FORMATION OF THE "VARIABLE" CONCEPT USING MATHEMATICAL GAMES

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Introduction

One of the problems in elementary algebra is the understanding of the "variable" concept and its role in mathematical models such as open phrases, functions etc. The problem is particularly serious among socially-deprived students, who have difficulty in grasping abstract explanations.

Most students, including socially-deprived, can correctly substitute a single number in a (simple) open phrase and find the image under a (simple) function. But, when the situation is slightly more complicated - for instance, given a set of numbers instead of just one, or when the inverse is required - the difficulties begin. They are noticeable even when we change instructions, such as when we ask for oral calculations instead of pencil and paper.

A fair amount of experience has accumulated in the use of games in school to overcome such problems. The main reason that makes the game useful in this context, is its potential to create an "external, concrete" situation. To take a simple example, given the open phrase 30-x, will substitution of a positive or a negative number always give a positive result? Instead of the abstract exercise, the student is given the same problem with two stacks of cards face down in front of him. He knows that one consists of positive and the other of negative numbers. In the first place, the very fact that the cards are there physically in front of him gives him a certain security. Also having guessed the answer, the fact that he can turn over and check his guess by substituting the number on the card, provides him with reinforcement. To understand better the contribution of the games we shall give a short description of five of them, all dealing with *open phrases (or functions)*.

Five games

1. The Steeplechase (2-4 players)

   This is among the most successful games we have developed (Friedlander, 1977). It consists of a board (Fig. 1), 4 runners and cards on which positive and negative numbers and zero are printed.

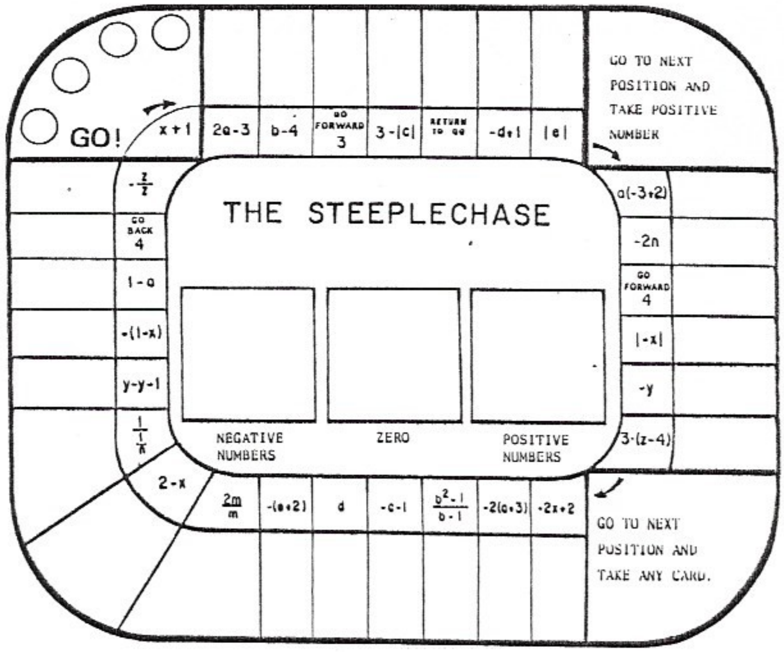


Fig. 1

The cards are placed face down in the appropriate places. Each player in his turn chooses a card from the pile which he thinks advantageous, according to the position of his runner on the board. He substitutes the number in the open phrase on which he stands and the result dictates his move: if positive he moves forward an appropriate number of places, if negative backward, and does not move if zero.

The winner is the one who first completes two circuits of the board.

This game can be played in several variations according to the level of the students (Ilani et al, 1982).

2. <u>Chip and Arrow</u> (2 players)

This game deals with the relation between a given number and the outcome of its substitution in an open phrase. The game contains a board (Fig. 2), 4 hexagonal chips with numbers (2, 3, -2, 1) which were used at the beginning of the game, and circular chips with various (well-chosen) numbers. The first player places one of the hexagonal chips on the hexagon on the board.

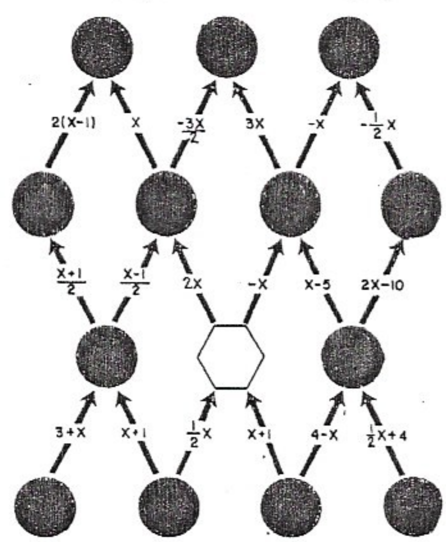


Fig. 2

Initially, each player receives 5 chips and moves by putting the appropriate number at the head or tail of an arrow, at the other end of which a number has already been placed. If a player does not have an appropriate chip, he takes from the bank until it is exhausted. The winner is the one who first places all his chips.

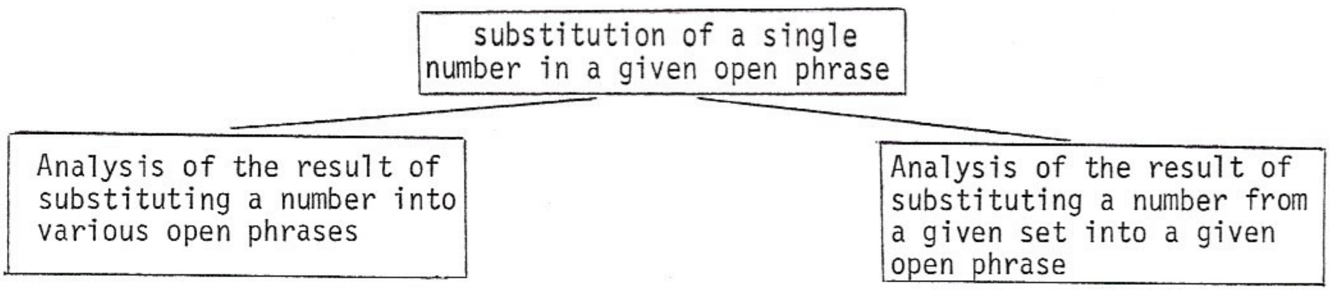
3. Bigger is Better: Mark I - (2 players)
   Each player is given a pile of cards with open phrases face down. There is another pile of cards with numbers, also face down, in the middle. At each turn, both players turn over one of their own cards and one of the number cards in the middle, each substitutes the number in his open phrase, and the one who gets the bigger result take the cards. The winner is the one who has the most cards at the end.

4. Bigger is Better: Mark II - (2 players)
   The game consists of 16 cards with open phrases and a die with numbers from -3 to +3.
   Each player takes 2 cards. The die is thrown and the players have to consider into which of their two open phrases to substitute to obtain the bigger result The player who has the bigger number takes the cards. The game continues until the cards are exhausted. The winner is the one with the most cards at the end of the game.

5. Bigger is Better: Mark III - (2 players)
   Here the role of the numbers and open phrases is reversed. The players hold numbers and have to choose which to substitute in a given open phrase in order to get the most advantageous result.

Discussion

The various aspects of the variable concept with which the games deal are illustrated in the following diagram.



Further, the games also deal with the converse problem, estimating the "source", given the result of the substitution, for the cases mentioned in the diagram.

All the games deal with the simple act of substitution of a given number in a given open phrase. Nevertheless, even in this simple aspect, the games are useful for weak students. If a number is written on a card the student can touch it and move it about. For example, in the game *Bigger is Better: Mark I,* we noticed that weak students have a tendency to put the dice physically on the card. This seems to provide the students with an intermediary stage between pen and paper exercises and oral exercises, and thus they gain confidence and "get a feeling" for the meaning of algebraic expressions.

In the *Steeplechase* and *Bigger is Better: Mark III* we have a situation in which the player has to choose from *a set of numbers*. In the latter the student has 5 cards, in other words a limited choice. It was interesting to observe that students were substituting less and less as the game progressed. First, they eliminated a number of possibilities by estimation or prediction. In the case of the *Steeplechase*, however, the player does not have any numbers in front of him, but has to choose a number from one of three piles, whose contents he knows but cannot see. In this case students who have difficulties, came to the conclusion by trial and error. Thus, after a little while they began to assume certain mathematical rules of their own, which in some cases were wrong. One such frequently recurring rule was, if a negative sign appears in the expression one must substitute a negative number in order to get a positive result. Therefore, in the case of $z - 4$ one must substitute a negative number!

In some cases we noticed a progressive development of these rules, wrong ones being corrected and others being modified in the light of experience.

In almost all observed cases the students learned from experience which was the "correct" pile from which to choose.

An interesting case was a student who deliberately picked a number from the "wrong" pile and said that he knew it was wrong, but he did not know how to handle negative numbers.

This latter example shows that the student did understand that he should choose a negative number, but lacked the technical skill, or confidence, to work with them.

The feeling of what an open phrase does to numbers is created in the *Steeplechase, Bigger is Better: Marks II and III,* and *Chip and Arrow*. In these games the students learn, while playing, the significance of "-" in front of the variable, or the influence of a number between zero and one as the coefficient of the variable. Again, in *Bigger is Better: Mark II* the student has 2 open phrases and if he cannot decide which of the two to use he can substitute in both, whereas in the other games he has only one open phrase in each case but he needs to make a decision which number of a set of numbers to use. This encourages the student to deduce "rules" of his own and to ask himself what is the effect of the open phrase.

Remarks such as "in b^2 it is better to substitute -3 than 1, in order to get a larger number" and similar remarks are evidence of decision making. Many such remarks were overheard. Another remark concerns the expression $3 - b^2$, where the student was interested in a positive result. "A pity I haven't a zero!" (What is interesting about this remark is that he did not say - a pity I have no negative numbers - as one might have predicted would happen.)

These sort of remarks, which changed and "corrected" themselves from stage to stage, are evidence of the fact that the combination of the possibility to make immediate checks and to "handle" the numbers, or, to relate to a given pile of numbers, leads the students from the level of pure technique to a higher stage of analysis.

The "reverse" direction is mainly treated in *Chip and Arrow*. Most of the time the game can be played in the direction of substitute ⟶ result. But sooner or later he reaches a more difficult stage, when the only unoccupied spaces are at the heads of arrows. In which case, he has to pick a number such that when substituted into the open phrase he will get the given number. Here we have a classical classroom difficulty in which the student muddles up the outcome with the number to be substituted. And this also occurred initially in the game. Even though the number was at the head of the arrow, all the students did was to substitute in the expression and place a counter correspondingly at the tail of the arrow.

Here we have another of the advantages of using games. The student who makes this mistake is almost inevitably corrected by his opponent. The latter has only to check (and usually does so) whether the number just placed, when substituted in the appropriate expression, gives the correct answer. This is, of course, a much easier task than that of the player whose turn it was. This gave us a situation in which either the students could correct themselves, as happened with the more able students, or the teacher could effectively intervene.

Essentially, the student faced with this situation in the game, has three possibilities. He can guess what number to substitute, check his guess and then see whether he has that number. He can use pencil and paper, write down the appropriate equation, calculate and find the number to substitute. And the simplest of all the possibilities to substitute each of the numbers in his possession and check whether one of them gives the desired result. Gradually, the players develop one of these techniques.

Conclusion

The results described were mainly obtained by observation and interviews with students across a wide ability spectrum, ranging from relatively very able (students) to extremely weak, and taking in socially deprived students. In all these groups we found that with an appropriate mix of games one could achieve at least partially, the following:

i - A pleasant way of practising basic algebraic skills, leading to mastery.

ii - A feel for simple algebraic expressions in the sense that, a student is aware of possible outcomes of various substitutions and the symbols thus having a meaning for him.

iii - The strategies which the games encourage in order to be played successfully, also encourage the student to go beyond the level of pure technical skills to the level of analysis.

Reference

Friedlander, A. "The Steeplechase", *Mathematics Teaching* (September 1977): 37-39.

Ilani, B., Taizi, N. and Bruckheimer, M. "Variations of a Game as a Strategy for Teaching Skills". In *Mathematics for the Middle Grades* (NCTM 1982 Yearbook): pp. 220-225.