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Some pedagogic applications of the calculator in the junior-high school mathematics curriculum

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Introduction

There seem to be two major ways of using the calculator in the mathematics classroom at any level, a functional use and a pedagogical use (Etlinger, 1974; Gawronski and Coblentz, 1976). There are correspondingly two types of articles in the gradually increasing literature on the subject. In addition there is a third type of article which reviews, exhorts or worries about the present state of the art. The articles in this third category often refer to the lack of useful pedagogical calculator material (Bell (1976 and 1978), Pollak (1977) and Fielker (1979)).

The pedagogical use of the calculator can express itself in a number of ways. Thus it can have a didactic impact on material already within the curriculum, changing the mathematical approach or the teaching strategy. Alternatively, it can be used to enrich the curriculum by including new topics or advancing topics which were previously taught later. These ideas are discussed in several places and, in particular in the NIE/NSF report (1977).

The purpose of this paper is to describe some work done within an on-going junior-high school curriculum development and implementation project, whose purpose was to develop and test materials which would use the calculator in a non-trivial way. The materials developed substantially enrich a pre-calculus course, as we shall indicate in the following. The facility to do extensive numerical calculations provides totally new possibilities at a relatively early age, and the motivation for the introduction of algorithmic methods.

The curriculum materials within this project are subject to a process of gradual development and intensive implementation procedures (Bruckheimer, 1979). It is thus possible not only to create new materials, but to gradually integrate them into the regular curriculum and to ensure adequate teacher training (see recommendation 18 in the NIE/NSF report).

Before we describe some of the major features of the calculator material we developed, one general point about its structure is worth making here. In general, even where calculators are freely available, in the absence of suitable materials, they tend to be used in the most trivial arithmetic ways only, and often far from efficiently. We therefore designed material which develops two themes in parallel: actual calculator skills (that is, how make full and efficient use of the calculator) and, at the same time, material with mathematical content which enriches the present curriculum (see Table 1).

Table 1 Overview of the structure of calculator materials

Chapter	Calculator skills	Position in existing curriculum
1. Approximation and Errors	First acquaintance with the calculator: four basic arithmetic operations only.	After a first discussion of rational and irrational numbers.
2. Graphing functions	Systematic development of calculator skills: efficiency.	As a continuation of the chapter dealing with the graphs of simple functions.
3. Approximating roots of equations	Organisation of calculations: application of skills acquired.	Before or after solution of quadratic equation (see discussion in text).

Description of the calculator materials

In the following we describe the major features of the three chapters comprising the materials, discuss the background reasoning which directed their development and some experience of their use in the classroom.

1. Approximation and Errors

In the original curriculum materials for Grade 8, there is a section devoted to rational and irrational numbers, the conversion of rational numbers from fractional to decimal form and vice-versa, recurring decimals, etc. This was our point of entry. We started with a discussion problem:

Three students were asked to cut a copper wire of length 1 m. into 3 equal pieces. Before cutting they calculated and wrote down the length of each piece.

Joe wrote	*33.333 cm.*
Fred wrote	*33.33 cm.*
Malcolm wrote	*33.3 cm.*

All three knew that the length of each piece is $33.\dot{3}$ *cm. They also knew that if a piece was measured using a ruler graded in cm and mm., one end would fall between two grade marks.*
Which of them gave the most realistic answer?

In the corresponding teacher guide we presented the background for the discussion including the origins of "error", both mathematical and physical.

The chapter continues with examples of errors arising from measurement and ways of recording them, leading to the concept of absolute and relative error, significant figures, etc. The exercises invite the use of the calculator but involve the four basic arithmetic operations only, so that the student can get thoroughly familiar with his particular machine.

Probably even more important at this stage is that the teacher does not get demoralised. In our experience, there is usually more than one type of machine in use, and the teacher will need to be in a position to advise on all of them. The problems of the teacher in curriculum innovation are crucial to success or failure and we shall have more to say on the point, as it effects the project being described, as we go on. Here, we just remark that, on average, the written teacher guide is more than twice as long as the student materials, and this is reinforced by workshops, individual guidance, etc. all aimed at the teacher.

We found that all the chapters, not only enriched the curriculum in the sense that they introduced concepts meaningfully, which had previously not been included, but they enriched the already existing material. Thus, this chapter on approximation and error, caused the student to consider again irrational square roots, rational numbers whose decimal form is non-terminating, etc. This deepened the understanding of real numbers and the various "equivalent" forms of writing the same number.

2. <u>Graphing functions</u>

The topic of graphing functions is severely restricted in the regular curriculum, because of the excessive calculation involved in the graphing of any but the simplest functions.

On the other hand, the graphing of more implicated functions does not require the acquisition of new mathematical concepts, thus making it an ideal topic for learning to use the calculator efficiently. It can also give a certain aesthetic pleasure, and one or two "surprises".

The major problem in the design of this chapter was how to "teach" the proper and efficient use of the calculator, given that we could not rely overmuch on the teacher, and all sorts of machines would be in use. We decided to restrict the problem somewhat by advising on the use of machines using algebraic notation. Then we began the chapter by posing the first exercise

"Draw the graph of the equation

$$y = x^3 + 2x^2 - x - 2 ."$$

The student text is designed in a set of carefully graded steps, so that the student can work his way through with minimum teacher guidance. The first step here was to make a table of integer values of x and the corresponding y, plot the points and decide where we lacked sufficient information to sketch the graph with confidence. To find the values of y for intermediate values of x invites the use of the calculator. This use involves a number of small points such as the use of the y^x button to calculate $(-1.5)^3$, for example. On some calculators, if we start with -1.5, the error signal appears. Or the fact that $(1.5)^3$ can be calculated on many calculators by the sequence

1.5, x, = , =

All these points can be covered by using the manufacturers handbook (if available in a language the students and teachers readily understand, which is not the case for our schools), but more often than not the handbook has got lost, the point does not sink in when there is no immediate use for it, etc. It seems to us better to deal with such points as they arise and are meaningful.

But, more important than these small points, is the necessity, now obvious to the student, to repeat the same sequence of calculations many times using different initial numbers. Efficiency is thus important. Hence the calculator gives us an opportunity to enrich the students' selection of mathematical strategies by introducing algorithmic methods in a natural way. Usually, at this stage, the introduction of algorithms is somewhat forced.

Whereas in the first chapter we suggested that the student needs little help in using the calculator for the four basic arithmetic operations, here the use of the memory and the "algebraic" keys requires some guidance. We, therefore, presented a scheme for the calculation using a stylised version of a calculator as in Fig. 1.

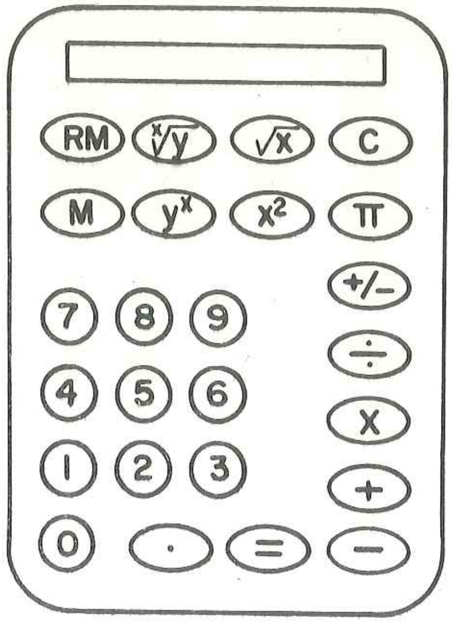

fig.1.

The beginning of the scheme is as follows:

Operation scheme		Typical result (with x = -1.8)
1. Enter absolute value of x		1.8
2. Raise to power 3	(y^x) (3) (=)	5.832
3. Change sign, if necessary	(+/-)	-5.832
4. Store in memory	(M)	

Etc.

The student is invited to follow through the scheme and then to write a scheme for his own machine which is as short as possible.

The corresponding written teachers' guide is of necessity extensive and deals with such points as types of machine, organising calculations and efficiency, general concepts in drawing graphs and the qualitative investigation of a given function as an aid in graphing it.

If properly developed, a sequence of graphing exercises using the calculator, can be a useful preparation for the differential calculus: thus we touch on such topics as increasing and decreasing functions, determination of local maxima and minima, symmetry, etc.

As part of the commentary on the first exercise we give the equation in the form

$$y = ((x + 2)x - 1)x - 2$$

which is, in general, the most efficient algorithmic way to calculate any polynomial. After working the exercise and having struggled with the problem of efficiency, this "non-obvious" step has considerable impact and emphasises the value of using one's head before rushing into calculations.

The final exercise in this chapter is to draw the graph of

$$y = 2x^2 \pm \sqrt{1 - x^2}$$

for $-1 < x < 1$. (This equation gives two values of y for all x between -1 and 1, and does not represent a function.) The result is a beautiful closed curve; a fine reward for the student's labour.

(For further possible developments of this section see the Appendix to this paper. Suggestions for other graphing activities are given by Russakoff (1978).)

3. Approximating roots of equations

In the process of graphing functions, it is natural to look for points where the graph crosses the x-axis, that is to look for the zeros. This provides a natural introduction to the third and last chapter we developed for Grade 9.

The main method for approximating zeros of functions which is developed is interval bisection, which is also strongly intuitive. The concept of approximation as a method of solution is novel at this stage. Efficient organisation of the calculation is now extremely important and thus we begin to use flowcharts and introduce implicitly the algorithm concept. The two previous chapters have an important role in that we deal with accuracy and error of the approximation methods exemplified, and the graph is used as a visual guide to the methods, their relative advantages in efficiency and pitfalls. Thus the three chapters are brought to a unified conclusion.

The chapter can be integrated into the existing curriculum at two different points:

(a) After quadratic functions and the quadratic equation:
After learning to solve the quadratic equation, it is natural to consider the solution of other equations, especially polynomial equations of higher degree.

(b) After linear functions, but before quadratics:
The algebraic solution of the quadratic equation is a very special case within the overall subject of the solution of equations. On the whole, equations are solved by numerical methods, of which some are very intuitive. Therefore, it may be argued that the some of these methods might well precede the quadratic equation, which would then be presented as a special case for whose solution a formula exists.

These two arguments have many parallels. For example, in the teaching of the quadratic equation itself, whether to teach first the method by factorising (special method), or the formula (general method). Very often the teaching order is a semi-historical relic: thus the usual high-school or college curriculum considers first methods of integration using expressions for the indefinite integral and then numerical methods, whereas the reverse might be more reasonable. (Pollack (1977) discusses the subject of topic ordering within the curriculum as affected by the advent of the calculator. Some ideas relevant to the content of this chapter are contained in a paper by Henry (1977).)

Discussion and Conclusion

The material described above was tested in five classes (Grade 8 and 9). It's impact was evaluated by a pre-post test and a cognitive attitude test. The results were extremely encouraging; the students who took part did very well on the achievement test and indicated high motivation.

The main part - the cognitive attitude test was centred around the question whether students accept approximation as a legitimate part of mathematics. This test was also given to a control group and there was a significant difference in the responses. In general, the control group took the view that mathematics is an accurate science and hence there is no such thing as "approximately" in mathematics. The experimental group, on the other hand, tended to see no contradiction in approximation being part of an accurate science.

If properly used with properly designed materials and adequate teacher preparation, the calculator has much to offer in the mathematics classroom. We have here tried to show how it can be used to enrich the junior high mathematics curriculum, both in terms of content, didactics and the mathematical strategies available to the student (such as the concept of approximation and algorithmic methods).

Appendix

In the stage before we began work on writing the chapters, we designed some worksheets for teacher workshops which we also used subsequently as part of our in-service training program. The exercise to draw the graph of

$$y = 2x^2 \pm \sqrt{1 - x^2}$$

which was mentioned in the section on graphing functions above, appeared there in another form which is of more than passing interest, so we give the full text.

Worksheet 3

Given the equation of a curve

$$(y - mx^2)^2 = 1 - x^2$$

a) Use your calculator to draw the curve for two different values of m

$m =$, $m =$.

b) Use the points of the graph you have plotted to calculate approximations to the areas enclosed by the curves. (It is worth organising the steps in the calculation before you begin.)

c) Compare the two areas you found.

Before we distributed the sheets we filled in values of m, so that each pair of teachers working together got different values of m. By comparing one with another it was clear how m influences the shape of the curve.
Section b) was the interesting one. The teachers were invited to find upper and lower bounds for the areas of their two curves. This presented little conceptual difficulty even though many had not studied calculus. The surprise came when they came to compare their results, not only for their two values of m, but with those obtained by others - everyone had, in fact, found an approximation to π - the area is independent of m, and when $m = 1$, we have a circle of unit radius

Although the finding of such areas was not included in the material subsequently developed for classroom use, there is no doubt that the calculator makes this feasable at an early pre-calculus stage. That is, we can develop material which provides the intuitive basis for the semi-formal and formal integration theory, which comes later. This intuitive stage, which is so important for a properly designed learning sequence (Griffiths and Howson, 1974, pp.158-159) has rarely been given adequate treatment. The calculator can thus be given here a non-trivial didactic role.

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