

The contribution of academic mathematics studies to teaching mathematics in secondary schools: The case of knowledge about the discipline of mathematics

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by

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Summary

In many countries, including Israel, secondary mathematics teacher certification programs involve academic mathematics courses. The underlying assumption is that these courses contribute to the mathematical knowledge needed for high quality teaching. However, empirical research, while scarce, shows that teachers often do not see a significant contribution of academic mathematics courses to their teaching practice.

The current study investigates the relevance and contribution of academic mathematics courses, taught by research mathematicians, to secondary school mathematics teaching. The study examines this topic from the perspective of

knowledge about the discipline of mathematics. Unlike knowledge of specific mathematical content, this is knowledge about knowledge, also called meta-mathematical knowledge or epistemological knowledge about the discipline.

The study consists of two parts. The first part focuses on mathematicians who teach academic mathematics to mathematics teachers and on the question what these mathematicians want to teach teachers about the discipline of mathematics. The second part focuses on mathematics teachers who learn academic mathematics from mathematicians and on the questions: what these teachers learn about the discipline of mathematics in academic mathematics courses; and how this knowledge contributes to their teaching.

The research is situated in the mathematics track of the Rothschild Weizmann program. It is a unique Master's program intended for experienced secondary mathematics teachers. A uniqueness of the program is that about half of the courses (eight courses) are devoted to high-level academic mathematics studies (including postgraduate mathematics content) taught by prominent research mathematicians.

It is a qualitative study based on semi-structured interviews. In the first part, I conducted interviews with five mathematicians who teach in the Rothschild Weizmann program. The purpose was to find out what, if at all, they want to teach teachers about the discipline of mathematics in the academic mathematics courses. This part of the study used grounded theory methods. In the second part, I conducted interviews with 14 teachers who graduated from the program. The purpose was to find out what, if at all, the teachers learned about the discipline of mathematics in the academic mathematics courses and how this knowledge contributes to their teaching. I analyzed data using content analysis methods.

The part of the study focusing on the mathematicians suggests that expanding teachers' knowledge about the discipline of mathematics was one of the participating mathematicians' main objectives. They referred to nine characteristics that can be grouped into three key aspects: (1) the essence of mathematics, (2) doing mathematics, and (3) the worth of mathematics. Based on these results, a conceptual framework was developed for the contribution of academic mathematics courses to secondary school mathematics teaching with respect to knowledge about the discipline of mathematics.

In the second part of the study, I analyzed the teacher interviews using the conceptual framework that was developed in the first part of the study. I found that there was a certain congruence between what mathematicians said they wanted to teach teachers about the discipline of mathematics in academic mathematics courses and what teachers said they learned about it. Teacher reports were related to all aspects and characteristics of the analytical framework, and no other aspects or characteristics were identified. However, while all of the participating mathematicians addressed all the three aspects and almost all the characteristics (except for one mathematician who addressed only seven of the nine characteristics), a diversity was found in the aspects and characteristics that were identified in different teacher interviews, and some characteristics were identified in more interviews than others .

Almost all the teachers who participated in the study (13 out of 14) reported the contribution of academic mathematics studies to their knowledge about the discipline of mathematics and changes in their teaching that concerns their new knowledge. In particular, in almost all cases (54 out of 57) where a report of a contribution to knowledge was identified in relation to some characteristic of knowledge about the discipline of mathematics in a certain interview, a report of a contribution to teaching related to that characteristic was also identified. In general, teachers reported changes in their teaching that present mathematics more in line with the way the interviewed mathematicians saw the discipline.

Additionally, the study showed that not all teachers attributed the same importance to the contribution of academic mathematics studies to their knowledge about the discipline of mathematics and teaching related to this knowledge. In most cases there was a connection between the attributed degree of contribution and the learning experience in the academic mathematics courses.

This study contributes to the research literature in a subject where empirical research is limited, namely the contribution of academic mathematics studies to teaching. It provides a conceptual framework describing what might be the benefit of such studies to teachers' knowledge about the discipline of mathematics. Based on empirical findings, the research presents a new and comprehensive view of the role of academic mathematics studies in teaching in the case of practicing secondary school teachers. This research might be of interest to education researchers, mathematicians, mathematics teachers and their educators, and policy makers.

