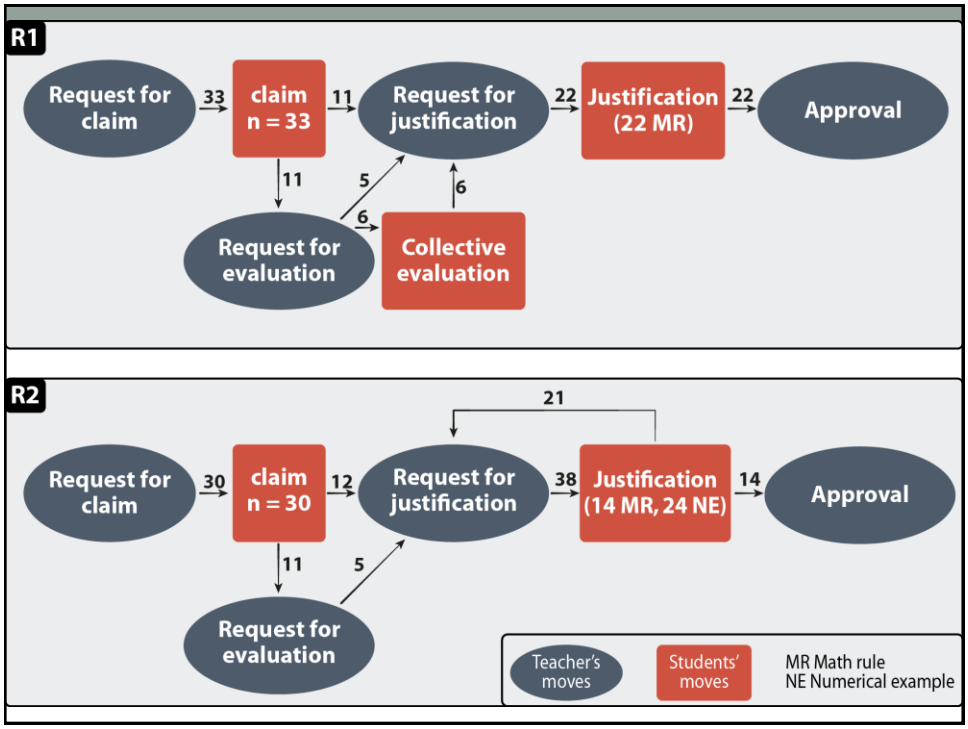
Data analysis: **comparing** classroom argumentative schemas

1. Overall structure.
2. Teacher's contribution.
3. Students' contribution.
4. Types of justification suggested.



Argumentative schemas in Sarah's two classes

1. Overall structure: Similar.
2. Teacher's contribution: Similar.
3. Students' contribution: Similar.
4. Types of justification suggested: Similar.



R_1: Collective evaluation of a student's claim that $4 \cdot k – 2 – 8$ is equivalent to $10 – 4 \cdot k$

R: What do you think, is it okay?

S_1: No.

R: Why?

S_2: You have to do it the other way round.

S_3: Because 8 plus 2. Because it is plus 10.

R: Okay. 8 plus 2. And here?

S_3: Minus.

S_4: No. Why minus four?

S_5: The minus refers to the 4, and then there is the commutative law.

S_4: Ah. Yes.

R_2: Teacher's pressing students for different justification that $x+7$ is equivalent to $8 \cdot x - 7 \cdot x + 7$

R: Is it equivalent to $x + 7$?
S: It is equivalent.
R: How can I prove that they are equivalent?
S: Substitute a number.
R: Okay, I will substitute a number. Which number?
S: 2.
R: [Substitutes…]. So maybe it is by coincidence? What do you think, should we substitute more numbers to prove that they are equivalent? I can try to substitute more numbers into x. What do you think?
S: Try it.
R: Try it how many times?
S: Try substituting 7.
R: OK, I will substitute 7… So is this by coincidence?

Argumentative schemas in Rebecca's two classes

1. Overall structure: Similarities and Differences.
2. Teacher's contribution: Similarities and Differences.
3. Students' contribution: Similarities and Differences.
4. Types of justification suggested: Different.

Students' opportunities to engage in argumentative activity

- Same textbook
- Similar lesson plans
- Same teacher in two classes

S1 vs. S2	R1 vs. R2
Similar	Similar
Similar	**Different**

Similarity between Sarah's two classes in students' opportunities to engage in argumentative activity

Teaching approach

Direct teaching

Students' participation

No significant contribution

Difference between Sarah's two classes in students' opportunities to engage in argumentative activity

Teaching approach

Inquiry, no-telling, no appraisal of students' suggestions

Students' participation

R_1: Adequate means to prove equivalence.

R_2: Inadequate means to prove equivalence.

Factors involved in shaping students' opportunities to engage in argumentative activity

- Textbook
- Teacher
- Classmates
- (Math topic)
- …
- Their interplay…

Future potential collaboration

- How students' opportunities to engage in math are shaped by the interplay of textbook, teacher, and classmates.
- Implications for curriculum development and teacher education/development.
- Potential research design: