Reading Mathematical Explanations

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Abstract

In this thesis, I investigate the ways in which students deal with the reading of mathematical explanations while solving a problem. I also examine whether the students use what they read, when solving similar problems.

The idea for this research originated from the SHAINET project, which was carried out in 2001 at the Department of Science Teaching of the Weizmann Institute of Science. The goal of this project was to help high school students who are studying mathematics at the three units level to overcome difficulties in their study of mathematics. An e-mail address was established with the aim of helping the students to fill the gaps in the studied material or to prepare for exams. For this purpose SHAINET team members answered the questions e-mailed by the students. Since e-mail communication requires reading without instructor guidance and since not much research has been carried out about such reading, I decided to investigate how students deal with mathematical problems in conditions similar to those encountered in the project.

During the project, criteria for formulating an e-mail solution to a problem were defined. These criteria follow both, the pedagogical approach of the project and the environment of e-mail communication. The conditions of the e-mail environment are

- No information is available about the students and their prior knowledge and experience besides the information reflected in the formulation of the question.
- In most cases, there was no feedback by the student, that is there was only a single opportunity to write to the student .

These conditions brought about the need to investigate whether and how the written material sent to the students supported their learning. For this reason a suitable problem from the three-unit level calculus course was chosen and two solutions for it were written. Both solutions satisfy the SHAINET criteria but they differ with respect to the activity required from the student. The first solution is explanatory and does not require the students to actively produce part of the solution. The material is presented in a continuous manner and includes all stages of the solution. The second solution is explanatory as well but does require student activity. It includes questions, which the students have to answer in the process of reading and as a part of this process. The effect of these two types of solutions was investigated in individual meetings with three-unit level students. The students were first asked to solve the

problem. If they didn’t succeed, they received one of the two types of solution and were asked to work through it without intervention on the part of the researcher. The next part of the meeting consisted of an interview requiring the solution of similar problems and the identification of their relation to the solution that the student had read.

Twenty-two students participated in the research. It was found that the students read the first type of solution only cursorily and then frequently consulted it when solving the similar problems; on the other hand, the students read the second (active) type of solution carefully and in detail and referred back to it without needing to consult it when solving similar problems. Most students, especially those who received the second type of solution, effectively used the reading for the solution of similar questions. Likewise, the students emphasized two elements of studying that were prominent in the solution they had read: The connections between various representations of the problem and the reasonableness of the obtained solution. Details of the research and its results as well as conclusions and educational implications are presented in this thesis.