THE EVERCHANGING (MATHEMATICS) CURRICULUM

Maxim Bruckheimer

With a compelling glance at the not-too-distant, but already quaint, past, Professor Maxim Bruckheimer, recently appointed Head of the Institute's Science Teaching Department and former Dean and Director of Studies of Britain's pioneering Open University, sheds some light on the why's and wherefore's of curricula reform.

Modern mathematics, traditional mathematics, contemporary mathematics, mathematics for the disadvantaged, mathematics for the slow learner, accelerated mathematics program and so on. Open any professional journal, or even an occasional newspaper, and you find fierce arguments for this and against that. A popular German weekly which I glanced at recently had a feature devoted to attacking the teaching of sets in school. One might be forgiven for thinking that there is something new about all this fuss, but there certainly is not. There have been major curricular upheavals at various periods, and minor reverberations in between. For example, an upheaval of earthquake proportions took place in England about the turn of the century, followed by a period of relative calm during the period of the two world wars, and then another major upheaval which began in the Fifties. It is difficult to say where we are at the moment, but I suspect we are almost after the storm, in a period of salvage, consolidation and readjustment.

Within the framework of curriculum changes, there are some fundamental questions to be answered; or, as I would prefer to say in an educational context, fundamental questions to be considered. I do not believe that one answers questions in education, but that does not mean that we are entitled to ignore them. Their consideration may deepen our insight into the problems and make our responses more conscious, intelligent and efficient.

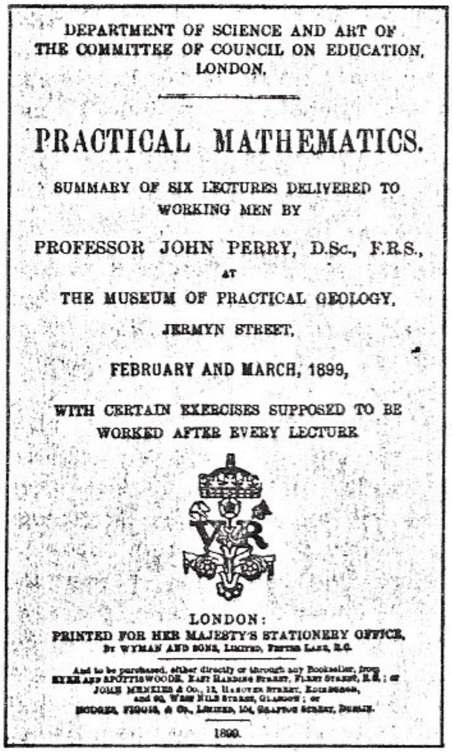
Some questions

First let us mention some of the questions. Why change a mathematics curriculum? Or any other curriculum for that matter? What makes us prefer one curriculum to another? Why, in a small country like Israel, do we spend any money at all on developing science curricula? After all science is international. Newton's laws of motion or Pythagoras' theorem have a certain independence of the language in which they are expressed. "Obviously" the thing for us to do is to take a "successful" curriculum from Europe or the U.S.A. and translate it. For the rest of the time we can engage in pure educational research for its own sake, within the limits of our means, just as we do in other fields.

Curriculum change: a first glance

Why change a curriculum? The obvious and simplistic answer is that the subject changes. Historians lead us to believe that Pythagoras had trouble with what we today regard as commonplace in secondary school. The square root of 2 is still today called an irrational number, which is not exactly the expression you would use to de-

Opposite: Prof. Bruckheimer


DEPARTMENT OF SCIENCE AND ART OF
THE COMMITTEE OF COUNCIL ON EDUCATION,
LONDON.

PRACTICAL MATHEMATICS.

SUMMARY OF SIX LECTURES DELIVERED TO
WORKING MEN BY

PROFESSOR JOHN PERRY, D.Sc., F.R.S.,

AT

THE MUSEUM OF PRACTICAL GEOLOGY,

JERMYN STREET,

FEBRUARY AND MARCH, 1899,

WITH CERTAIN EXERCISES SUPPOSED TO BE
WORKED AFTER EVERY LECTURE.

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1899.

scribe a good friend. One of the major proponents and exponents of mathematical reform in England at the turn of the century, Professor John Perry, F.R.S., a student of Lord Kelvin and not a mathematician as such, wrote:

> "The fifth book (of Euclid) is equivalent to two or three lines of the most elementary algebra . . ."

In Euclid's time, algebra, in the sense of symbolic or abstract arithmetic, was unknown. By the beginning of the twentieth century it was high time to replace parts of the geometry of Euclid by effectively equivalent algebraic results.

These changes, as we have said, are a "natural" outcome of the development of the subject. And within this aspect of curriculum change we can include such semi-external factors as the advent of computers and, more recently, the cheap electronic pocket calculator, whose effect on curricula is surely at an early stage. But even at this early stage, it would seem reasonable to forecast the demise of logarithms as a computational tool (as opposed to the theoretical concept, which is still very much alive).

Curriculum change: a specific example

At a slightly deeper level we see the more subtle changes in subject matter, which do not invalidate previous approaches, but suggest to us more satisfactory, or even more creative ways of teaching a subject. It is difficult in a relatively non-technical article to illustrate this point, but the following example should help to make sense of it. We use Perry's work again as representative of the curriculum revolution at the beginning of the twentieth century, but begin with a brief example of the status quo before Perry.

Take any nineteenth century school algebra and you find chapter after chapter devoted to the classification and analysis of equations and formulae. The one I have before me has 18 closely printed pages on ratio and proportion and then another 7 on variation. The whole book, which is more than 500 pages long, deals with school algebra (in the classical sense) only. One topic is particularly worthy of note. It is a section of six pages on graphs, about which the author writes in his preface:

> "The short section on Graphs will, it is hoped be found both interesting and useful."

This algebra book was published in 1898 and is probably fairly representative of its day and many days thereafter.

In this environment then we find Perry charging into battle.

Here is a section from his second lecture entitled "Algebra": "The average man who has worked through many rules in complex arithmetic, and algebra, and engineering, very quickly forgets them all, except the one or two that he constantly needs. It is only a teacher who remembers hundreds of rules. But, if at the beginning a man knows that his rules are *all one rule;* all his separate rules are mere examples of one general principle; he never can forget it, for every common-sense calculation that he makes only fixes the general principle more firmly in his mind. . . . My one simple rule is to treat all numerical calculation as work with some formula. . . ."

In the next four pages he goes on to consider many problems covering many of the topics to which the contemporary algebra book devotes more chapters than he pages. What Perry is here suggesting is not, in the strictest sense, a change in the topics included within the curriculum, but a fundamental change in their teaching arising from a changed conception of the topics themselves. He wants to de-emphasize their differences and concentrate on their common ground. As he puts it, ". . . strip them of their technical terms and artificial complexities. . ." He wants to wipe away the accumulation of years of special and separate cases and to replace them with a unified and practical approach. Here we see Perry exhibiting the aspect of curriculum change which we might call "weeding and pruning". The rose grows better next year if you weed the rose-bed and cut it drastically.

But there is another aspect

which, to take the horticultural metaphor a little further, might be called adding fertilizer and replacing old worn-out plants by new stock. We noted above the traditional algebra book's almost apologetic mention of graphs and the relatively small amount of space devoted to them (without any mention of squared—or graph—paper). Now compare Perry! His next three lectures are devoted to "Squared Paper." It is of more than incidental interest to quote Perry's introduction to his third lecture. It is not easy for us to conceive of school mathematics without graph paper, and we have here an example of the elementary but immensely creative development of the curriculum.

LECTURE III.

The use of Squared Paper.

25. A sheet of squared paper is covered with equidistant horizontal and vertical lines. Every tenth line is very distinct, so that it is easy to measure off horizontal and vertical distances without using a scale. The paper has its scales on it everywhere, in fact.

Before 1876 sheets of squared paper were very expensive; they were only used by a few people in important work. In that year Prof. Ayrton and I began to use it extensively in Japan, and when we returned to London and introduced it at the Finsbury Technical College our methods of teaching Mechanical and Electrical Engineering and laboratory work which have now become so common, we saw that one essential thing was the manufacture of cheap squared paper. It can now be bought for 7½d. a quire instead of 8d. per sheet. Our students treat it almost like scribbling paper. This year the candidates in three important subjects of the Science and Art Department will, for the first time, write their answers upon books of squared paper. It is of importance that the student should use many sheets of squared paper, use them lavishly. It used to be that many men knew how squared

paper *might* be used, but they really never used it, or if they did use it, they used it not for solving problems but for illustrating methods of solving problems.

26. In this and my next lecture I mean to show you some of the uses of squared paper. It would be easy to divide this subject up into 150 propositions and lead you on from one to the next, and so build up a science; but here at the very beginning, I want you to understand that, just as I said when describing the slide rule, all the following exercises are really one exercise. A student ought, after doing one or two of them, to see the general idea underlying them all, and if he will only practice by himself and exercise his common sense, he will be able to solve any such problem, and furthermore, he will need no elaborate proofs, things will be so self-evident as to require no proof.

Proofs indeed! Some people need proofs that they themselves exist. The mathematicians will tell you that this subject ought to be called Co-ordinate Geometry or Analytical Geometry. They will tell you that nobody ought to be allowed to begin it until he has mastered the most elaborate Algebraic and Trigonometric investigations. Now I want you to understand that I have known it to be used, and used very wisely and well, by a man who could neither read nor write.

There are many gems in this work by Perry, but I must try to return to the subject in hand. Before I do, I just want to quote one more piece from the next lecture which may interest any scientists who might come to read this article:

I may say that plotting log. x and log. y on squared paper is so often successful, especially in gas and steam engine work, that an old pupil of mine (Mr. Human) induced a stationer to manufacture sheets of *logarithmic paper,* so that one might lay out log. x and log. y without using a table. Here is a sheet of such paper. . . .

And a little later, in another problem:

In work like this there would evidently be a great saving of labor if we had paper ruled logarithmically one way and with equal divisions the other way. It is my intention to get some paper of this kind manufactured.

But to return to the subject. Here we see Perry in the role of constructive reformer. One aspect of his thesis is particularly noteworthy and was also present in the first example we gave: The idea that it is all "really one exercise." This is an aspect which reappears throughout curriculum reform, and was particularly noticeable in the reform which began in the Fifties. As the subjects continue to expand, there is an increasing pressure to include more in the curriculum for any particular age group. By the law of conservation of curricula, this increasing pressure can be accommodated without disaster if the teaching becomes more efficient. One of the ways of making it more efficient is to teach two subjects simultaneously; that is, to teach that subject which is the common part of the two, and which supports both. This always presupposes that these "unifying" topics can be taught to that age group. And here we leave Perry, an engineer, who represents the consumer force on curriculum reform. He was certainly interested in the desirability as well as the feasibility of teaching a subject to a particular group of students, but his interest was that of an experimentalist in the sense that we find them in industry. Behind him are the research experimentalist and the theoretician.

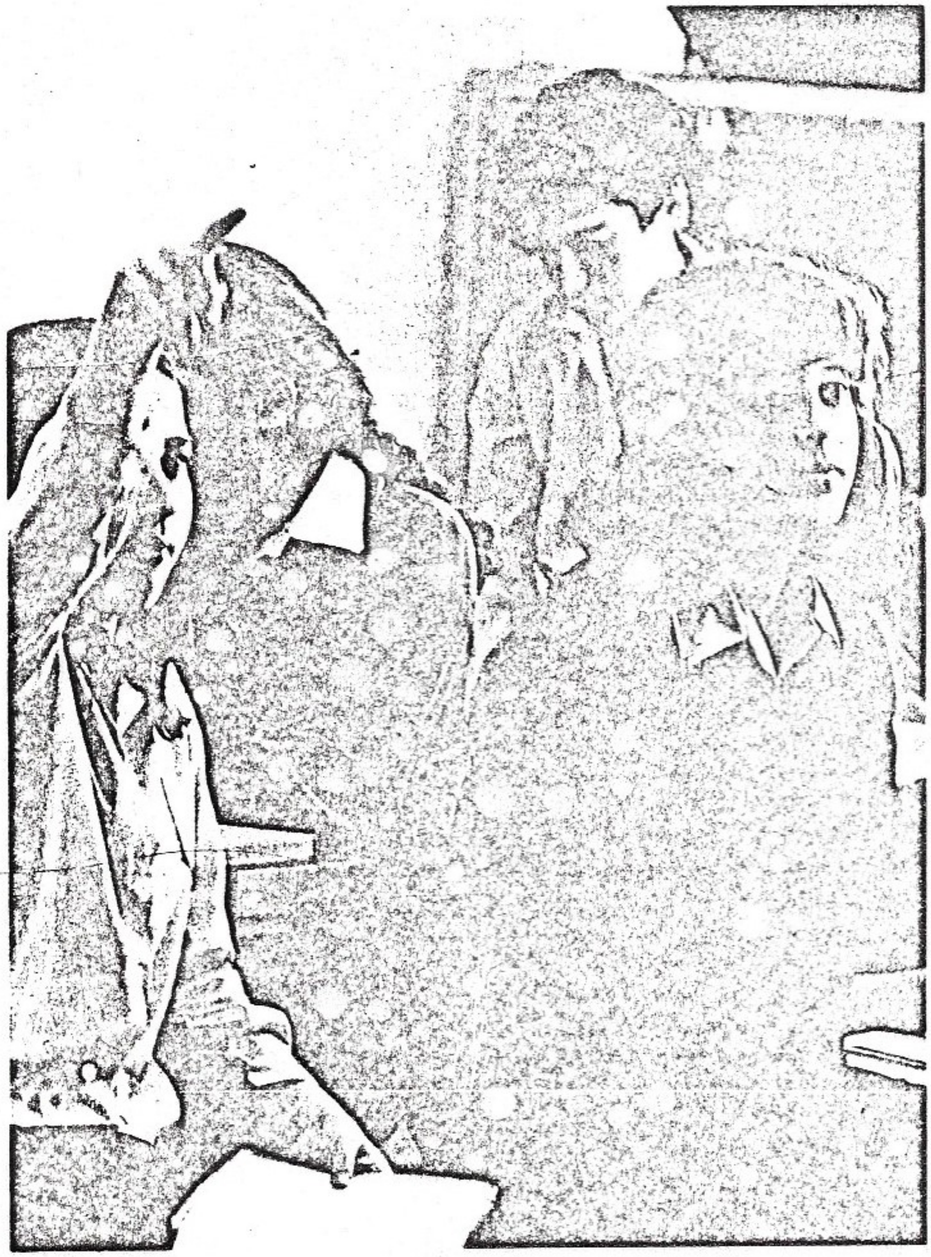
Education research as a factor in curriculum reform

It might be considered bad planning on my part to devote so much space to the influence of someone like Professor Perry, when I still have to consider the influence of the educational researchers, be they mathematicians, psychologists, philosophers or something else. But it seems to me that, compared with the effect on curriculum reform of people like Perry, representing the consumer, and the gifted mathematicians representing more intrinsic interests, the educational researchers have, so far, had a very minor impact, and the greater part of the impact which they have had has been at the primary school level.

I do not want to give the impression of dismissing this aspect of curriculum change. There is no doubt that, even where there has been no direct influence, the indirect effect *via* the reformers of Perry's type, has not been inconsiderable. Thus when Perry writes about the slide rules,

> "Read no book of elaborate instructions, you can find out everything for yourself by using the rule."

he obviously was subscribing to the virtues of the "discovery method." Whether he was explicitly aware of this as an educational theory or whether it was part of his intuitive skill as an educator, I do not know. But it would seem to me at least arguable that his views were influenced by his environment, which included the educational theorists. It would not surprise me if he had heard of Mary Boole (Ed: the wife of George Boole) and even read some of her work, which had a strong "discovery" bias. She certainly knew about him and approved of his views, as recorded in the *Journal of Education,* May 1903. The educational researchers and educational research are important, but more to sharpen and deepen the approach of the actual curriculum reformers, than as a directly applicable tool for curriculum change. G. H. Hardy, Britain's most distinguished mathematician in the first half of the twentieth century, revolutionized the teaching of analysis in English universities with his book "Pure Mathematics."



There is no question that his mathematical insight was the driving force and not educational theories. Nevertheless, I suspect that traces of contemporary didactical influence can be found in his work. (Maybe some bright student will someday make it the subject of a doctoral dissertation.)

The environment and curriculum change

The environment is not only an influence on those who create curriculum changes, but must be considered an intrinsic factor in the design of such change. Again there are many aspects to the environmental factors, and we can treat them to various depths. In an article of this sort some brief indicators must be sufficient. The following exercise might seem a little absurd:

"A hare makes 5 leaps in the time in which a greyhound makes 4, but three of the greyhound's leaps are equal to 4 of the hare's. How many leaps must the greyhound make before he catches the hare, if she is 100 of her own leaps before him at starting?"

It is taken however from a "Practical Arithmetic and Book-Keeping" of the middle of the nineteenth century. Clearly such an exercise is meaningless in our environment today, and the hare and the greyhound may actually be confusing and distracting unknowns to an

Israeli child. The example is, in a sense, trivial since, if we wanted to keep this type of question, we should obviously replace the hare and the greyhound by more familiar objects. But we should also look at a more positive effect that can be gained from the student's environment. Perry obviously understood it:

"A student ought at once to use squared paper for himself and use it with numbers in which he is interested. Many of you are in the watch trade. Well then, there are numbers published concerning your trade which, if you plot them on squared paper, will show you the rate and the yearly change of rate at which your trade is leaving Clerkenwell and going to America, and Birmingham, and Coventry, and the Continent. If this is too sad a subject. . . ."

I imagine his students sat up a little at this point.

At a somewhat deeper level, we may understand the influence of the environment as follows. Whenever a curriculum is developed consciously, among other things it takes into account the past experiences of both student and teacher and the conceptual and physical environment in which they now live. This, in itself, makes it very unlikely for a curriculum developed in, say, America to find immediate applicability in Israel. Not only will the examples and exercises have to be changed, but there is probably a large conceptual and experiential gap between children and teachers in the two environments. For example, the Israeli secondary school system is still dominated by frontal teaching. Any curriculum which is designed to make an effective and major use of discovery methods or individualized instruction will need considerable preparation which is unlikely to be practicable. Essentially this is not only a problem of transfer from one country to another, it is a problem within the one society, which in most countries is heterogeneous.

The so-called culturally deprived part of society, which is the majority in Israel, probably requires enriched curriculum materials, including enriched teacher training, if some of the educational lag between the two parts of society are to be reduced. These are not grounds for the adoption of the curriculum materials of others, but for the development of special materials using the experiences and achievements of others. The impossible ideal is the individualized curriculum for each child-teacher pair. The Weizmann group is deeply involved in these problems and currently is working on two projects whose major feature is that they take the present curriculum as the starting point, and by working with the children and the teachers in school, are evolving a new curriculum in small stages.

Conclusion

It is arguable (and, in fact, argued) whether it is better to reform curricula in a small number of large steps or more numerous small steps. My own feeling is in favor of the "continuous reform," that is, evolution rather than revolution. But there can be little argument about the necessity for curricula to change. Beside the arguments presented above, there is, to my mind, the powerful argument of teacher stagnation and staleness. It will take a very inspired and gifted teacher to retain his freshness and excitement after ten years of teaching from the same texts with the same methods. The Hawthorne effect exists within education as well.

In this article I have touched briefly on some constituents and causes of curriculum change. The discerning reader may have noticed that I never said what I meant by curriculum, but he may have formed an impression of it, nevertheless.