

Ion Channel Stochasticity May Be Critical in Determining the Reliability and Precision of Spike Timing

Elad Schneidman

Department of Neurobiology, Institute of Life Sciences, Institute of Computer Science, and Center for Neural Computation, Hebrew University, Jerusalem 91904, Israel

Barry Freedman

Idan Segev

Department of Neurobiology, Institute of Life Science and Center for Neural Computation, Hebrew University, Jerusalem 91904, Israel

The firing reliability and precision of an isopotential membrane patch consisting of a realistically large number of ion channels is investigated using a stochastic Hodgkin-Huxley (HH) model. In sharp contrast to the deterministic HH model, the biophysically inspired stochastic model reproduces qualitatively the different reliability and precision characteristics of spike firing in response to DC and fluctuating current input in neocortical neurons, as reported by Mainen & Sejnowski (1995). For DC inputs, spike timing is highly unreliable; the reliability and precision are significantly increased for fluctuating current input. This behavior is critically determined by the relatively small number of excitable channels that are opened near threshold for spike firing rather than by the total number of channels that exist in the membrane patch. Channel fluctuations, together with the inherent bistability in the HH equations, give rise to three additional experimentally observed phenomena: subthreshold oscillations in the membrane voltage for DC input, “spontaneous” spikes for subthreshold inputs, and “missing” spikes for suprathreshold inputs. We suggest that the noise inherent in the operation of ion channels enables neurons to act as “smart” encoders. Slowly varying, uncorrelated inputs are coded with low reliability and accuracy and, hence, the information about such inputs is encoded almost exclusively by the spike rate. On the other hand, correlated presynaptic activity produces sharp fluctuations in the input to the postsynaptic cell, which are then encoded with high reliability and accuracy. In this case, information about the input exists in the exact timing of the spikes. We conclude that channel stochasticity should be considered in realistic models of neurons.

1 Introduction

Following the formulation of the Hodgkin-Huxley (HH) equations for modeling spike initiation in the squid giant axon (Hodgkin & Huxley, 1952), research into the electrical activity of single neurons followed two main paths: the attempt to discover further macroscopic equations governing different membrane currents (e.g., Yamada, Koch, & Adams, 1989), and the attempt to investigate, and mathematically describe, the behavior of the ion channels underlying these currents (see Hille, 1992; Sakmann & Neher, 1995). Although part of the same general problem, mathematically these two areas of investigation are entirely different. In the HH formulation, the ion conductances are modeled by means of deterministic differential equations, and their values range continuously from zero to a given maximum. However, because individual ion channels are discrete elements whose properties can be given only probabilistically, the electrical activity of nerve cells is most accurately described as resulting from the interaction of stochastic, discrete units. It is commonly assumed that a large collection of such discrete units practically forms a continuous deterministic system, as is the case in numerous large physical systems. Because the number of excitable channels in the axon's spike initiation zone is estimated to be large (on the order of tens of thousands of ion channels; Hille, 1992), models for spike generation in neurons typically use deterministic rather than stochastic equations (Mainen, Joerges, Huguenard, & Sejnowski, 1995; Rinzel & Ermentrout, 1989; Rapp, Yarom, & Segev, 1996).

A few theoretical studies did consider the effect of channel stochasticity, focusing on the question, "When does the stochastic model converge to the corresponding deterministic model?" The pioneering work of Fitzhugh (1965) used a kinetic (stochastic) model for the conductance change associated with the HH equations; others used stochastic HH equations to investigate the effect of various parameters (e.g., number of channels, membrane area) on the dynamics of the membrane voltage. The main message of these studies is that the stochastic system differs considerably from the deterministic HH system when a small number (a few hundred) of channels and small membrane areas are involved (Skaugen & Walløe, 1979; Strassberg & DeFelice, 1993; DeFelice & Isaac, 1992). Other aspects of this problem—spontaneous spiking due to channel noise and the effect of channel stochasticity on spike propagation in axons—were recently explored (Rubinstein, 1995; Chow & White, 1996; Horikawa, 1991, 1993). In a more general perspective, the effect of different kinds of noise on the firing threshold of neurons was examined by Lecar and Nossal (1971a,b). Recently Jensen and Gartner (1997) have dealt with the effect of additive white noise on the firing reliability of different neuron models (see also Longtin & Hinzer, 1996, and Braun, Huber, Dewald, Schafer, & Voigt, 1998).

A recent experimental study by Mainen and Sejnowski (1995) on the reliability of spike firing time in neocortical pyramidal cells motivated us

to readdress the question, “How good an approximation is the deterministic model, given a large number of excitable channels?” focusing on the timing of the spikes during the train. Mainen and Sejnowski showed that spike timing is highly unreliable for repeated DC current inputs, whereas fluctuating current inputs significantly improve the firing reliability and the firing precision up to a millisecond range (see also the recent studies by Nowak, Sanches-Vives, & McCormick, 1997 and Tang, Bartels, & Sejnowski, 1997 and studies on the reliability of neuronal spike firing times in behaving animals, e.g., de Ruyter van Steveninck, Lewen, Strong, Koberle & Bialek, 1997, and Reich, Victor, Knight, Ozaki, & Kaplan, 1997). We were interested in exploring if, for realistic membrane area and number of excitable channels, a biophysically inspired noise that is generated by channel stochasticity plays an important role in determining the reliability of spike firing times in the spike initiation zone. In this context, it is noteworthy that channel noise was shown to be significant for the macroscopic behavior of neurons (Volgushev, Chistiakova, & Singer, 1998), and in other excitable membranes (Berzakov & Vodyanoy, 1995). We therefore modeled membrane patches of areas of a few hundred square micrometers, comprising a total of a few tens of thousands ion channels that receive both DC inputs and the more biologically realistic fluctuating current inputs. We show that for a broad range of inputs, the stochastic equations generate results that are strikingly different from those obtained from the corresponding deterministic HH equations. In addition to its significant effect on the timing of spike firing, channel noise also produces three additional experimental observations: voltage-dependent subthreshold membrane voltage oscillations for DC input, occasional “missing” spikes for suprathreshold inputs and “spontaneous” spikes for subthreshold inputs. In section 4, we speculate on the functional implications of channel stochasticity for neural coding. A preliminary account of this work has appeared as an abstract (Schneidman, Freedman, & Segev, 1997).

2 Basic Model and Simulation Scheme

The membrane dynamics of the HH equations is given by

$$C_m \frac{dV}{dt} = -g_L(V - V_L) - g_K(V - V_K) - g_{Na}(V - V_{Na}) + I, \quad (2.1)$$

where V is the membrane potential; V_L , V_K , V_{Na} are the reversal potentials of the leakage, potassium, and sodium currents, respectively. g_L , g_K , g_{Na} are the corresponding specific ion conductances; C_m is the specific membrane capacitance; and I is the specific current injected into this membrane patch. The voltage-dependent conductances for the potassium and sodium channels are given by

$$g_K(V, t) = \bar{g}_K n^4; \quad g_{Na}(V, t) = \bar{g}_{Na} m^3 h, \quad (2.2)$$

Table 1: Hodgkin-Huxley Parameters and Rate Functions Used in the Simulations.

C_m	Specific membrane capacitance	$1 \mu\text{F}/\text{cm}^2$
T	Temperature	6.3°C
V_L	Leakage reversal potential	10.6 mV
g_L	Leakage conductance	$0.3 \text{ mS}/\text{cm}^2$
V_K	Potassium reversal potential	-12 mV
\bar{g}_K	Maximal potassium conductance	$36 \text{ mS}/\text{cm}^2$
γ_K	Potassium channel conductance	20 pS
D_K	Potassium ion channel density	$18 \text{ channels}/\mu\text{m}^2$
V_{Na}	Sodium reversal potential	115 mV
\bar{g}_{Na}	Maximal sodium conductance	$120 \text{ mS}/\text{cm}^2$
γ_{Na}	Sodium channel conductance	20 pS
D_{Na}	Sodium ion channel density	$60 \text{ channels}/\mu\text{m}^2$
$\alpha_n(V)$		$\frac{0.01(10-V)}{\exp[(10-V)/10]-1}$
$\alpha_m(V)$		$\frac{0.1(25-V)}{\exp[(25-V)/10]-1}$
$\alpha_h(V)$		$0.07 \exp[-V/20]$
$\beta_n(V)$		$0.125 \exp[-V/80]$
$\beta_m(V)$		$4.0 \exp[-V/18]$
$\beta_h(V)$		$\frac{1}{\exp[(30-V)/10]+1}$

where the dynamics of n (and similarly for m and h) is given by

$$\frac{dn}{dt} = \alpha_n(1 - n) - \beta_n n. \quad (2.3)$$

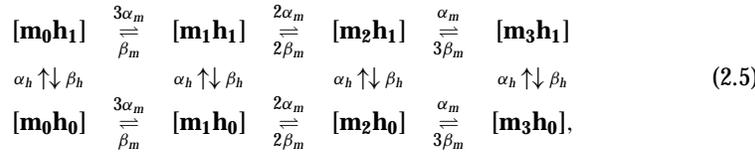
The rate functions α 's, β 's, and the maximal conductances \bar{g} 's, as formulated by Hodgkin and Huxley, are given in Table 1.

Equations 2.1 through 2.3 give an extremely successful description of the mean behavior of the voltage and currents in the squid giant axon, without the need to treat the activity of individual ion channels that underlie this behavior. Hodgkin and Huxley suggested a physical interpretation of their equations—namely, that the term n^4 indicates that there are four separate gates and that a K^+ channel is opened only when all these gates are open. The term m^3h indicates that there are three m -gates and one h -gate that must be open to enable the Na^+ current to flow via the sodium channel. A corresponding kinetic model (Fitzhugh, 1965; Clay & DeFelice, 1983) can be formalized as a Markov model that explicitly incorporates the internal workings of the ion channels. The model, which is equivalent in the limit of large number of channels to the HH model (Skaugen & Walløe, 1979), suggests that the K^+ channel can exist in five different states and that the kinetic scheme describing the behavior of this channel is given by

$$[\mathbf{n}_0] \xrightleftharpoons[\beta_n]{4\alpha_n} [\mathbf{n}_1] \xrightleftharpoons[2\beta_n]{3\alpha_n} [\mathbf{n}_2] \xrightleftharpoons[3\beta_n]{2\alpha_n} [\mathbf{n}_3] \xrightleftharpoons[4\beta_n]{\alpha_n} [\mathbf{n}_4], \quad (2.4)$$

where $[\mathbf{n}_i]$ is the number of the channels with i open gates and, hence, $[\mathbf{n}_4]$ labels the single open state of the K^+ channel. α_n, β_n are identical to the original HH rate functions.

Similarly, in this description, each Na^+ channel can exist in eight different states, as in the following scheme,



where $[\mathbf{m}_i\mathbf{h}_j]$ refers to the number of channels within the population that are currently in the state $m_i h_j$, $[\mathbf{m}_3\mathbf{h}_1]$ labels the single open state of the Na^+ channel, and $\alpha_h, \beta_h, \alpha_m,$ and β_m are the rate functions in HH formalism. The potassium and sodium membrane conductances are given by

$$g_K(V, t) = \gamma_K [\mathbf{n}_4]; \quad g_{Na}(V, t) = \gamma_{Na} [\mathbf{m}_3\mathbf{h}_1], \tag{2.6}$$

where γ_K and γ_{Na} are the conductances of the single potassium and sodium ion channel at their open state, respectively.

By switching from the standard HH model to Fitzhugh's (1965) model, channel stochasticity is incorporated into the voltage dynamics. Instead of keeping track of each of the channels separately, we have used a more efficient scheme to track only the total populations of channels in each of their possible states (see Skaugen & Walløe, 1979, and Chow & White, 1996, for a discussion of possible simulation methods for populations of channels). Specifically, if at time t there are n_A channels in state A and n_B channels in state B and the transfer rate of channels from state A to state B is r , then each of the channels in state A might transfer to state B between time t and $t + \Delta t$ with probability $p = r\Delta t$. Hence, for each time step, we determine Δn_{AB} , the number of channels that move from A to B , by choosing a random number from a binomial distribution (Press, Teukolsky, Vetterling, & Flannery, 1992),

$$\text{Prob}(\Delta n_{AB}) = \binom{n_A}{\Delta n_{AB}} p^{\Delta n_{AB}} (1 - p)^{(n_A - \Delta n_{AB})}. \tag{2.7}$$

In this study, we used the forward Euler integration method with $\Delta t = 0.01$ msec, as in Chow and White (1996). It is important to note that the spatially independent (space-clamped) HH equations were utilized; we simulate isopotential membrane patches of varying areas. Clearly, this is a severe oversimplification of the realistic case, and its implications will be addressed in section 4. Finally, in order to make the transition from the deterministic to the stochastic model, we need to know how many channels there are in the modeled membrane patch. Once we choose the conductance of the

individual channel, the number of channels can be calculated directly from the channel densities and from the maximal conductances, \bar{g} 's, given in the HH model (see Table 1).

3 Results

Before we proceed to the actual simulation results, we first try to estimate the effect of introducing stochasticity into the HH model. Suppose that the area of the membrane patch is $200 \mu m^2$. With the parameters of Table 1, this membrane patch bears 3600 K^+ channels and 12,000 Na^+ channels. Considering the large number of modeled channels, one would naively estimate the number of fluctuating channels about the mean to be on the order of $\frac{\sqrt{N}}{N}$ channels. Hence, for $N = 3600$ K^+ channels, the size of the fluctuation is 1.7%, and we would expect rather small deviations from the deterministic model. An even smaller effect would be expected for the Na^+ channels. Surprisingly, this is not the case, as shown in Figure 1.

3.1 Encoding Reliability and Precision: Input Current Versus Channel Fluctuations. The response of a stochastic isopotential HH compartment to repeated presentation of suprathreshold currents is shown in Figure 1. When the same suprathreshold DC current pulse ($10 \mu A/cm^2$, 250 msec; A, top frame) is repeatedly presented to the modeled membrane patch, the resulting spike trains vary considerably from trial to trial; the spike firing time is neither reliable nor accurate (see Figure 1A, bottom frame). This should be compared with the response of the corresponding deterministic model shown in the middle frames. In contrast, when the stimulus is fluctuating (simulating the current that presumably reaches the site of spike generation following the activation of many synaptic inputs impinging on the dendritic tree, B top frame; see the caption of Figure 1 for details), the reliability and accuracy of the spike train in the stochastic HH model are improved compared to DC case (see Figure 1B, bottom frame).

As in the study of Mainen and Sejnowski (1995), two measures of the spike timing, the reliability and the precision, were calculated from the peristimulus time histogram (PSTH, not shown) for a wide range of input patterns (see caption of Figure 2 for details). The reliability and precision of the spike patterns were strongly correlated with the amplitude of the fluctuations in the input current, σ_{input} (see Figures 2A and 2B); the reliability and precision dropped as the input was filtered with larger time constants (see Figures 2C and 2D). In the stochastic HH model, both the reliability and the precision, which for most of the responses was in the range of 1 to 2 msec, are in close agreement with the results of Mainen and Sejnowski (1995). It is noteworthy that there is no clear dependence of the reliability and precision on the mean value of the injected current, as was also found experimentally by Mainen and Sejnowski (personal communication).

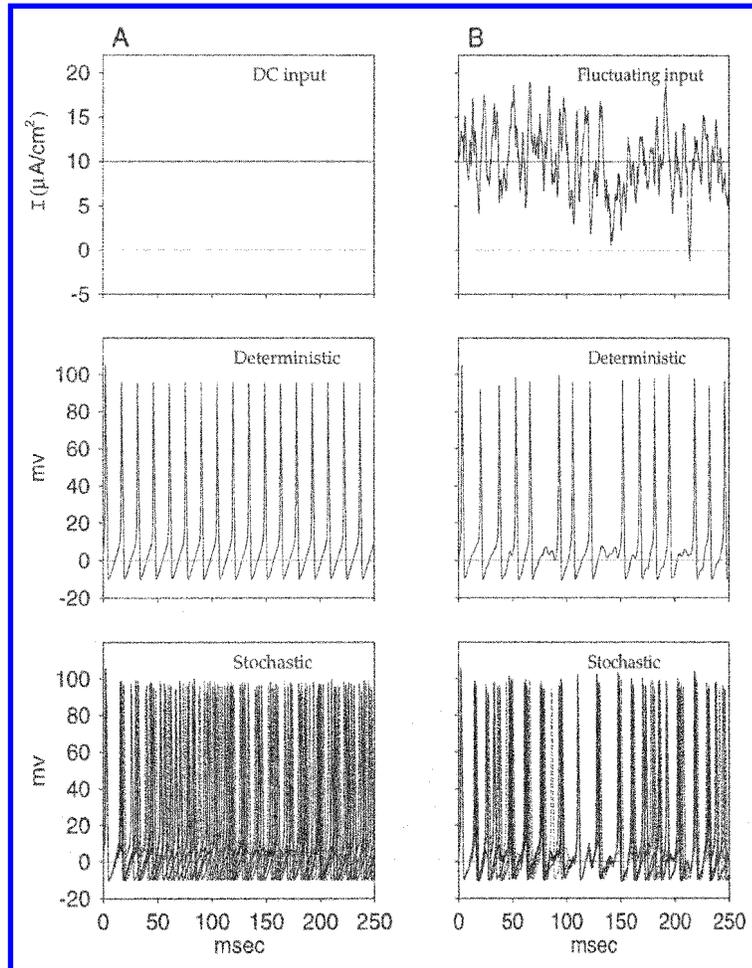


Figure 1: Reliability of firing patterns in a model of an isopotential Hodgkin-Huxley membrane patch in response to both DC and fluctuating current input. (A) Ten superimposed responses to repeated suprathreshold DC current pulses ($10 \mu\text{A}/\text{cm}^2$, 250 msec; top frame) evoked a train of regular firing in the deterministic HH model (middle frame) and a jitter in the firing in the stochastic HH model (bottom frame). (B) The same patch was again stimulated 10 times repeatedly, this time with a fluctuating stimulus (low-pass gaussian white noise with a mean \bar{I} , of $10 \mu\text{A}/\text{cm}^2$, and a standard deviation σ_{input} of $7 \mu\text{A}/\text{cm}^2$, which was convolved with an alpha function with a time constant $\tau_{input} = 1$ msec, top frame; see Mainen & Sejnowski, 1995). As can be clearly seen, the jitter in spike timing in the stochastic model is significantly smaller in B than in A (i.e., increased accuracy for the fluctuating current input). Patch area used was $200 \mu\text{m}^2$, with 3,600 K^+ channels and 12,000 Na^+ channels. (Compare to Figure 1 in Mainen & Sejnowski, 1995.)

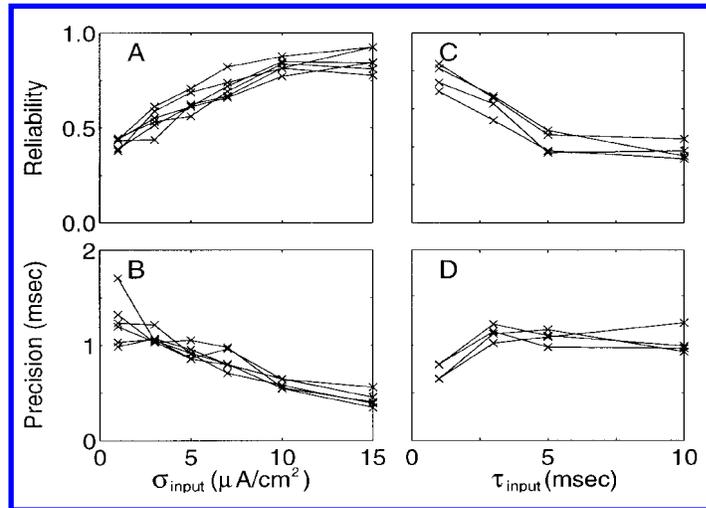


Figure 2: Dependence of reliability and precision on stimulus parameters. The reliability and accuracy of the spike train was calculated in a similar manner to that of Mainen & Sejnowski (1995). The peristimulus time histogram (PSTH) of 20 successive presentations of a particular stimulus was smoothed using an adaptive filter, yielding an estimate for the instantaneous firing rate. Significant elevations in the instantaneous firing rate (“events”) were selected from the PSTH using a threshold of two times the mean firing rate over a given block of responses. The reliability of the response to a particular stimulus is defined as the average of the fraction of spikes that occurred in the events in that stimulus’ PSTH. The temporal precision of the response is defined by the average of standard deviation of spike timing within the events in that stimulus’s PSTH. (A) Estimates of the reliability of the spike train in a $200\mu\text{m}^2$ stochastic HH membrane patch, for stimuli with various fluctuation amplitudes, σ_{input} . Each curve is for a different mean value of the stimulus ($\bar{I} = 7 - 20 \mu\text{A}/\text{cm}^2$, $\tau_{\text{input}} = 1 \text{ msec}$). (B) The temporal precision of the same responses as in A. (C) The reliability for stimuli filtered with different time constants ($\tau_{\text{input}} = 1 - 10 \text{ msec}$). Each curve is for a different mean value of the stimulus and a given σ_{input} . ($\bar{I} = 7 - 20 \mu\text{A}/\text{cm}^2$, $\sigma_{\text{input}} = 3 - 12 \mu\text{A}/\text{cm}^2$). (D) The temporal precision of the same responses as in C.

Hence, with a realistically large number of channels, when incorporating their unavoidable stochasticity, one obtains an effect that is qualitatively similar to the behavior of real neurons and is significant from both biophysical and computational viewpoints. Clearly, the effect of stochasticity depends on the number of ion channels and the membrane area. It increases

when decreasing the number of channels and decreases when increasing the membrane area. Still, the effect of channel stochasticity was significant even when the membrane area was increased by a factor of five (to $1000 \mu\text{m}^2$) as well as when, for a given membrane patch, the channel density was increased by the same factor (not shown; see section 4). But why is the result of the stochastic model so different from that obtained from the corresponding deterministic HH model?

The apparent error in the previous estimation (see the beginning of section 3) of the size of the effect of channel stochasticity lies in failing to realize that the relevant number of channels is not the total number of channels in the membrane patch, but rather the number of channels that are *open near the threshold for spike firing*. If this number is relatively small, the size of the fluctuations in the number of open channels in this regime is not negligible. Mathematically, the correct estimation for the size of the fluctuation should rely on the binomial statistics. For a total population of N channels and a probability p of a channel to be open, the size of the fluctuations is $\sqrt{Np(1-p)}$ and the fluctuation relative to the mean, Np , is $\sqrt{\frac{(1-p)}{Np}}$. If p is small, as in the case of near threshold for spike firing, the relative size of the fluctuations is rather large. In this case, the inherent stochasticity of the channels is expected to have a significant effect on the voltage dynamics and, specifically, on the time of threshold crossing. When this is the case, the firing behavior of the stochastic model is expected to be considerably different from that of the corresponding deterministic model.

Figure 3 shows that this is indeed the situation. As in Figure 1A, 10 repeated $10 \mu\text{A}/\text{cm}^2$ DC current inputs were applied, this time to a $600 \mu\text{m}^2$ membrane patch consisting of a total of 10,800 K^+ channels and 36,000 Na^+ channels. The voltage response is shown in Figure 3B, whereas the numbers of open K^+ and Na^+ channels near the threshold for spike firing are shown in panels C and D, respectively. A surprisingly small number of ion channels—approximately 300 K^+ channels and 50 Na^+ channels—are opened in this voltage regime. With these small numbers, channel fluctuations become significant and critically determine the exact time in which a sufficient number of *additional* Na^+ channels is recruited to initiate a regenerative response. When injecting the same DC current repeatedly, the fluctuations vary significantly from one trial to the other, and, consequently, the time of spike firing for this input is unreliable.

In principle, this channel-noise-induced unreliability can be mostly overridden by injecting a current that fluctuates significantly. If the input fluctuations are sufficiently large, the voltage dynamics will be dominated by the transients in the current input rather than by the channel noise. This effect is demonstrated in Figure 4, where the response to a fluctuating input in a specific time window is shown. The current input is depicted in A, the voltage response is shown in panel B, and the number of open K^+ and Na^+ channels is shown in panels C and D, respectively. In contrast to the DC-

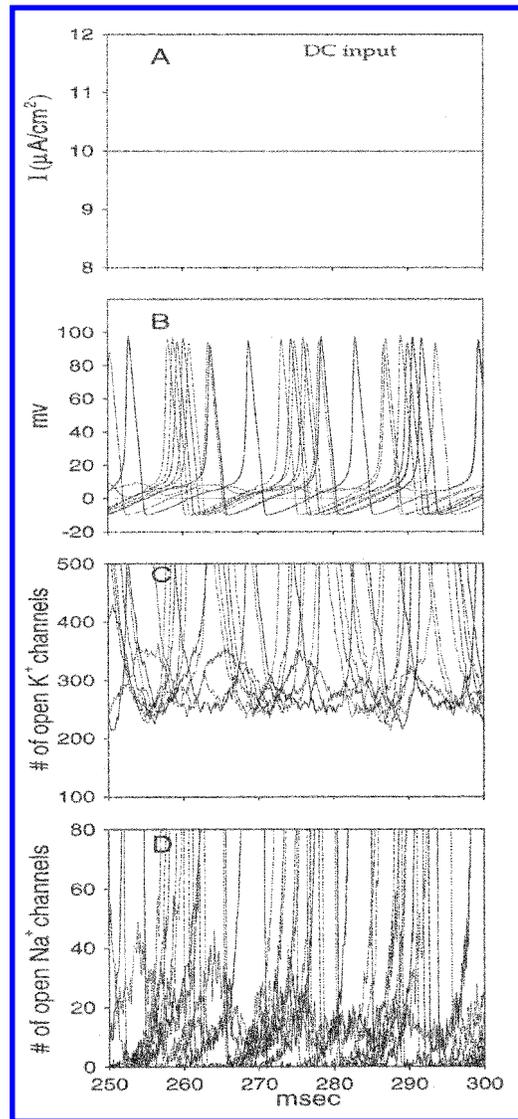
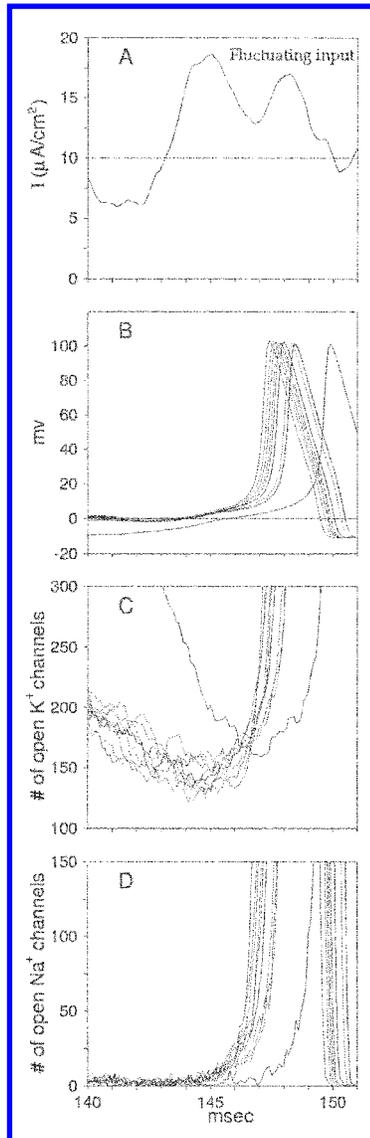


Figure 3: Channel fluctuations ruin the reliability of spike timing in the case of DC current input. (A) $10 \mu\text{A}/\text{cm}^2$ DC current injected to a $600 \mu\text{m}^2$ stochastic HH model (10, 800 K^+ channels and 36, 000 Na^+ channels) results with dispersed spike timings for repeated simulation (10 superimposed voltage traces) in B. (C, D) the number of open Na^+ and K^+ channels, respectively, corresponding to the voltage traces presented in B.

input case (see Figure 3), here the transients in the input current partially overcome the channel fluctuations and enforce 9 out of the 10 spikes to occur within an approximately 1 msec time window (see Figure 4B). The reason for this relatively high reliability of spike timing becomes clear by observing panels C and D. The accuracy is determined by two parameters. The first is the variability in the time where the number of open K^+ channels reaches a sufficiently small value (note that a large outward K^+ current impedes the initiation of the spike). This variability should be small in order to obtain high accuracy. Indeed, in 9 of 10 repetitions, this condition is satisfied (see Figure 4C). The second parameter is the rate of the buildup of the Na^+ channel population toward threshold. For an accurate spike timing, this buildup, which is determined by the amplitude and rate of the depolarizing input current, should be sufficiently large to overcome the channel fluctuations (see Figure 4D). For a given voltage, the size of the channel fluctuation is as large in the fluctuating-input case as in the DC-input case, but in the former these channels fluctuations are “lost in the crowd.”

To examine the relative contribution of the K^+ and Na^+ channels to the reliability and precision, we simulated a hybrid system in which one of the channel populations was stochastic and the other was deterministic. Both channel types contribute to the complex reliability nature of the system. However, as expected from the larger number of K^+ channels that are open near threshold for spike firing (see Figures 3C and 3D and Figures 4C and 4D), as well as from their slower kinetics, the noise introduced by the K^+ channels is more dominant in determining the reliability and accuracy of this system (not shown).

3.2 Subthreshold Oscillations, “Spontaneous” Spikes, and “Missing” Spikes. Along with the effect of channel fluctuations on spike timing, incorporating channel stochasticity in the HH model gives rise to three additional phenomena that were observed experimentally: (1) considerable subthreshold oscillations in the membrane voltage for DC inputs, (2) “spontaneous” spikes for “subthreshold” inputs, and (3) “missing” spikes for suprathreshold inputs. These phenomena cannot be reproduced in the deterministic HH model. In the stochastic model, oscillations in the membrane voltage are already observed for zero current input (see Figure 5A). Occasionally these oscillations are sufficiently large to generate “spontaneous” spikes, which would not have occurred in the corresponding deterministic model (current threshold for spike firing in the deterministic model is $I = 7 \mu A/cm^2$). An example of “spontaneous” spikes in the case of $I = 4 \mu A/cm^2$ is shown in Figure 5B; detailed analysis of spontaneous spiking in the stochastic HH model for zero current input was recently performed by Chow and White (1996). Compared to the deterministic model, where regular repetitive firing occurs (see Figure 1), “missing” spikes (with respect to the corresponding deterministic model) are observed in the stochastic model for suprathreshold currents (see Figures 5C and 5D).



It is important to note that both the amplitude and the frequency of the membrane oscillations observed in the stochastic model are voltage dependent (e.g., compare A to B in Figure 5). This is also the case with the membrane voltage oscillations in neocortical neurons reported by Gutfreund, Yarom, and Segev (1995), Klink and Alonso (1993), as well as in other neuron types, for example, Hutcheon, Miura, Yarom, and Pail (1994) and Lampl and Yarom (1997). We suggest that in addition to the deterministic macroscopic mechanisms that were proposed to explain the generation of the subthreshold oscillations, the stochastic nature (and the limited number) of the ion channels may have a dominant effect on the nature of these oscillations (see also Longtin & Hinzer, 1996, and Braun et al., 1998).

Channel stochasticity has such a dramatic effect on the voltage dynamics because it exploits a peculiar, and largely neglected, aspect of the deterministic HH equations: its two stable states for suprathreshold current input (see the discussion of the bistability in the HH equations in Cooley, Dodge, and Cohen, 1965, and Guttman, Lewis, and Rinzel, 1980). For a DC input, one state is the well-known repetitive firing behavior (the light trace in Figure 6A) whereas the other state is a nonfiring behavior of early damped voltage oscillations that converges to a steady voltage (see Figure 6A, dark trace). In both cases, a $7 \mu\text{A}/\text{cm}^2$ DC current was injected, and the marked difference between the two curves is the result of minute perturbation in the initial conditions (see the caption of Figure 6 for details). These two different behaviors can be better appreciated in the phase plane diagram in panels B and C of Figure 6. Translating the ion conductances to the corresponding

Figure 4: *Facing page*. Fluctuating input current partially overrides the channel stochasticity and increases the reliability of spike timing. A small time window of the system behavior for the fluctuating input case is presented. (A) The input current with a mean value of $10 \mu\text{A}/\text{cm}^2$ (horizontal dotted line) and with $\sigma_{input} = 5 \mu\text{A}/\text{cm}^2$ and $\tau_{input} = 1 \text{ msec}$, injected to a $600 \mu\text{m}^2$ stochastic HH membrane patch is depicted. (B) Ten superimposed voltage-traces responses to repeated injection of the fluctuating current in A. In 9 out of 10 of the cases, a spike was fired within approximately 1 millisecond time window. (C, D) The number of open Na^+ and K^+ channels, respectively, for the voltage traces presented in B, reflecting how the fluctuations of both Na^+ and K^+ channels are overridden by the fluctuations in the input current. When a sufficient number of K^+ channels close (C), the depolarizing transient in the input current, starting at $t = 143 \text{ msec}$, results in the nearly synchronous buildup of Na^+ channels at $t = 146 \text{ msec}$ (D). The result is spike firing at $t = 147.3 - 148.4 \text{ msec}$. In the one case where an insufficient number of K^+ channels was closed in time, the spike is initiated somewhat later due to the next fluctuation in the input current.

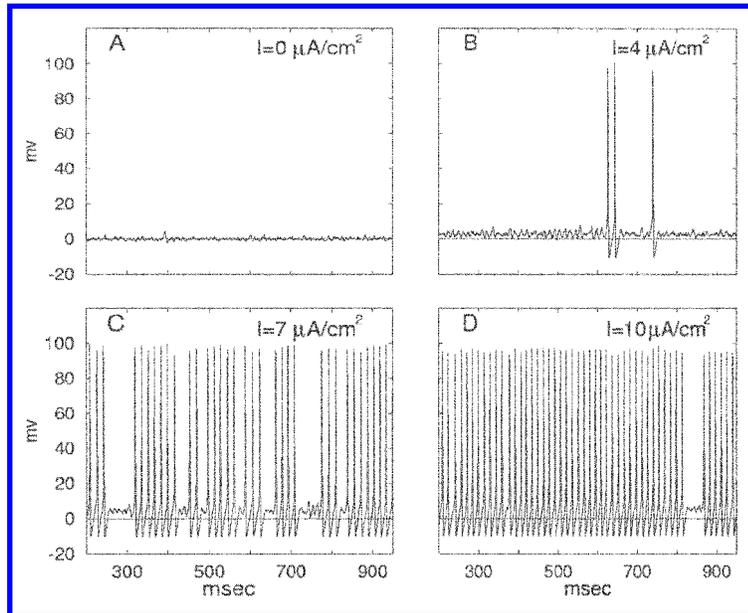
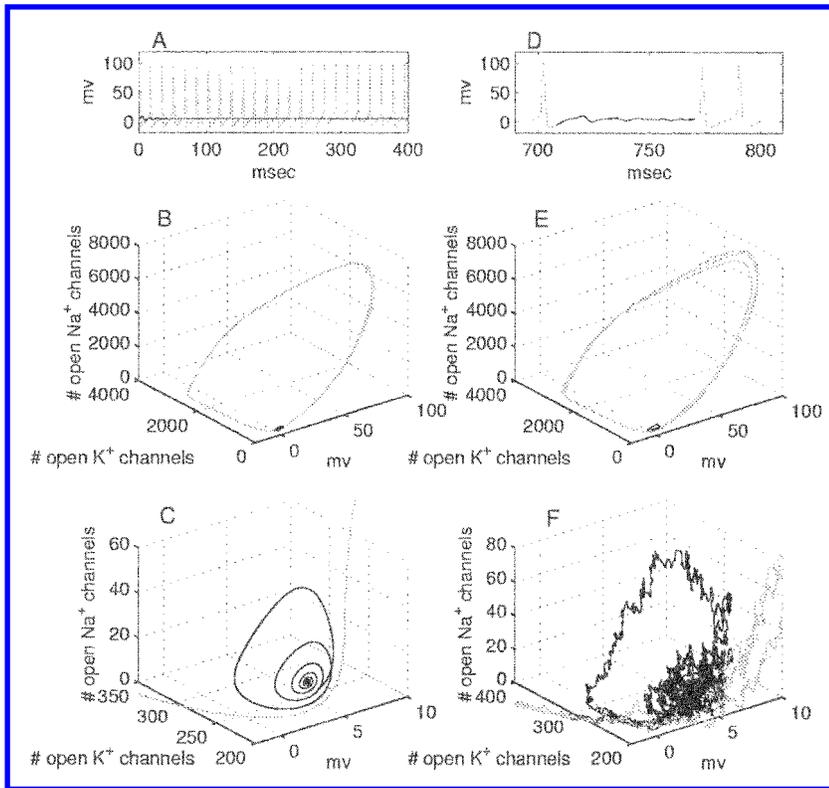


Figure 5: The response of the stochastic model to injected DC input currents. DC currents of different amplitudes were injected to a stochastic HH model of an isopotential membrane patch of area $600 \mu\text{m}^2$ (10,800 K^+ channels and 36,000 Na^+ channels). (A, B) Membrane voltage oscillations are the dominant effect of the stochastic nature of the ion channels, with occasional spontaneous spiking. (C, D) Suprathreshold DC input currents result with irregular spiking, occasional “missing” spikes and membrane voltage oscillations. This is not expected in the corresponding deterministic HH model where the threshold is $7 \mu\text{A}/\text{cm}^2$. Below this value, smooth subthreshold voltage response is observed in the deterministic model (not shown); above this value, regular firing is obtained (not shown).

number of open ion channels, these panels show the very different paths in phase space taken by the firing (light curve) and the nonfiring (dark curve) trajectories. The bottom panel shows the convergence of the nonfiring behavior to a fixed point. It also shows that the distance, in terms of number of open channels, between the continuous firing cycle and the nonfiring voltage behavior is very small. Although small, the deterministic nature of the HH equations implies that for a DC input, the system remains in one stable state or the other. However, introduction of channel noise could, in principle, flip the system between these two states.

Figures 6D–F show that in the stochastic model, channel fluctuations occasionally bridge the small distance in phase space between the two stable states. The stochastic opening (or closing) of a few extra K^+ and/or Na^+ channels pushes the system spontaneously from the continuous firing stable state (light line) to the nonfiring stable state (dark line), where it stays for a while, and vice versa (at $t = 772$ msec in Figure 6D). This spontaneous transition between the two states is the cause for the “missing” spikes and the subthreshold membrane voltage as well as for occasional “spontaneous spikes” (see Figure 5B). Panels E and F depict the corresponding phase-plane behavior of the system. It clearly shows that fluctuations due to only



a few channels are responsible for the transition between these two stable states. We conclude that the nonfiring stable state in the deterministic HH model becomes a key player in the stochastic HH model. Experimentally, the coexistence of the two stable solutions in the squid giant axon, as well as in the corresponding HH model, was demonstrated by Guttman et al. (1980) (see also Cooley et al., 1965).

Considering the subthreshold membrane oscillations, the role of channel fluctuations is twofold. First, they drive the system from the firing state into the basin of attraction of the nonfiring stable state. Second, the fluctuations prevent the system from converging into the fixed point of the nonfiring stable state of the corresponding deterministic model. As a result, the system is cycling around this fixed point, and the subthreshold membrane voltage oscillations thus emerge. The frequency of the subthreshold oscillations is set by the period of these cycles. Based on this observation, we can predict analytically the power spectrum of the oscillations with a fair degree of accuracy, and we can also quantify the rate of transfer between the two states in the case of DC input (unpublished observations). Questions regard-

Figure 6: *Previous page*. Channel fluctuations cause flipping between firing and nonfiring stable states in the stochastic HH model. (A–C) The two stable states of the deterministic HH model. In A, two traces of the membrane potential are shown for a $600 \mu\text{m}^2$ membrane patch, injected with a $7 \mu\text{A}/\text{cm}^2$ DC current. The difference between the light and the dark traces results from the minute difference in the initial conditions. In the continuous firing case (light trace), the initial values are: $V = 4.21 \text{ mV}$; $g_K = 4121 \text{ pS}$ (corresponds to 206.05 open K^+ channels) and $g_{\text{Na}} = 195.8 \text{ pS}$ (corresponding to 9.79 open Na^+ channels) assuming a single-channel conductance of 20 pS (Table 1). In the nonfiring case (dark trace) the initial values are: $V = 4.23 \text{ mV}$; $g_K = 4399.4 \text{ pS}$ (corresponds to 219.97 open K^+ channels) and $g_{\text{Na}} = 197.4 \text{ pS}$ (corresponding to 9.87 open Na^+ channels). (B) The 3D phase plane of these two behaviors of the system. The light curve is for the spiking behavior, and the dark curve is for the nonfiring stable state. A magnification of B is presented in C, reflecting the small basin of attraction of the nonfiring state and the short distance in terms of number of open channels between the two states. (D–F) The corresponding behavior of A–C in the stochastic model. Channel fluctuations in the stochastic model spontaneously flip the system between the firing and the nonfiring states. (D) a typical voltage trace of the stochastic HH patch. Light and dark lines were used to emphasize the different segments of the trace. The corresponding phase-plane traces are shown in E and F. As can be seen in F, the system flips from the firing stable state (light trace) to the nonfiring stable state (dark trace), where it stays for a few cycles. Hence, the subthreshold oscillations in the top trace translate to small-size loops in the phase-plane. The system then flips back to the firing stable state (light trace).

ing the effect of various parameters on the subthreshold oscillations, such as the area of the membrane patch and the properties of the channels, will be addressed briefly in section 4, but a more complete study is yet to be performed.

4 Discussion

4.1 Stochastic Versus Deterministic HH Model. We have shown that with a realistically large number of ion channels, the inherent noise in channel operation critically determines the timing and dynamics of spike firings for the stochastic HH model. The reason for this strong effect of channel stochasticity is that near the threshold for spike firing, only a very small percentage of Na^+ and a small percentage of K^+ channels is open (the activation variables, m and n , are small near threshold). Consequently, the variability in membrane voltage near threshold for excitation is large, and this is reflected in the variability of spike firing time. We conclude that for a wide range of input parameters, the stochastic model captures important features of real neurons; these features are neglected in the deterministic model.

In agreement with the experimental results Mainen and Sejnowski (1995) and Nowak et al. (1997), the reliability and precision of spike timing in the stochastic HH model are very sensitive to the properties of the current input. The reliability and precision of the spike timing are high for strongly fluctuating inputs and decreases for smoother (e.g., DC) inputs. This study shows that this effect could be explained in terms of the relation between the instantaneous shape and amplitude of the input signal and the amplitude of channel fluctuations. Strongly fluctuating inputs “override” the inherent channel fluctuations, and the spike timing is dictated primarily by the input rather than by channel stochasticity. In contrast, channel fluctuations become relatively more significant for smooth inputs, and spike firing time becomes less reliable.

In addition to its effect of spike timing, channel stochasticity produces three additional phenomena that do not occur in the deterministic HH model but were all observed experimentally (e.g., see Guttman et al., 1980). Voltage membrane oscillations are seen for subthreshold current inputs, and they also occur between spikes for suprathreshold inputs. A detailed study of the amplitude and frequency of these voltage-dependent oscillations will be performed elsewhere. “Spontaneous” spikes (for subthreshold inputs) and “missing” spikes (for suprathreshold inputs) were also observed in this model. These three phenomena result from the “unmasking” of the nonfiring stable state in the HH model by the channel fluctuations. This state, which was largely neglected in the framework of the deterministic HH model, becomes a key player in determining voltage dynamics in the stochastic model.

4.2 Toward a More Realistic Stochastic Model of Neurons. This study gives only a qualitative explanation for the reliability behaviors of neurons, in particular, of neocortical pyramidal cells. First, the spatial domain of neurons was completely neglected. It is especially important to consider the filtering effect and the impedance load imposed by the soma and dendrites, as well as by the axon, on the excitable channels at the spike initiation zone. A multicompartmental model (possibly composed of an axon with several highly excitable nodes of Ranvier, separated by passive internodes and a few dendritic compartments) should be used to understand better the effect of channel stochasticity on the reliability and accuracy of spiking in neurons. In such a model, the input should impinge onto the dendritic compartments and be simulated by a barrage of synaptic conductance changes (rather than by current inputs). In this context, it is important to emphasize that in many neuron types, the dendritic membrane is endowed with excitable channels in low density, and this may imply a large variability (fluctuations) already in the receptive region of the neuron. In contrast, we expect that in the axon, most of the variability will arise in the compartment where the spikes are initiated and that, downstream along the axon, spike timing would be encoded very reliably and with high precision.¹ Still, complete failure may occur in axons at regions with a low safety factor for propagation (e.g., see Grossman, Parnas, & Spira, 1979).

The other severe simplification in this study is that it relies exclusively on the standard HH equations. Although important as a reference, one ought to inquire to what extent the results can be generalized to other excitable systems. After all, most neurons consist of a large variety of ion channel types (e.g., A current, persistent and slow-inactivating Na^+ currents, low threshold Ca^{+2} current) each with different density and kinetics. Moreover, based on direct measurements of single ion channels, different kinetic models could be assigned to each of the channel types (see Patlak, 1991; Vandenberg & Bezanilla, 1991; Marom, Salman, Lyakhov, & Braun, 1996) although their main features are usually similar. Indeed, a more realistic model of the spike-generating mechanism that takes into account various channel types, kinetic schemes, and voltage dependence, as well as long-term (memory) effects, should be explored (see Vandenberg & Bezanilla, 1991; Sigworth, 1993;

¹ One might wonder if the uncertainty engendered by the bistability in the HH model would make spike propagation along the axon impossible. If, at each site in the axon, there is some probability that the system will go into a stable nonfiring state, the spike may fail somewhere along the axon. Also, a significant noise in the axon (see Rubinstein, 1995) may destroy temporal correlations between the output synapses. However, except for the compartment where the spike may, or may not, be initiated as a result of the depolarizing synaptic current, all other axonal compartments downstream receive relatively sharp and large current input from the spike in the previous node. For such inputs, channel stochasticity will be masked, and, consequently, the axon is expected to act as a highly reliable delay line, adding only relatively small jitter (see Horikawa, 1991, 1993; Abeles & Lass, 1975; Lass & Abeles, 1975).

Toib, Lyakhov, & Marom, 1998; Abbott, Turrigiano, LeMasson, & Marder, 1996; Fleidervish, Friedman, & Gutnick, 1996).

Still, we can state with confidence that the surprisingly large effect of channel stochasticity is likely to persist for other models. The important parameter that determines the size of fluctuations near threshold for spike firing is the number of open channels in this voltage regime. To the best of our knowledge, in all existing models of excitability, only a small percentage of the total number of excitable channels is open near threshold. Consequently, a large variability in spike firing time is also expected in these models. Clearly the exact nature of spike firing reliability will depend on channel properties.

What about the subthreshold membrane oscillations, spontaneous spikes, and missing spikes? The nature of the bistability of the HH model, which is set by its inverted-Hopf bifurcation, is what “enables” the channel noise to switch the system spontaneously between its two stable states. This is the source of the subthreshold membrane oscillations, the “missing” and the “spontaneous” spikes. These phenomena may not occur in models with different types of stabilities (e.g., those with saddle-node bifurcation), and other phenomena may then arise (see Rinzel & Ermentrout, 1989; White, Budde, & Kay, 1995; Longtin & Hinzer, 1996). Indeed, our initial simulations of the spike initiation zone in a cortical pyramidal neurons using parameters taken from Mainen et al. (1995) and Rapp et al. (1996) better replicate the responses of cortical pyramidal cells to the different stimuli given in Mainen and Sejnowski (1995) and Nowak et al. (1997).

4.3 Sources of Noise in Neurons. In this study we considered only the effect of one source of noise in neurons: the intrinsic stochastic nature of the ion channels. A variety of other sources of noise exists, such as spontaneous synaptic release and variability in the number of transmitter molecules and the number of available receptors. Other possible sources of neuronal noise are changes in intracellular and extracellular ion concentrations and in the concentration of neuromodulators, as well as in the activity of ion pumps. Ephaptic interactions (electric field effect) of one neuron on other neurons is yet another possible source of noise.

Our study shows that the intrinsic channel stochasticity should be considered as a key source of the variability of action potential timing. Clearly the other possible sources of noise should also be considered in order to quantify the relative contribution of each of these sources or their possible synergistic effect. Experimental studies should be performed to clarify this issue by using different manipulations—for example, blocking synaptic receptors (as did Mainen & Sejnowski, 1995), using a dynamic clamp to “replace” the noisy channel conductance with a deterministic conductance (Sharp, O’Neil, Abbott, & Marder, 1993), blocking specific ion channels and observing the resultant changes in membrane noise under voltage-clamp conditions (see initial results in this direction by Volgushev et al., 1998),

and blocking ion pumps. From a theoretical viewpoint, as shown by Jensen and Gartner (1997), a simple additive noise could qualitatively reproduce the differences in reliability and accuracy of spike timing in response to DC versus the fluctuating input found experimentally. However, because the neuronal noise is both voltage and activity dependent, it is clear that a simple additive noise is only a first-order approximation to the real case. The biophysically inspired model of channel noise is inherently voltage and activity dependent. The difference between models with simple additive noise and models with more realistic noise in terms of the fine temporal structure of spike firing requires further exploration.

4.4 Implication for Neural Coding. The reliability and accuracy of the neuron firing, in both the stochastic HH model and in cortical neurons (Mainen & Sejnowski, 1995; Bair & Koch, 1996), as well as in other neurons (de Ruyter van Steveninck et al., 1997), range between an unreliable response to DC inputs and a very reliable response to large-amplitude, highly fluctuating inputs. The actual current that reaches the site of spike initiation in neurons varies between these two extreme input patterns; its exact nature is determined by the degree of correlation among the synaptic inputs that impinge onto the neuron. Highly correlated synaptic activity gives rise to sharp current transients, whereas uncorrelated synaptic activity gives rise to smoother current traces. Our modeling results suggest that the neuron's most basic machinery—the ion channels—enables it to act as a “smart” encoder. Slowly varying inputs are coded with low reliability and accuracy, and, hence, the information about such inputs is encoded almost exclusively by the spike *rate*. Trying to decode information about such an input, using the exact temporal structure of the spike train, would result in decoding the internal noise of the cell rather than decoding the input. On the other hand, correlated inputs are encoded with higher reliability and accuracy, giving more of a “temporal” code, that is, information about the input exists in the exact timing of the spikes.

It is clear that in such a system, correlated activity of a population of neurons is likely to propagate within the network with high temporal precision, as suggested by Abeles (1991) in his “synfire” model and by the recent work by Marsalek, Koch, & Maunsell, (1997). In contrast, weakly correlated activity would propagate in an imprecise temporal manner and is more likely to decay within the network. The fact that the intrinsic noise of neurons may serve as a mechanism to destroy propagation of random correlations and, at the same time, allow for an accurate chains of activity to persist within the network has no bearing on the question of whether such chains do exist. Still, it is tempting to hypothesize that neurons do use their intrinsic channel stochasticity to exploit temporal code in addition to using rate code (see Bialek, Rieke, de Ruyter van Steveninck, & Warland, 1991; de Ruyter van Steveninck et al., 1997; Hopfield, 1995; Theunissen & Miller, 1995; Abbott, 1994; Softky, 1995; Heller, Hertz, Kjaer, & Richmond, 1995).

In addition to its significance for information coding, the relatively small size of the channel pool in the spike initiation zone has further computational implications. One clear advantage of such a limited channel pool was demonstrated in the work of Toib et al. (1998) (see also Marom, 1998), which shows that channel inactivation and reactivation kinetics have a significant, long-lasting (minutes) effect on the “availability” of channels, providing the neuron with an effective memory. Thus, the output spike train depends on both the properties of the instantaneous synaptic input and the history of the presynaptic and postsynaptic activity. This memory is embedded in the distribution of channel states in the spike initiation site. The nature and resolution of this memory depend on the size of the channel pool and on the kinetics and number of states of the channels. We hypothesize that the number of channels in the spike initiation zone may be optimized in some sense to give the reliability and accuracy discussed above, together with a short-term memory of the neuron’s activity. In this context, it is interesting to mention the work of Marder, Abbott, Turrigiano, Liu, and Golowasch (1996) and Abbott et al. (1996), which demonstrates activity-dependent long-term changes in the properties of intrinsic membrane currents.

Another important effect of stochasticity in a limited pool of channels are the subthreshold and suprathreshold membrane oscillations. Such oscillations were observed in neocortical neurons (see Gutfreund et al., 1995) as well as in other neuron types (Hutcheon et al., 1994; Lampl & Yarom, 1997) and were suggested to serve as the underlying clock for neurons firing and even as a synchronizing and binding mechanism for neuronal activity (Hopfield, 1995; Volgushev et al., 1997). In the stochastic HH model, these voltage oscillations result from the channel noise; in other systems, other mechanisms may be responsible for these oscillations (e.g., Gutfreund et al., 1995; White et al., 1995).

Finally, we suggest that channel stochasticity is likely to be a key player in setting neurons’ firing patterns, and thus it should be incorporated in models that explore the firing variability and spike timing of cortical neurons. It seems that channel stochasticity would be dominant in models that assume balanced excitation-inhibition (see Shadlen & Newsome, 1994, 1995; Softky, 1995; Bell, Mainen, Tsodyks, & Sejnowski, 1995), in which the effective resting membrane voltage of the cell is near threshold. Bell et al. (1995) and Troyer and Miller (1997) suggest complex repolarization and refractoriness schemes as another source for the high firing variability in neocortical neurons, which in many ways coincide with the effect of channel fluctuations. Whatever the correct model might be, the main message of this study is that the noise inherent in the activity of ion channels must be considered if one wishes to understand what determines the firing patterns of neurons and, consequently, the nature of the neural code.

Acknowledgments

We are grateful to our friends and colleagues, Miki London, Yosi Yarom, John Rinzel, David Hansel, Shimon Marom, Zach Mainen, Henry Markram, Tali Tishby, and Moshe Abeles, for their insightful input during various stages of this work. This study was supported by grants from the ONR, the Human Frontiers, and the Israel Academy of Science.

References

- Abbott, L. (1994). Decoding neuronal firing and modeling neural networks. *Quarter. Rev. Biophys.*, *27*, 291–331.
- Abbott, L., Turrigiano, G., LeMasson, G., & Marder, E. (1996). Activity-dependent conductances in model and biological neurons. In D. Waltz (Ed.), *Natural and artificial parallel computing* (pp. 43–68). Philadelphia: SIAM.
- Abeles, M. (1991). *Corticonics: Neural circuits of the cerebral cortex*. Cambridge: Cambridge University Press.
- Abeles, M., & Lass, Y. (1975). Transmission of information by the axon: II. The channels capacity. *Biol. Cybern.*, *19*, 121–125.
- Bair, W., & Koch, C. (1996). Temporal precision of spike trains in extrastriate cortex of the behaving macaque monkey. *Neural Comp.*, *8*, 1185–1202.
- Bell, A., Mainen, Z., Tsodyks, M., & Sejnowski, T. (1995). “Balancing” of conductances may explain irregular cortical spiking (Tech. Rep. No. INC-9502). San Diego: Institute for Neural Computation, University of California at San Diego.
- Berzukov, S., & Vodyanoy, I. (1995). Noise-induced enhancement of signal transduction across voltage-dependent ion channels. *Nature*, *378*, 362–364.
- Bialek, W., Rieke, F., de Ruyter van Steveninck, R., & Warland, D. (1991). Reading a neural code. *Science*, *252*, 1854–1857.
- Braun, H., Huber, M., Dewald, M., Schafer, K., & Voigt, K. (1998). Computer simulations of neuronal signal transduction: The role of nonlinear dynamics and noise. *Int. J. Bifurc. Chaos*, in press.
- Chow, C., & White, J. (1996). Spontaneous action potentials due to channel fluctuations. *Biophys. J.*, *71*, 3013–3021.
- Clay, J., & DeFelice, L. (1983). Relationship between membrane excitability and single channel open-close kinetics. *Biophys. J.*, *42*, 151–157.
- Cooley, J., Dodge, F., & Cohen, H. (1965). Digital computer solutions for excitable membrane models. *J. Cell. Comp. Physiol.*, *66*, 99–100.
- de Ruyter van Steveninck, R., Lewen, G., Strong, S., Koberle, R., & Bialek, W. (1997). Reproducibility and variability in neural spike trains. *Science*, *275*, 1805–1808.
- DeFelice, L., & Isaac, A. (1992). Chaotic states in random world. *J. Stat. Phys.*, *70*, 339–352.
- Fitzhugh, R. (1965). A kinetic model of the conductance changes in nerve membrane. *J. Cell. Comp. Physiol.*, *66*, 111–118.
- Fleidervish, I., Friedman, A., & Gutnick, M. (1996). Slow inactivation of Na⁺

- current and slow cumulative spike adaptation in mouse and guinea-pig neocortical neurons in slices. *J. Physiol.*, *493*, 83–97.
- Grossman, Y., Parnas, I., & Spira, M. (1979). Differential conduction block in branches of a bifurcating axon. *J. Physiol.*, *295*, 283–305.
- Gutfreund, Y., Yarom, Y., & Segev, I. (1995). Subthreshold oscillations and resonant frequency in guinea-pig cortical neurons: Physiology and modeling. *J. Physiol.*, *483*, 621–640.
- Guttman, R., Lewis, S., & Rinzel, J. (1980). Control of repetitive firing in squid axon membrane as a model for a neuroneoscillator. *J. Physiol.*, *305*, 377–395.
- Heller, J., Hertz, J., Kjaer, T., & Richmond, B. (1995). Information flow and temporal coding in primate pattern vision. *J. Comput. Neurosci.*, *2*, 175–193.
- Hille, B. (1992) *Ionic channels of excitable membrane* (2nd ed.). Sunderland, MA: Sinauer Associates.
- Hodgkin, A., & Huxley, A. (1952). A quantitative description of membrane current and its application to conduction and excitation in nerve. *J. Physiol. (London)*, *117*, 500–544.
- Hopfield, J. (1995). Pattern recognition computation using action potential timing for stimulus representation. *Nature*, *376*, 33–36.
- Horikawa, H. (1991). Noise effects on spike propagation in the stochastic Hodgkin Huxley model. *Biol. Cybern.* *66*, 19–30.
- Horikawa, H., (1993). Simulation study on effects of channel noise on differential conduction at an axon branch. *Biophys. J.*, *65*, 680–686.
- Hutcheon, B., Miura, R., Yarom, Y., & Pail, E. (1994). Low threshold calcium current and resonance in thalamic neurons: A model of frequency preference. *J. Neurophysiol.*, *71*, 583–594.
- Jensen, R., & Gartner, D. (1997). *Synchronization of randomly driven nonlinear oscillators and the reliable firing of cortical neurons*. Paper presented at the Computational Neuroscience Meeting, Big Sky, MT.
- Klink, R., & Alonso, A. (1993). Ionic mechanisms for the subthreshold oscillations and differential electroresponsiveness of medial entorhinal cortex layer II neurons. *J. Neurophysiol.*, *70*, 144–157.
- Lampl, I., & Yarom, Y. (1997). Subthreshold oscillations and resonant behavior: Two manifestations of the same mechanism. *Neuroscience*, *78* (2), 325–341.
- Lass, Y., & Abeles, M. (1975). Transmission of information by the axon: I. Noise and memory in the myelinated nerve fiber of the frog. *Biol. Cybern.*, *19*, 61–67.
- Lecar, H., & Nossal, R. (1971a). Theory of threshold fluctuations in nerves. I. Relationships between electrical noise and fluctuations in axon firing. *Biophys. J.*, *11*, 1048–1067.
- Lecar, H. & Nossal, R. (1971b). Theory of fluctuations in nerves. II. Analysis of various sources of membrane noise. *Biophys. J.*, *11*, 1068–1084.
- Longtin, A., & Hinzer, K. (1996). Encoding with bursting, subthreshold oscillations, and noise in mammalian cold receptors. *Neural Comp.*, *8*, 215–255.
- Mainen, Z., Joerges, J., Huguenard, J., & Sejnowski, T. (1995). A model of spike initiation in neocortical pyramidal neurons. *Neuron*, *15*, 1427–1439.
- Mainen, Z. F., & Sejnowski, T. (1995). Reliability of spike timing in neocortical neurons. *Science*, *268*, 1503–1508.

- Marder, E., Abbott, L., Turrigiano, G., Liu, Z., & Golowasch, J. (1996). Memory from the dynamics of intrinsic membrane currents. *Proc. Natl. Acad. Sci. USA*, *93*, 13481–13486.
- Marom, S. (1998). Slow changes in the availability of voltage-gated ion channels: Effects on the dynamics of excitable membranes. *J. Mem. Biol.*, *161*, 105–113.
- Marom, S., Salman, H., Lyakhov, V., & Braun, E. (1996). Effects of density and gating delayed-rectifier potassium channels on resting membranes potential and its fluctuations. *J. Membrane Biol.*, *154*, 267–274.
- Marsalek, P., Koch, C., & Maunsell, J. (1997). On the relationship between synaptic input and spike output jitter in individual neurons. *Proc. Natl. Acad. Sci. USA*, *94*, 735–740.
- Nowak, L., Sanches-Vives, M., & McCormick, D. (1997). Influence of low and high frequency inputs on spike timing in visual cortical neurons. *Cerebral Cortex*, *7*, 487–501.
- Patlak, J. (1991). Molecular kinetics of voltage-dependent Na⁺ channels. *Physiological Rev.*, *71*(4), 1047–1080.
- Press, W., Teukolsky, S., Vetterling, W., & Flannery, B. (1992). *Numerical recipes in C: The art of scientific computing* (2nd ed.). Cambridge: Cambridge University Press.
- Rapp, M., Yarom, Y., & Segev, I. (1996). Modeling back propagating action potential in weakly excitable dendrites of neocortical pyramidal cells. *Proc. Natl. Acad. Sci. USA*, *93*, 11985–11990.
- Reich, D., Victor, J., Knight, B., Ozaki, T., & Kaplan, E. (1997). Response variability and timing precision of neuronal spike trains in vivo. *J. Neurophysiol.*, *77*, 2836–2841.
- Rinzel, J., & Ermentrout, B. (1989). Analysis of neural excitability and oscillations. In C. Koch & I. Segev (Eds.), *Methods in neuronal modeling* (pp. 135–169). Cambridge, MA: MIT Press.
- Rubinstein, J. (1995). Threshold fluctuations in an N sodium channel model of the node of ranvier. *Biophys. J.*, *68*, 779–785.
- Sakmann, B., & Neher, E. (1995). *Single-channel recording* (2nd ed.). New York: Plenum.
- Schneidman, E., Freedman, B., & Segev, I. (1997). *Spike timing reliability in a stochastic Hodgkin-Huxley model*. Paper presented at the Computational Neuroscience Meeting, Big Sky, MT.
- Shadlen, M., & Newsome, W. (1994). Noise, neural codes and cortical organization. *Curr. Opin. Neurobiol.*, *4*, 569–579.
- Shadlen, M., & Newsome, W. (1995). Is there a signal in the noise? *Curr. Opin. Neurobiol.*, *5*, 248–250.
- Sharp, A., O'Neil, M., Abbott, L., & Marder, E. (1993). The dynamic champ: Artificial conductances in biological neurons. *Trends in Neurosci.*, *16*, 389–394.
- Sigworth, F. (1993). Voltage gating of ion channels. *Quarter. Rev. Biophys.*, *27*, 1–40.
- Skaugen, E., & Walløe, L. (1979). Firing behavior in a stochastic nerve membrane model based upon the Hodgkin-Huxley equations. *Acta Physiol. Scand.*, *107*, 343–363.

- Softky, W. (1995). Simple codes versus efficient codes. *Curr. Opin. Neurobiol.*, *5*, 239–247.
- Strassberg, A., & DeFelice, L. (1993). Limitations of the Hodgkin-Huxley formalism: Effects of single channel kinetics on transmembrane voltage dynamics. *Neural Computation*, *5*, 843–856.
- Tang, A., Bartels, A., & Sejnowski, T. (1997). Cholinergic modulation preserves spike timing under physiologically realistic fluctuating input. In M. C. Mozer, M. I. Jordan, & T. Petsche (Eds.), *Advances in neural information processing systems*, *9* (pp. 866–872). Cambridge, MA: MIT Press.
- Theunissen, F., & Miller, J. (1995). Temporal encoding in nervous systems: A rigorous definition. *J. Comp. Neurosci.*, *2*, 149–162.
- Toib, A., Lyakhov, V., & Marom, S. (1998). Interaction between duration of activity and rate of recovery from slow inactivation in mammalian brain Na⁺ channels. *J. Neurosci.*, *18*, 1893–1903.
- Troyer, T., & Miller, K. (1997). Physiological gain leads to high ISI variability in a simple model of a cortical regular spiking cell. *Neural Computation*, *9*, 733–745.
- Vandenberg, C., & Bezanilla, F. (1991). A sodium channel gating model based on single channel, macroscopic ionic, and gating currents in the squid axon. *Biophys. J.*, *60*, 1511–1533.
- Volgushev, M., Christiakova, M., & Singer, W. (1998). Modification of discharge patterns of neocortical neurons by induced oscillations of the membrane potential. *Neuroscience*, *83*, 15–25.
- White, J., Budde, T., & Kay, A. (1995). A bifurcation analysis of neuronal sub-threshold oscillations. *Biophys. J.*, *69*, 1203–1217.
- Yamada, W., Koch, C., & Adams, P. (1989). Multiple channels and calcium dynamics. in C. Koch & I. Segev (Eds.), *Methods in neuronal modeling* (pp. 97–134). Cambridge, MA: MIT Press.

This article has been cited by:

1. Xinmeng Guo, Haitao Yu, Nathan X. Kodama, Jiang Wang, Roberto F. Galán. 2020. Fluctuation Scaling of Neuronal Firing and Bursting in Spontaneously Active Brain Circuits. *International Journal of Neural Systems* 30:01, 1950017. [[Crossref](#)]
2. Alexandre Melanson, André Longtin. 2019. Data-driven inference for stationary jump-diffusion processes with application to membrane voltage fluctuations in pyramidal neurons. *The Journal of Mathematical Neuroscience* 9:1. . [[Crossref](#)]
3. Zahra Vahdat, Z. Xu, Abhyudai Singh. Modeling and characterization of neuronal synapses using stochastic hybrid systems 4729-4734. [[Crossref](#)]
4. Aleksandra Świetlicka, Krzysztof Kolanowski, Rafał Kapela. 2019. Training the Stochastic Kinetic Model of Neuron for Calculation of an Object's Position in Space. *Journal of Intelligent & Robotic Systems* 3. . [[Crossref](#)]
5. Xin Fu, Yuguo Yu. 2019. Reliable and efficient processing of sensory information at body temperature by rodent cortical neurons. *Nonlinear Dynamics* 98:1, 215-231. [[Crossref](#)]
6. Ram Kuber Singh, Ying Xu, Runchun Wang, Tara Julia Hamilton, Susan L. Denham, Andre van Schaik. 2019. CAR-Lite: A Multi-Rate Cochlear Model on FPGA for Spike-Based Sound Encoding. *IEEE Transactions on Circuits and Systems I: Regular Papers* 66:5, 1805-1817. [[Crossref](#)]
7. Seyed Ali Sadegh Zadeh, Chandra Kambhampati. A Computational Investigation of the Role of Ion Gradients in Signal Generation in Neurons 291-304. [[Crossref](#)]
8. Sebastian Reinartz. Long-Term Activity Dynamics of Single Neurons and Networks 331-350. [[Crossref](#)]
9. Jan-Hendrik Schleimer, Susanne Schreiber. 2018. Phase-response curves of ion channel gating kinetics. *Mathematical Methods in the Applied Sciences* 41:18, 8844-8858. [[Crossref](#)]
10. Zhilai Yang, Qilian Tan, Dan Cheng, Lei Zhang, Jiqian Zhang, Er-wei Gu, Weiping Fang, Xianfu Lu, Xuesheng Liu. 2018. The Changes of Intrinsic Excitability of Pyramidal Neurons in Anterior Cingulate Cortex in Neuropathic Pain. *Frontiers in Cellular Neuroscience* 12. . [[Crossref](#)]
11. Srdjan D. Antic, Michael Hines, William W. Lytton. 2018. Embedded ensemble encoding hypothesis: The role of the "Prepared" cell. *Journal of Neuroscience Research* 96:9, 1543-1559. [[Crossref](#)]
12. Alessandra Paffi, Francesca Camera, Chiara Carocci, Francesca Apollonio, Micaela Liberti. 2018. Stimulation Strategies for Tinnitus Suppression in a Neuron Model. *Computational and Mathematical Methods in Medicine* 2018, 1-9. [[Crossref](#)]
13. Pavol Bauer, Stefan Engblom, Sanja Mikulovic, Aleksandar Senek. 2018. Multiscale modelling via split-step methods in neural firing. *Mathematical and Computer Modelling of Dynamical Systems* 24:4, 426-445. [[Crossref](#)]
14. Yujiang Liu, Yuan Yue, Yuguo Yu, Liwei Liu, Lianchun Yu. 2018. Effects of channel blocking on information transmission and energy efficiency in squid giant axons. *Journal of Computational Neuroscience* 44:2, 219-231. [[Crossref](#)]
15. Aleksandra Świetlicka, Krzysztof Kolanowski, Rafał Kapela, Mirosław Galicki, Andrzej Rybarczyk. 2018. Investigation of generalization ability of a trained stochastic kinetic model of neuron. *Applied Mathematics and Computation* 319, 115-124. [[Crossref](#)]
16. Christopher J. Lingle, Pedro L. Martinez-Espinosa, Laura Guarina, Emilio Carbone. 2018. Roles of Na⁺, Ca²⁺, and K⁺ channels in the generation of repetitive firing and rhythmic bursting in adrenal chromaffin cells. *Pflügers Archiv - European Journal of Physiology* 470:1, 39-52. [[Crossref](#)]
17. Rodrigo Cofré, Cesar Maldonado. 2018. Information Entropy Production of Maximum Entropy Markov Chains from Spike Trains. *Entropy* 20:1, 34. [[Crossref](#)]
18. Adrianna Loback, Jason Prentice, Mark Ioffe, Michael Berry II. 2017. Noise-Robust Modes of the Retinal Population Code Have the Geometry of "Ridges" and Correspond to Neuronal Communities. *Neural Computation* 29:12, 3119-3180. [[Abstract](#)] [[Full Text](#)] [[PDF](#)] [[PDF Plus](#)]
19. Alison I. Weber, Jonathan W. Pillow. 2017. Capturing the Dynamical Repertoire of Single Neurons with Generalized Linear Models. *Neural Computation* 29:12, 3260-3289. [[Abstract](#)] [[Full Text](#)] [[PDF](#)] [[PDF Plus](#)]
20. Simone Orcioni, Alessandra Paffi, Francesca Camera, Francesca Apollonio, Micaela Liberti. 2017. Automatic decoding of input sinusoidal signal in a neuron model: Improved SNR spectrum by low-pass homomorphic filtering. *Neurocomputing* 267, 605-614. [[Crossref](#)]
21. Rajeshwari Iyer, Mark A. Ungless, Aldo A. Faisal. 2017. Calcium-activated SK channels control firing regularity by modulating sodium channel availability in midbrain dopamine neurons. *Scientific Reports* 7:1. . [[Crossref](#)]
22. M. A. Lopes, K.-E. Lee, A. V. Goltsev. 2017. Neuronal network model of interictal and recurrent ictal activity. *Physical Review E* 96:6. . [[Crossref](#)]

23. Abhyudai Singh. Noise mechanisms in synaptic transmission and their impact on spike-timing precision 5925-5930. [[Crossref](#)]
24. Francesca Scarsi, Jacopo Tessadori, Michela Chiappalone, Valentina Pasquale. 2017. Investigating the impact of electrical stimulation temporal distribution on cortical network responses. *BMC Neuroscience* **18**:1. . [[Crossref](#)]
25. Lianchun Yu, Yuguo Yu. 2017. Energy-efficient neural information processing in individual neurons and neuronal networks. *Journal of Neuroscience Research* **95**:11, 2253-2266. [[Crossref](#)]
26. Rukiye Uzun, Ergin Yilmaz, Mahmut Ozer. 2017. Effects of autapse and ion channel block on the collective firing activity of Newman–Watts small-world neuronal networks. *Physica A: Statistical Mechanics and its Applications* **486**, 386-396. [[Crossref](#)]
27. Milos Radivojevic, Felix Franke, Michael Altermatt, Jan Müller, Andreas Hierlemann, Douglas J Bakkum. 2017. Tracking individual action potentials throughout mammalian axonal arbors. *eLife* **6**. . [[Crossref](#)]
28. Ashesh K. Dhawale, Maurice A. Smith, Bence P. Ölveczky. 2017. The Role of Variability in Motor Learning. *Annual Review of Neuroscience* **40**:1, 479-498. [[Crossref](#)]
29. Chandan Singh, William B. Levy. 2017. A consensus layer V pyramidal neuron can sustain interpulse-interval coding. *PLOS ONE* **12**:7, e0180839. [[Crossref](#)]
30. Nicola Romanò, Heather McClafferty, Jamie J. Walker, Paul Le Tissier, Michael J. Shipston. 2017. Heterogeneity of Calcium Responses to Secretagogues in Corticotrophs From Male Rats. *Endocrinology* **158**:6, 1849-1858. [[Crossref](#)]
31. Meili Lu, Xile Wei. Desynchronizing of noisy neuron networks using reinforcement learning 296-299. [[Crossref](#)]
32. Rukiye Uzun, Mahmut Ozer. Effects of autaptic coupling strength and ion channel blockage on firing regularity in a Hodgkin-Huxley neuron 1-4. [[Crossref](#)]
33. Jason Boulet, Ian C. Bruce. 2017. Predictions of the Contribution of HCN Half-Maximal Activation Potential Heterogeneity to Variability in Intrinsic Adaptation of Spiral Ganglion Neurons. *Journal of the Association for Research in Otolaryngology* **18**:2, 301-322. [[Crossref](#)]
34. Haitao Yu, Roberto F. Galán, Jiang Wang, Yibin Cao, Jing Liu. 2017. Stochastic resonance, coherence resonance, and spike timing reliability of Hodgkin–Huxley neurons with ion-channel noise. *Physica A: Statistical Mechanics and its Applications* **471**, 263-275. [[Crossref](#)]
35. Marijn B. Martens, Arthur R. Houweling, Paul H. E. Tiesinga. 2017. Anti-correlations in the degree distribution increase stimulus detection performance in noisy spiking neural networks. *Journal of Computational Neuroscience* **42**:1, 87-106. [[Crossref](#)]
36. Mohammad Saeed Feali, Arash Ahmadi. 2017. Realistic Hodgkin–Huxley Axons Using Stochastic Behavior of Memristors. *Neural Processing Letters* **45**:1, 1-14. [[Crossref](#)]
37. Xinmeng Guo, Jiang Wang, Jing Liu, Haitao Yu, Roberto F. Galán, Yibin Cao, Bin Deng. 2017. Optimal time scales of input fluctuations for spiking coherence and reliability in stochastic Hodgkin–Huxley neurons. *Physica A: Statistical Mechanics and its Applications* **468**, 381-390. [[Crossref](#)]
38. Rishi R. Dhingra, Mathias Dutschmann, Roberto F. Galán, Thomas E. Dick. 2017. Kölliker-Fuse nuclei regulate respiratory rhythm variability via a gain-control mechanism. *American Journal of Physiology-Regulatory, Integrative and Comparative Physiology* **312**:2, R172-R188. [[Crossref](#)]
39. Kevin Burrage, Pamela Burrage, Andre Leier, Tatiana Marquez-Lago. A Review of Stochastic and Delay Simulation Approaches in Both Time and Space in Computational Cell Biology 241-261. [[Crossref](#)]
40. Aleksandra Świetlicka, Karol Gugala, Witold Pedrycz, Andrzej Rybarczyk. 2017. Development of the deterministic and stochastic Markovian model of a dendritic neuron. *Biocybernetics and Biomedical Engineering* **37**:1, 201-216. [[Crossref](#)]
41. Haitao Yu, Xinmeng Guo, Jiang Wang. 2017. Stochastic resonance enhancement of small-world neural networks by hybrid synapses and time delay. *Communications in Nonlinear Science and Numerical Simulation* **42**, 532-544. [[Crossref](#)]
42. Malu Zhang, Hong Qu, Xiurui Xie, Jürgen Kurths. 2017. Supervised learning in spiking neural networks with noise-threshold. *Neurocomputing* **219**, 333-349. [[Crossref](#)]
43. Haitao Yu, Rishi R. Dhingra, Thomas E. Dick, Roberto F. Galán. 2017. Effects of ion channel noise on neural circuits: an application to the respiratory pattern generator to investigate breathing variability. *Journal of Neurophysiology* **117**:1, 230-242. [[Crossref](#)]
44. James C. Schaff, Fei Gao, Ye Li, Igor L. Novak, Boris M. Slepchenko. 2016. Numerical Approach to Spatial Deterministic-Stochastic Models Arising in Cell Biology. *PLOS Computational Biology* **12**:12, e1005236. [[Crossref](#)]
45. F. Grassia, T. Kohno, T. Levi. 2016. Digital hardware implementation of a stochastic two-dimensional neuron model. *Journal of Physiology-Paris* **110**:4, 409-416. [[Crossref](#)]

46. Elie Bertrand Megam Ngounkadi, Hilaire Bertrand Fotsin, Martial Kabong Nono, Patrick Herve Louodop Fotso. 2016. Noise effects on robust synchronization of a small pacemaker neuronal ensemble via nonlinear controller: electronic circuit design. *Cognitive Neurodynamics* **10**:5, 385-404. [[Crossref](#)]
47. Aleena M. Notary, Matthew J. Westacott, Thomas H. Hraha, Marina Pozzoli, Richard K. P. Benninger. 2016. Decreases in Gap Junction Coupling Recovers Ca²⁺ and Insulin Secretion in Neonatal Diabetes Mellitus, Dependent on Beta Cell Heterogeneity and Noise. *PLoS Computational Biology* **12**:9, e1005116. [[Crossref](#)]
48. Anqi Ling, Yandong Huang, Jianwei Shuai, Yueheng Lan. 2016. Channel based generating function approach to the stochastic Hodgkin-Huxley neuronal system. *Scientific Reports* **6**:1. . [[Crossref](#)]
49. Philipe RF Mendonça, Mariana Vargas-Caballero, Ferenc Erdélyi, Gábor Szabó, Ole Paulsen, Hugh PC Robinson. 2016. Stochastic and deterministic dynamics of intrinsically irregular firing in cortical inhibitory interneurons. *eLife* **5**. . [[Crossref](#)]
50. Lianchun Yu, Chi Zhang, Liwei Liu, Yuguo Yu. 2016. Energy-efficient population coding constrains network size of a neuronal array system. *Scientific Reports* **6**:1. . [[Crossref](#)]
51. Christopher M. Lee, Ahmad F. Osman, Maxim Volgushev, Monty A. Escabí, Heather L. Read. 2016. Neural spike-timing patterns vary with sound shape and periodicity in three auditory cortical fields. *Journal of Neurophysiology* **115**:4, 1886-1904. [[Crossref](#)]
52. Ergin Yilmaz, Veli Baysal, Matjaž Perc, Mahmut Ozer. 2016. Enhancement of pacemaker induced stochastic resonance by an autapse in a scale-free neuronal network. *Science China Technological Sciences* **59**:3, 364-370. [[Crossref](#)]
53. Ergin Yilmaz, Veli Baysal, Mahmut Ozer, Matjaž Perc. 2016. Autaptic pacemaker mediated propagation of weak rhythmic activity across small-world neuronal networks. *Physica A: Statistical Mechanics and its Applications* **444**, 538-546. [[Crossref](#)]
54. Moira L. Steyn-Ross, D. A. Steyn-Ross. 2016. From individual spiking neurons to population behavior: Systematic elimination of short-wavelength spatial modes. *Physical Review E* **93**:2. . [[Crossref](#)]
55. Mihai Alexandru Petrovici. Probabilistic Inference in Neural Networks 219-346. [[Crossref](#)]
56. Paul C. Bressloff. 2016. Diffusion in Cells with Stochastically Gated Gap Junctions. *SIAM Journal on Applied Mathematics* **76**:4, 1658-1682. [[Crossref](#)]
57. Wahiba Taouali, Giacomo Benvenuti, Pascal Wallisch, Frédéric Chavane, Laurent U. Perrinet. 2016. Testing the odds of inherent vs. observed overdispersion in neural spike counts. *Journal of Neurophysiology* **115**:1, 434-444. [[Crossref](#)]
58. Christoph Hartmann, Andreea Lazar, Bernhard Nessler, Jochen Triesch. 2015. Where's the Noise? Key Features of Spontaneous Activity and Neural Variability Arise through Learning in a Deterministic Network. *PLoS Computational Biology* **11**:12, e1004640. [[Crossref](#)]
59. Yandong Huang, Sten Rüdiger, Jianwei Shuai. 2015. Accurate Langevin approaches to simulate Markovian channel dynamics. *Physical Biology* **12**:6, 061001. [[Crossref](#)]
60. M. Uzuntarla, M. Ozer, U. Ileri, A. Calim, J. J. Torres. 2015. Effects of dynamic synapses on noise-delayed response latency of a single neuron. *Physical Review E* **92**:6. . [[Crossref](#)]
61. David T. Lee, Jehoshua Bruck. 2015. Algorithms for Generating Probabilities with Multivalued Stochastic Relay Circuits. *IEEE Transactions on Computers* **64**:12, 3376-3388. [[Crossref](#)]
62. Elad Ganmor, Ronen Segev, Elad Schneidman. 2015. A thesaurus for a neural population code. *eLife* **4**. . [[Crossref](#)]
63. Aleksandra Świetlicka. 2015. Trained stochastic model of biological neural network used in image processing task. *Applied Mathematics and Computation* **267**, 716-726. [[Crossref](#)]
64. Ergin Yilmaz, Veli Baysal, Mahmut Ozer. 2015. Enhancement of temporal coherence via time-periodic coupling strength in a scale-free network of stochastic Hodgkin-Huxley neurons. *Physics Letters A* **379**:26-27, 1594-1599. [[Crossref](#)]
65. Francesca Scarsi, Jacopo Tessadori, Valentina Pasquale, Michela Chiappalone. Impact of stimuli distribution on neural network responses 4761-4764. [[Crossref](#)]
66. Heather A. Brooks, Paul C. Bressloff. 2015. Quasicycles in the stochastic hybrid Morris-Lecar neural model. *Physical Review E* **92**:1. . [[Crossref](#)]
67. Alessandra Paffi, Francesca Camera, Francesca Apollonio, Guglielmo d'Inzeo, Micaela Liberti. 2015. Restoring the encoding properties of a stochastic neuron model by an exogenous noise. *Frontiers in Computational Neuroscience* **9**. . [[Crossref](#)]
68. Ergin Yilmaz, Mahmut Ozer. 2015. Delayed feedback and detection of weak periodic signals in a stochastic Hodgkin-Huxley neuron. *Physica A: Statistical Mechanics and its Applications* **421**, 455-462. [[Crossref](#)]
69. Alex Bukoski, D. A. Steyn-Ross, Moira L. Steyn-Ross. 2015. Channel-noise-induced critical slowing in the subthreshold Hodgkin-Huxley neuron. *Physical Review E* **91**:3. . [[Crossref](#)]

70. Yongguo Mei, Adria Carbo, Stefan Hoops, Raquel Hontecillas, Josep Bassaganya-Riera. 2015. ENISI SDE: A New Web-Based Tool for Modeling Stochastic Processes. *IEEE/ACM Transactions on Computational Biology and Bioinformatics* **12**:2, 289-297. [[Crossref](#)]
71. Wolfgang G. Bywalez, Dinu Patirniche, Vanessa Rupprecht, Martin Stemmler, Andreas V.M. Herz, Dénes Pálfi, Balázs Rózsa, Veronica Egger. 2015. Local Postsynaptic Voltage-Gated Sodium Channel Activation in Dendritic Spines of Olfactory Bulb Granule Cells. *Neuron* **85**:3, 590-601. [[Crossref](#)]
72. Katarína Bodová, David Paydarfar, Daniel B. Forger. 2015. Characterizing spiking in noisy type II neurons. *Journal of Theoretical Biology* **365**, 40-54. [[Crossref](#)]
73. David Golomb. 2014. Mechanism and Function of Mixed-Mode Oscillations in Vibrissa Motoneurons. *PLoS ONE* **9**:10, e109205. [[Crossref](#)]
74. Yadin Dudai, Kathinka Evers. 2014. To Simulate or Not to Simulate: What Are the Questions?. *Neuron* **84**:2, 254-261. [[Crossref](#)]
75. H. Watanabe, H. Tsubokawa, M. Tsukada, T. Aihara. 2014. Frequency-dependent signal processing in apical dendrites of hippocampal CA1 pyramidal cells. *Neuroscience* **278**, 194-210. [[Crossref](#)]
76. Benjamin Dummer, Stefan Wieland, Benjamin Lindner. 2014. Self-consistent determination of the spike-train power spectrum in a neural network with sparse connectivity. *Frontiers in Computational Neuroscience* **8**. . [[Crossref](#)]
77. Katerina D. Oikonomou, Mandakini B. Singh, Enas V. Sterjanaj, Srdjan D. Antic. 2014. Spiny neurons of amygdala, striatum, and cortex use dendritic plateau potentials to detect network UP states. *Frontiers in Cellular Neuroscience* **8**. . [[Crossref](#)]
78. Cian O'Donnell, Mark C. W. van Rossum. 2014. Systematic analysis of the contributions of stochastic voltage gated channels to neuronal noise. *Frontiers in Computational Neuroscience* **8**. . [[Crossref](#)]
79. Anton Spanne, Pontus Geborek, Fredrik Bengtsson, Henrik Järntell. 2014. Spike generation estimated from stationary spike trains in a variety of neurons in vivo. *Frontiers in Cellular Neuroscience* **8**. . [[Crossref](#)]
80. Pavel A. Puzerey, Roberto F. Galán. 2014. On how correlations between excitatory and inhibitory synaptic inputs maximize the information rate of neuronal firing. *Frontiers in Computational Neuroscience* **8**. . [[Crossref](#)]
81. Biswa Sengupta, Martin B. Stemmler. 2014. Power Consumption During Neuronal Computation. *Proceedings of the IEEE* **102**:5, 738-750. [[Crossref](#)]
82. Veli Baysal, Ergin Yilmaz, Mahmut Ozer. Impact of time-periodic coupling strength on the firing regularity of a scale-free network 1958-1961. [[Crossref](#)]
83. Ravinder Jerath, Kyler Harden, Molly Crawford, Vernon A. Barnes, Mike Jensen. 2014. Role of cardiorespiratory synchronization and sleep physiology: effects on membrane potential in the restorative functions of sleep. *Sleep Medicine* **15**:3, 279-288. [[Crossref](#)]
84. Rukiye Uzun, Mahmut Ozer, Matjaž Perc. 2014. Can scale-freeness offset delayed signal detection in neuronal networks?. *EPL (Europhysics Letters)* **105**:6, 60002. [[Crossref](#)]
85. Biswa Sengupta, Simon Barry Laughlin, Jeremy Edward Niven. 2014. Consequences of Converting Graded to Action Potentials upon Neural Information Coding and Energy Efficiency. *PLoS Computational Biology* **10**:1, e1003439. [[Crossref](#)]
86. Bruce Graham. Modeling the Axon 1-9. [[Crossref](#)]
87. Cian O'Donnell, Matthew F. Nolan. Stochastic Ion Channel Gating and Probabilistic Computation in Dendritic Neurons 397-414. [[Crossref](#)]
88. Klaus M. Stiefel, Benjamin Torben-Nielsen. Optimized Dendritic Morphologies for Noisy Inputs 147-158. [[Crossref](#)]
89. Ram Krips, Miriam Furst. 2014. Prediction of Human's Ability in Sound Localization Based on the Statistical Properties of Spike Trains along the Brainstem Auditory Pathway. *Computational Intelligence and Neuroscience* **2014**, 1-11. [[Crossref](#)]
90. Deena R Schmidt, Peter J Thomas. 2014. Measuring Edge Importance: A Quantitative Analysis of the Stochastic Shielding Approximation for Random Processes on Graphs. *The Journal of Mathematical Neuroscience* **4**:1, 6. [[Crossref](#)]
91. Asaf Gal, Shimon Marom. 2013. Self-organized criticality in single-neuron excitability. *Physical Review E* **88**:6. . [[Crossref](#)]
92. Yongguo Mei, Adria Carbo, Raquel Hontecillas, Josep Bassaganya-Riera. ENISI SDE: A novel web-based stochastic modeling tool for computational biology 392-397. [[Crossref](#)]
93. Martin Boerlin, Christian K. Machens, Sophie Denève. 2013. Predictive Coding of Dynamical Variables in Balanced Spiking Networks. *PLoS Computational Biology* **9**:11, e1003258. [[Crossref](#)]
94. Qingyun Wang, Yanhong Zheng, Jun Ma. 2013. Cooperative dynamics in neuronal networks. *Chaos, Solitons & Fractals* **56**, 19-27. [[Crossref](#)]
95. Brett A. Schmerl, Mark D. McDonnell. 2013. Channel-noise-induced stochastic facilitation in an auditory brainstem neuron model. *Physical Review E* **88**:5. . [[Crossref](#)]

96. Muhammet Uzuntarla, John R. Cressman, Mahmut Ozer, Ernest Barreto. 2013. Dynamical structure underlying inverse stochastic resonance and its implications. *Physical Review E* **88**:4. . [[Crossref](#)]
97. Marifi Güler. 2013. An Investigation of the Stochastic Hodgkin-Huxley Models Under Noisy Rate Functions. *Neural Computation* **25**:9, 2355-2372. [[Abstract](#)] [[Full Text](#)] [[PDF](#)] [[PDF Plus](#)]
98. Jay M. Newby, Paul C. Bressloff, James P. Keener. 2013. Breakdown of Fast-Slow Analysis in an Excitable System with Channel Noise. *Physical Review Letters* **111**:12. . [[Crossref](#)]
99. Lei Wang, Pu-Ming Zhang, Pei-Ji Liang, Yi-Hong Qiu. 2013. Enhancement of the Neuronal Dynamic Range by Proper Intensities of Channel Noise. *Chinese Physics Letters* **30**:7, 070506. [[Crossref](#)]
100. Karol Guęała, Aleksandra Świetlicka, Michał Burdajewicz, Andrzej Rybarczyk. 2013. Random number generation system improving simulations of stochastic models of neural cells. *Computing* **95**:S1, 259-275. [[Crossref](#)]
101. Aleksandra Świetlicka, Karol Guęała, Marta Kolasa, Jolanta Pauk, Andrzej Rybarczyk, Rafał Długosz. 2013. A New Model of the Neuron for Biological Spiking Neural Network Suitable for Parallel Data Processing Realized in Hardware. *Solid State Phenomena* **199**, 217-222. [[Crossref](#)]
102. S. Y. Zeng, Z. Z. Zhang, D. Q. Wei, X. S. Luo, W. Y. Tang, S. W. Zeng, R. F. Wang. 2013. Optimal physiological structure of small neurons to guarantee stable information processing. *EPL (Europhysics Letters)* **101**:3, 38005. [[Crossref](#)]
103. Marifi Güler. 2013. Stochastic Hodgkin-Huxley Equations with Colored Noise Terms in the Conductances. *Neural Computation* **25**:1, 46-74. [[Abstract](#)] [[Full Text](#)] [[PDF](#)] [[PDF Plus](#)]
104. YUBING GONG, LI WANG, BO XU. 2012. PERIODIC COUPLING STRENGTH-ENHANCED COHERENCE RESONANCE INDUCED BY A PARTICULAR KIND OF NON-GAUSSIAN NOISE IN NEURONAL NETWORKS. *Fluctuation and Noise Letters* **11**:04, 1250029. [[Crossref](#)]
105. Nicolaus T. Schmandt, Roberto F. Galán. 2012. Stochastic-Shielding Approximation of Markov Chains and its Application to Efficiently Simulate Random Ion-Channel Gating. *Physical Review Letters* **109**:11. . [[Crossref](#)]
106. David Lee, Jehoshua Bruck. Modeling biological circuits with urn functions 2756-2760. [[Crossref](#)]
107. Haroon Anwar, Iain Hepburn, Erik De Schutter. 2012. A computational study of stochastic mechanisms in dendritic calcium spike generation. *BMC Neuroscience* **13**:S1. . [[Crossref](#)]
108. Dmitry Zarubin, Ekaterina Zhuchkova, Susanne Schreiber. 2012. Effects of cooperative ion-channel interactions on the dynamics of excitable membranes. *Physical Review E* **85**:6. . [[Crossref](#)]
109. YUBING GONG, LI WANG, XIU LIN. 2012. EFFECT OF NON-GAUSSIAN CHANNEL NOISE ON COHERENCE RESONANCE IN COUPLED STOCHASTIC HODGKIN-HUXLEY NEURONS. *Fluctuation and Noise Letters* **11**:02, 1250013. [[Crossref](#)]
110. Patricio Orio, Daniel Soudry. 2012. Simple, Fast and Accurate Implementation of the Diffusion Approximation Algorithm for Stochastic Ion Channels with Multiple States. *PLoS ONE* **7**:5, e36670. [[Crossref](#)]
111. Ciara E. Dangerfield, David Kay, Kevin Burrage. 2012. Modeling ion channel dynamics through reflected stochastic differential equations. *Physical Review E* **85**:5. . [[Crossref](#)]
112. Jian-Ping Li, Lian-Chun Yu, Mei-Chen Yu, Yong Chen. 2012. Zero-Lag Synchronization in Spatiotemporal Chaotic Systems with Long Range Delay Couplings. *Chinese Physics Letters* **29**:5, 050501. [[Crossref](#)]
113. Gilles Wainrib, Michèle Thieullen, Khashayar Pakdaman. 2012. Reduction of stochastic conductance-based neuron models with time-scales separation. *Journal of Computational Neuroscience* **32**:2, 327-346. [[Crossref](#)]
114. Tanuj Aggarwal, Donatello Materassi, Robert Davison, Thomas Hays, Murti Salapaka. 2012. Detection of Steps in Single Molecule Data. *Cellular and Molecular Bioengineering* **5**:1, 14-31. [[Crossref](#)]
115. Dmitry R. Lyamzin, Jose A. Garcia-Lazaro, Nicholas A. Lesica. 2012. Analysis and modelling of variability and covariability of population spike trains across multiple time scales. *Network: Computation in Neural Systems* **23**:1-2, 76-103. [[Crossref](#)]
116. Xiu Lin, YuBing Gong, Li Wang, XiaoGuang Ma. 2012. Coherence resonance and bi-resonance by time-periodic coupling strength in Hodgkin-Huxley neuron networks. *Science China Chemistry* **55**:2, 256-261. [[Crossref](#)]
117. Wei-Lian Ning, Zheng-Zhen Zhang, Shang-You Zeng, Xiao-Shu Luo, Jin-Lin Hu, Shao-Wen Zeng, Yi Qiu, Hui-Si Wu. 2012. Coherence resonance in globally coupled neuronal networks with different neuron numbers. *Chinese Physics B* **21**:2, 028702. [[Crossref](#)]
118. A. Aldo Faisal. Noise in Neurons and Other Constraints 227-257. [[Crossref](#)]
119. Ergin Yilmaz, Mahmut Özer, Abdullah Cavuşoęlu. 2012. Impact of The Ion Channel Blockage on the Collective Spiking Regularity of a Scale-Free Neuronal Network. *Procedia Technology* **1**, 199-204. [[Crossref](#)]

120. Eduard Kuriscak, Petr Marsalek, Julius Stroffek, Zdenek Wunsch. 2012. The Effect of Neural Noise on Spike Time Precision in a Detailed CA3 Neuron Model. *Computational and Mathematical Methods in Medicine* **2012**, 1-16. [[Crossref](#)]
121. Nooraini Yusoff, Ioana Sporea, André Grüning. Neural Networks in Cognitive Science 58-83. [[Crossref](#)]
122. Xiu Lin, Yubing Gong, Li Wang. 2011. Multiple coherence resonance induced by time-periodic coupling in stochastic Hodgkin-Huxley neuronal networks. *Chaos: An Interdisciplinary Journal of Nonlinear Science* **21**:4, 043109. [[Crossref](#)]
123. S. Coombes, R. Thul, J. Laudanski, A. R. Palmer, C. J. Sumner. 2011. Neuronal spike-train responses in the presence of threshold noise. *Frontiers in Life Science* **5**:3-4, 91-105. [[Crossref](#)]
124. LI WANG, YUBING GONG, XIU LIN. 2011. ENHANCEMENT OF INTRINSIC SPIKING COHERENCE BY EXTERNAL NON-GAUSSIAN NOISE IN A STOCHASTIC HODGKIN-HUXLEY NEURON. *Fluctuation and Noise Letters* **10**:04, 359-369. [[Crossref](#)]
125. YUBING GONG, XIU LIN, LI WANG. 2011. EXTERNAL NON-GAUSSIAN NOISE-ENHANCED COLLECTIVE INTRINSIC SPIKING COHERENCE IN AN ARRAY OF STOCHASTIC HODGKIN-HUXLEY NEURONS. *Fluctuation and Noise Letters* **10**:04, 395-404. [[Crossref](#)]
126. Joshua H. Goldwyn, Eric Shea-Brown. 2011. The What and Where of Adding Channel Noise to the Hodgkin-Huxley Equations. *PLoS Computational Biology* **7**:11, e1002247. [[Crossref](#)]
127. Marifi Güler. 2011. Persistent membranous cross correlations due to the multiplicity of gates in ion channels. *Journal of Computational Neuroscience* **31**:3, 713-724. [[Crossref](#)]
128. Yubing Gong, Yinghang Hao, Xiu Lin, Li Wang, Xiaoguang Ma. 2011. Influence of time delay and channel blocking on multiple coherence resonance in Hodgkin-Huxley neuron networks. *Biosystems* **106**:2-3, 76-81. [[Crossref](#)]
129. Philippe Gastrein, émilie Campanac, Célia Gasselín, Robert H. Cudmore, Andrzej Bialowas, Edmond Carlier, Laure Fronzaroli-Molinieres, Norbert Ankri, Dominique Debanne. 2011. The role of hyperpolarization-activated cationic current in spike-time precision and intrinsic resonance in cortical neurons in vitro. *The Journal of Physiology* **589**:15, 3753-3773. [[Crossref](#)]
130. Olivier Caillard. 2011. Pre & Postsynaptic Tuning of Action Potential Timing by Spontaneous GABAergic Activity. *PLoS ONE* **6**:7, e22322. [[Crossref](#)]
131. David Lee, Jehoshua Bruck. Generating probability distributions using multivalued stochastic relay circuits 308-312. [[Crossref](#)]
132. Demitre Serletis, Osbert C. Zalay, Taufik A. Valiante, Berj L. Bardakjian, Peter L. Carlen. 2011. Complexity in Neuronal Noise Depends on Network Interconnectivity. *Annals of Biomedical Engineering* **39**:6, 1768-1778. [[Crossref](#)]
133. Ergin Yilmaz, Mahmut Ozer, Abdullah Cavusoglu. Effects of channel blocking on the spontaneous firing regularity of scale-free Hodgkin-Huxley neuronal network 581-584. [[Crossref](#)]
134. Ude Lu, Dong Song, T W Berger. 2011. Nonlinear Dynamic Modeling of Synaptically Driven Single Hippocampal Neuron Intracellular Activity. *IEEE Transactions on Biomedical Engineering* **58**:5, 1303-1313. [[Crossref](#)]
135. Risa J. Lin, Dieter Jaeger. 2011. Using computer simulations to determine the limitations of dynamic clamp stimuli applied at the soma in mimicking distributed conductance sources. *Journal of Neurophysiology* **105**:5, 2610-2624. [[Crossref](#)]
136. Joshua H. Goldwyn, Nikita S. Imennov, Michael Famulare, Eric Shea-Brown. 2011. Stochastic differential equation models for ion channel noise in Hodgkin-Huxley neurons. *Physical Review E* **83**:4. . [[Crossref](#)]
137. Dominique Debanne, Emilie Campanac, Andrzej Bialowas, Edmond Carlier, Gisèle Alcaraz. 2011. Axon Physiology. *Physiological Reviews* **91**:2, 555-602. [[Crossref](#)]
138. Daniele Linaro, Marco Storage, Michele Giugliano. 2011. Accurate and Fast Simulation of Channel Noise in Conductance-Based Model Neurons by Diffusion Approximation. *PLoS Computational Biology* **7**:3, e1001102. [[Crossref](#)]
139. YUBING GONG, XIU LIN, YINGHANG HAO, XIAOGUANG MA. 2011. NON-GAUSSIAN NOISE- AND COUPLING-INDUCED FIRING TRANSITIONS OF NEWMAN-WATTS NEURONAL NETWORKS. *Fluctuation and Noise Letters* **10**:01, 1-11. [[Crossref](#)]
140. Tilo Schwalger, Karin Fisch, Jan Benda, Benjamin Lindner. 2010. How Noisy Adaptation of Neurons Shapes Interspike Interval Histograms and Correlations. *PLoS Computational Biology* **6**:12, e1001026. [[Crossref](#)]
141. Jungah Lee, HyungGoo R. Kim, Choongkil Lee. 2010. Trial-to-Trial Variability of Spike Response of V1 and Saccadic Response Time. *Journal of Neurophysiology* **104**:5, 2556-2572. [[Crossref](#)]
142. Yinghang Hao, Yubing Gong, Xiu Lin, Yanhang Xie, Xiaoguang Ma. 2010. Transition and enhancement of synchronization by time delays in stochastic Hodgkin-Huxley neuron networks. *Neurocomputing* **73**:16-18, 2998-3004. [[Crossref](#)]

143. A. Mboussi Nkomidio, P. Woafu. 2010. Effects of imperfection of ionic channels and exposure to electromagnetic fields on the generation and propagation of front waves in nervous fibre. *Communications in Nonlinear Science and Numerical Simulation* **15**:9, 2350-2360. [[Crossref](#)]
144. STEFANO BONACCORSI, DELIO MUGNOLO. 2010. EXISTENCE OF STRONG SOLUTIONS FOR NEURONAL NETWORK DYNAMICS DRIVEN BY FRACTIONAL BROWNIAN MOTIONS. *Stochastics and Dynamics* **10**:03, 441-464. [[Crossref](#)]
145. Robert C. Cannon, Cian O'Donnell, Matthew F. Nolan. 2010. Stochastic Ion Channel Gating in Dendritic Neurons: Morphology Dependence and Probabilistic Synaptic Activation of Dendritic Spikes. *PLoS Computational Biology* **6**:8, e1000886. [[Crossref](#)]
146. YingHang Hao, YuBing Gong, Xiu Lin, XiaoGuang Ma. 2010. Delay-induced coherence bi-resonance-like behavior in stochastic Hodgkin-Huxley neuron networks. *Science China Chemistry* **53**:8, 1762-1766. [[Crossref](#)]
147. Bor-Sen Chen, Cheng-Wei Li. 2010. On the Noise-Enhancing Ability of Stochastic Hodgkin-Huxley Neuron Systems. *Neural Computation* **22**:7, 1737-1763. [[Abstract](#)] [[Full Text](#)] [[PDF](#)] [[PDF Plus](#)] [[Supplemental Material](#)]
148. Wainrib Gilles, Thieullen Michèle, Pakdaman Khashayar. 2010. Intrinsic variability of latency to first-spike. *Biological Cybernetics* **103**:1, 43-56. [[Crossref](#)]
149. Elena Phoka, Hermann Cuntz, Arnd Roth, Michael Häusser. 2010. A New Approach for Determining Phase Response Curves Reveals that Purkinje Cells Can Act as Perfect Integrators. *PLoS Computational Biology* **6**:4, e1000768. [[Crossref](#)]
150. Rukiye Uzun, Mahmut Ozer. Effect of channel noise on spike propagation in myelinated axons 1-4. [[Crossref](#)]
151. Go Ashida, Masayoshi Kubo. 2010. Suprathreshold stochastic resonance induced by ion channel fluctuation. *Physica D: Nonlinear Phenomena* **239**:6, 327-334. [[Crossref](#)]
152. S. Kuang, J. Wang, T. Zeng. 2010. Intrinsic Rhythmic Fluctuation of Membrane Voltage Evoked by Membrane Noise in the Hodgkin-Huxley System. *Acta Physica Polonica A* **117**:3, 435-438. [[Crossref](#)]
153. Yanhang Xie, Yubing Gong, Yinghang Hao, Xiaoguang Ma. 2010. Synchronization transitions on complex thermo-sensitive neuron networks with time delays. *Biophysical Chemistry* **146**:2-3, 126-132. [[Crossref](#)]
154. Tatjana Tchumatchenko, Aleksey Malyshev, Theo Geisel, Maxim Volgushev, Fred Wolf. 2010. Correlations and Synchrony in Threshold Neuron Models. *Physical Review Letters* **104**:5. . [[Crossref](#)]
155. Yubing Gong, Yinghang Hao, Yanhang Xie. 2010. Channel block-optimized spiking activity of Hodgkin-Huxley neurons on random networks. *Physica A: Statistical Mechanics and its Applications* **389**:2, 349-357. [[Crossref](#)]
156. B. Sengupta, S. B. Laughlin, J. E. Niven. 2010. Comparison of Langevin and Markov channel noise models for neuronal signal generation. *Physical Review E* **81**:1. . [[Crossref](#)]
157. Mahmut Ozer, Muhammet Uzuntarla, Matjaž Perc, Lyle J. Graham. 2009. Spike latency and jitter of neuronal membrane patches with stochastic Hodgkin-Huxley channels. *Journal of Theoretical Biology* **261**:1, 83-92. [[Crossref](#)]
158. Yubing Gong, Yinghang Hao, Yanhang Xie, Xiaoguang Ma, Chuanlu Yang. 2009. Non-Gaussian noise optimized spiking activity of Hodgkin-Huxley neurons on random complex networks. *Biophysical Chemistry* **144**:1-2, 88-93. [[Crossref](#)]
159. Yubing Gong, Yanhang Xie, Yinghang Hao. 2009. Coherence resonance induced by non-Gaussian noise in a deterministic Hodgkin-Huxley neuron. *Physica A: Statistical Mechanics and its Applications* **388**:18, 3759-3764. [[Crossref](#)]
160. Katarina Bodova, Daniel B Forger. 2009. Statistical properties of noise-induced firing and quiescence in a Hodgkin-Huxley model. *BMC Neuroscience* **10**:S1. . [[Crossref](#)]
161. YanHang Xie, YuBing Gong, YingHang Hao. 2009. Enhancement of spike coherence by the departure from Gaussian noise in a Hodgkin-Huxley neuron. *Science in China Series B: Chemistry* **52**:8, 1186-1191. [[Crossref](#)]
162. Shangyou Zeng, Yi Tang. 2009. Effect of clustered ion channels along an unmyelinated axon. *Physical Review E* **80**:2. . [[Crossref](#)]
163. Ashok Patel, Bart Kosko. 2009. Error-probability noise benefits in threshold neural signal detection. *Neural Networks* **22**:5-6, 697-706. [[Crossref](#)]
164. Eugenio Urdapilleta, Inés Samengo. 2009. Quasistatic approximation of the interspike interval distribution of neurons driven by time-dependent inputs. *Physical Review E* **80**:1. . [[Crossref](#)]
165. Ashok Patel, Bart Kosko. Neural signal-detection noise benefits based on error probability 2423-2430. [[Crossref](#)]
166. Susanne Schreiber, Inés Samengo, Andreas V.M. Herz. 2009. Two Distinct Mechanisms Shape the Reliability of Neural Responses. *Journal of Neurophysiology* **101**:5, 2239-2251. [[Crossref](#)]
167. Mahmut Ozer, Matjaž Perc, Muhammet Uzuntarla. 2009. Controlling the spontaneous spiking regularity via channel blocking on Newman-Watts networks of Hodgkin-Huxley neurons. *EPL (Europhysics Letters)* **86**:4, 40008. [[Crossref](#)]

168. Yubing Gong, Yanhang Xie, Yinghang Hao. 2009. Coherence resonance induced by the deviation of non-Gaussian noise in coupled Hodgkin–Huxley neurons. *The Journal of Chemical Physics* **130**:16, 165106. [[Crossref](#)]
169. Mahmut Ozer, Matjaž Perc, Muhammet Uzuntarla. 2009. Stochastic resonance on Newman–Watts networks of Hodgkin–Huxley neurons with local periodic driving. *Physics Letters A* **373**:10, 964–968. [[Crossref](#)]
170. Joshua T. Dudman, Matthew F. Nolan. 2009. Stochastically Gating Ion Channels Enable Patterned Spike Firing through Activity-Dependent Modulation of Spike Probability. *PLoS Computational Biology* **5**:2, e1000290. [[Crossref](#)]
171. J.F. Storm, K. Vervaeke, H. Hu, L.J. Graham. Functions of the Persistent Na⁺ Current in Cortical Neurons Revealed by Dynamic Clamp 165–197. [[Crossref](#)]
172. YuBing Gong, YanHang Xie, Bo Xu, XiaoGuang Ma. 2009. Effect of gating currents of ion channels on the collective spiking activity of coupled Hodgkin–Huxley neurons. *Science in China Series B: Chemistry* **52**:1, 20–25. [[Crossref](#)]
173. C. M. Gómez. 2008. Numerical exploration of the influence of neural noise on the psychometric function at low stimulation intensity levels. *Journal of Biosciences* **33**:5, 743–753. [[Crossref](#)]
174. Yong Chen, Lianchun Yu, Shao-Meng Qin. 2008. Detection of subthreshold pulses in neurons with channel noise. *Physical Review E* **78**:5. . [[Crossref](#)]
175. Mahmut Ozer, Muhammet Uzuntarla, Temel Kayikcioglu, Lyle J. Graham. 2008. Collective temporal coherence for subthreshold signal encoding on a stochastic small-world Hodgkin–Huxley neuronal network. *Physics Letters A* **372**:43, 6498–6503. [[Crossref](#)]
176. T. A. Engel, L. Schimansky-Geier, A.V.M. Herz, S. Schreiber, I. Erchova. 2008. Subthreshold Membrane-Potential Resonances Shape Spike-Train Patterns in the Entorhinal Cortex. *Journal of Neurophysiology* **100**:3, 1576–1589. [[Crossref](#)]
177. D. Biron, S. Wasserman, J. H. Thomas, A. D. T. Samuel, P. Sengupta. 2008. An olfactory neuron responds stochastically to temperature and modulates *Caenorhabditis elegans* thermotactic behavior. *Proceedings of the National Academy of Sciences* **105**:31, 11002–11007. [[Crossref](#)]
178. Rosa H. M. Chan, Dong Song, Theodore W. Berger. Tracking temporal evolution of nonlinear dynamics in hippocampus using time-varying volterra kernels 4996–4999. [[Crossref](#)]
179. Gašper Tkačik, Thomas Gregor, William Bialek. 2008. The Role of Input Noise in Transcriptional Regulation. *PLoS ONE* **3**:7, e2774. [[Crossref](#)]
180. Mahmut Ozer, Muhammet Uzuntarla. 2008. Effects of the network structure and coupling strength on the noise-induced response delay of a neuronal network. *Physics Letters A* **372**:25, 4603–4609. [[Crossref](#)]
181. YuBing Gong, Bo Xu, XiaoGuang Ma, JiQu Han. 2008. Effect of channel block on the collective spiking activity of coupled stochastic Hodgkin–Huxley neurons. *Science in China Series B: Chemistry* **51**:4, 341–346. [[Crossref](#)]
182. A. Aldo Faisal, Luc P. J. Selen, Daniel M. Wolpert. 2008. Noise in the nervous system. *Nature Reviews Neuroscience* **9**:4, 292–303. [[Crossref](#)]
183. Aaditya V. Rangan, Gregor Kovačič, David Cai. 2008. Kinetic theory for neuronal networks with fast and slow excitatory conductances driven by the same spike train. *Physical Review E* **77**:4. . [[Crossref](#)]
184. Yifat Kovalsky, Ron Amir, Marshall Devor. 2008. Subthreshold oscillations facilitate neuropathic spike discharge by overcoming membrane accommodation. *Experimental Neurology* **210**:1, 194–206. [[Crossref](#)]
185. Irina Erchova, David J. McGonigle. 2008. Rhythms of the brain: An examination of mixed mode oscillation approaches to the analysis of neurophysiological data. *Chaos: An Interdisciplinary Journal of Nonlinear Science* **18**:1, 015115. [[Crossref](#)]
186. Antti Saarinen, Marja-Leena Linne, Olli Yli-Harja. 2008. Stochastic Differential Equation Model for Cerebellar Granule Cell Excitability. *PLoS Computational Biology* **4**:2, e1000004. [[Crossref](#)]
187. Christopher R. Butson, Gregory A. Clark. 2008. Random Noise Paradoxically Improves Light-Intensity Encoding in Hermissenda Photoreceptor Network. *Journal of Neurophysiology* **99**:1, 146–154. [[Crossref](#)]
188. Christopher R. Butson, Gregory A. Clark. 2008. Mechanisms of Noise-Induced Improvement in Light-Intensity Encoding in Hermissenda Photoreceptor Network. *Journal of Neurophysiology* **99**:1, 155–165. [[Crossref](#)]
189. Jacobo D. Sitt, J. Aliaga. 2007. Versatile biologically inspired electronic neuron. *Physical Review E* **76**:5. . [[Crossref](#)]
190. Etay Ziv, Ilya Nemenman, Chris H. Wiggins. 2007. Optimal Signal Processing in Small Stochastic Biochemical Networks. *PLoS ONE* **2**:10, e1077. [[Crossref](#)]
191. L. C. Yu, Y. Chen, Pan Zhang. 2007. Frequency and phase synchronization of two coupled neurons with channel noise. *The European Physical Journal B* **59**:2, 249–257. [[Crossref](#)]
192. Shangyou Zeng, Yi Tang, Peter Jung. 2007. Spiking synchronization of ion channel clusters on an axon. *Physical Review E* **76**:1. . [[Crossref](#)]

193. Peter Rowat. 2007. Interspike Interval Statistics in the Stochastic Hodgkin-Huxley Model: Coexistence of Gamma Frequency Bursts and Highly Irregular Firing. *Neural Computation* **19**:5, 1215-1250. [[Abstract](#)] [[PDF](#)] [[PDF Plus](#)]
194. Victor M. Rodriguez-Molina, Ad Aertsen, Detlef H. Heck. 2007. Spike Timing and Reliability in Cortical Pyramidal Neurons: Effects of EPSC Kinetics, Input Synchronization and Background Noise on Spike Timing. *PLoS ONE* **2**:3, e319. [[Crossref](#)]
195. Mahmut Ozer, Muhammet Uzuntarla, Sukriye Nihal Agaoglu. 2006. Effect of the sub-threshold periodic current forcing on the regularity and the synchronization of neuronal spiking activity. *Physics Letters A* **360**:1, 135-140. [[Crossref](#)]
196. G Schmid, I Goychuk, P Hänggi. 2006. Capacitance fluctuations causing channel noise reduction in stochastic Hodgkin-Huxley systems. *Physical Biology* **3**:4, 248-254. [[Crossref](#)]
197. David Paydarfar, Daniel B. Forger, John R. Clay. 2006. Noisy Inputs and the Induction of On-Off Switching Behavior in a Neuronal Pacemaker. *Journal of Neurophysiology* **96**:6, 3338-3348. [[Crossref](#)]
198. D. A. Steyn-Ross, Moira L. Steyn-Ross, M. T. Wilson, J. W. Sleigh. 2006. White-noise susceptibility and critical slowing in neurons near spiking threshold. *Physical Review E* **74**:5. . [[Crossref](#)]
199. Alan D. Dorval. 2006. The Rhythmic Consequences of Ion Channel Stochasticity. *The Neuroscientist* **12**:5, 442-448. [[Crossref](#)]
200. S.L. Ginzburg, M.A. Pustovoi. 2006. Response of Hodgkin-Huxley stochastic bursting neuron to single-pulse stimulus. *Physica A: Statistical Mechanics and its Applications* **369**:2, 354-368. [[Crossref](#)]
201. Joshua X. Gittelman, Bruce L. Tempel. 2006. Kv1.1-Containing Channels Are Critical for Temporal Precision During Spike Initiation. *Journal of Neurophysiology* **96**:3, 1203-1214. [[Crossref](#)]
202. Renaud Jolivet, Alexander Rauch, Hans-Rudolf Lüscher, Wulfram Gerstner. 2006. Predicting spike timing of neocortical pyramidal neurons by simple threshold models. *Journal of Computational Neuroscience* **21**:1, 35-49. [[Crossref](#)]
203. Antti Saarinen, Marja-Leena Linne, Olli Yli-Harja. 2006. Modeling single neuron behavior using stochastic differential equations. *Neurocomputing* **69**:10-12, 1091-1096. [[Crossref](#)]
204. Mahmut Ozer. 2006. Frequency-dependent information coding in neurons with stochastic ion channels for subthreshold periodic forcing. *Physics Letters A* **354**:4, 258-263. [[Crossref](#)]
205. Alan D. Dorval, John A. White. 2006. Synaptic input statistics tune the variability and reproducibility of neuronal responses. *Chaos: An Interdisciplinary Journal of Nonlinear Science* **16**:2, 026105. [[Crossref](#)]
206. Andreas T Schaefer, Kamilla Angelo, Hartwig Spors, Troy W Margrie. 2006. Neuronal Oscillations Enhance Stimulus Discrimination by Ensuring Action Potential Precision. *PLoS Biology* **4**:6, e163. [[Crossref](#)]
207. Björn Naundorf, Fred Wolf, Maxim Volgushev. 2006. Unique features of action potential initiation in cortical neurons. *Nature* **440**:7087, 1060-1063. [[Crossref](#)]
208. E. É. Saftenku. 2006. Presumed mechanisms of a long-term increase in the intrinsic excitability of cerebellar granule cells: A model study. *Neurophysiology* **38**:2, 101-110. [[Crossref](#)]
209. Matteo Gianni, Micaela Liberti, Francesca Apollonio, Guglielmo D'Inzeo. 2006. Modeling electromagnetic fields detectability in a HH-like neuronal system: stochastic resonance and window behavior. *Biological Cybernetics* **94**:2, 118-127. [[Crossref](#)]
210. Kamran Diba, Christof Koch, Idan Segev. 2006. Spike propagation in dendrites with stochastic ion channels. *Journal of Computational Neuroscience* **20**:1, 77-84. [[Crossref](#)]
211. Koen Vervaeke, Hua Hu, Lyle J. Graham, Johan F. Storm. 2006. Contrasting Effects of the Persistent Na⁺ Current on Neuronal Excitability and Spike Timing. *Neuron* **49**:2, 257-270. [[Crossref](#)]
212. Tim Gollisch. 2006. Estimating receptive fields in the presence of spike-time jitter. *Network: Computation in Neural Systems* **17**:2, 103-129. [[Crossref](#)]
213. Jose Luis Perez Velazquez. 2005. Brain, behaviour and mathematics: Are we using the right approaches?. *Physica D: Nonlinear Phenomena* **212**:3-4, 161-182. [[Crossref](#)]
214. Lijian Yang, Ya Jia. 2005. Effects of patch temperature on spontaneous action potential train due to channel fluctuations: Coherence resonance. *Biosystems* **81**:3, 267-280. [[Crossref](#)]
215. Pedro V. Carelli, Marcelo B. Reyes, José C. Sartorelli, Reynaldo D. Pinto. 2005. Whole Cell Stochastic Model Reproduces the Irregularities Found in the Membrane Potential of Bursting Neurons. *Journal of Neurophysiology* **94**:2, 1169-1179. [[Crossref](#)]
216. Ashok Patel, Bart Kosko. 2005. Stochastic resonance in noisy spiking retinal and sensory neuron models. *Neural Networks* **18**:5-6, 467-478. [[Crossref](#)]
217. Yubing Gong, Maosheng Wang, Zhonghuai Hou, Houwen Xin. 2005. Optimal Spike Coherence and Synchronization on Complex Hodgkin-Huxley Neuron Networks. *ChemPhysChem* **6**:6, 1042-1047. [[Crossref](#)]

218. A. Aldo Faisal, John A. White, Simon B. Laughlin. 2005. Ion-Channel Noise Places Limits on the Miniaturization of the Brain's Wiring. *Current Biology* **15**:12, 1143-1149. [[Crossref](#)]
219. John R. Clay. 2005. Axonal excitability revisited. *Progress in Biophysics and Molecular Biology* **88**:1, 59-90. [[Crossref](#)]
220. Mahmut Ozer, N. Hakan Ekmekci. 2005. Effect of channel noise on the time-course of recovery from inactivation of sodium channels. *Physics Letters A* **338**:2, 150-154. [[Crossref](#)]
221. Gilad A. Jacobson, Kamran Diba, Anat Yaron-Jakobovitch, Yasmin Oz, Christof Koch, Idan Segev, Yosef Yarom. 2005. Subthreshold voltage noise of rat neocortical pyramidal neurones. *The Journal of Physiology* **564**:1, 145-160. [[Crossref](#)]
222. M. Möller, E. Penner, M. Waring, C. Schöfl, K. Prank, G. Brabant, A. Bader, U. Ahlvers. 2005. Precision of intracellular calcium spike timing in primary rat hepatocytes. *Systems Biology* **2**:1, 31-34. [[Crossref](#)]
223. T. Tateno, A. Harsch, H. P. C. Robinson. 2004. Threshold Firing Frequency-Current Relationships of Neurons in Rat Somatosensory Cortex: Type 1 and Type 2 Dynamics. *Journal of Neurophysiology* **92**:4, 2283-2294. [[Crossref](#)]
224. Ramani Balu, Phillip Larimer, Ben W. Strowbridge. 2004. Phasic Stimuli Evoke Precisely Timed Spikes in Intermittently Discharging Mitral Cells. *Journal of Neurophysiology* **92**:2, 743-753. [[Crossref](#)]
225. Shangyou Zeng, Peter Jung. 2004. Mechanism for neuronal spike generation by small and large ion channel clusters. *Physical Review E* **70**:1. . [[Crossref](#)]
226. G Schmid, I Goychuk, P Hänggi. 2004. Effect of channel block on the spiking activity of excitable membranes in a stochastic Hodgkin-Huxley model. *Physical Biology* **1**:2, 61-66. [[Crossref](#)]
227. J. M. CASADO, J. P. BALTANÁS. 2004. RELIABILITY OF SPIKE TIMING IN A NEURON MODEL. *International Journal of Bifurcation and Chaos* **14**:06, 2061-2068. [[Crossref](#)]
228. Nikolai Axmacher, Richard Miles. 2004. Intrinsic cellular currents and the temporal precision of EPSP-action potential coupling in CA1 pyramidal cells. *The Journal of Physiology* **555**:3, 713-725. [[Crossref](#)]
229. Jason Ritt. 2003. Evaluation of entrainment of a nonlinear neural oscillator to white noise. *Physical Review E* **68**:4. . [[Crossref](#)]
230. Leonid M. Litvak, Zachary M. Smith, Bertrand Delgutte, Donald K. Eddington. 2003. Desynchronization of electrically evoked auditory-nerve activity by high-frequency pulse trains of long duration. *The Journal of the Acoustical Society of America* **114**:4, 2066-2078. [[Crossref](#)]
231. Leonid M. Litvak, Bertrand Delgutte, Donald K. Eddington. 2003. Improved temporal coding of sinusoids in electric stimulation of the auditory nerve using desynchronizing pulse trains. *The Journal of the Acoustical Society of America* **114**:4, 2079-2098. [[Crossref](#)]
232. Saul L. Ginzburg, Mark A. Pustovoit. 2003. Bursting Dynamics of a Model Neuron Induced by Intrinsic Channel Noise. *Fluctuation and Noise Letters* **03**:03, L265-L274. [[Crossref](#)]
233. Blaise Agüera y Arcas, Adrienne L. Fairhall, William Bialek. 2003. Computation in a Single Neuron: Hodgkin and Huxley Revisited. *Neural Computation* **15**:8, 1715-1749. [[Abstract](#)] [[PDF](#)] [[PDF Plus](#)]
234. John D. Hunter, John G. Milton. 2003. Amplitude and Frequency Dependence of Spike Timing: Implications for Dynamic Regulation. *Journal of Neurophysiology* **90**:1, 387-394. [[Crossref](#)]
235. David H. Goldberg, Arun P. Sripati, Andreas G. Andreou. 2003. Energy efficiency in a channel model for the spiking axon. *Neurocomputing* **52-54**, 39-44. [[Crossref](#)]
236. C. S. Zhou, J. Kurths, E. Allaria, S. Boccaletti, R. Meucci, F. T. Arecchi. 2003. Constructive effects of noise in homoclinic chaotic systems. *Physical Review E* **67**:6. . [[Crossref](#)]
237. M.C.W. van Rossum, B. J. O'Brien, R. G. Smith. 2003. Effects of Noise on the Spike Timing Precision of Retinal Ganglion Cells. *Journal of Neurophysiology* **89**:5, 2406-2419. [[Crossref](#)]
238. J. W. Shuai, P. Jung. 2003. Optimal ion channel clustering for intracellular calcium signaling. *Proceedings of the National Academy of Sciences* **100**:2, 506-510. [[Crossref](#)]
239. Yoshimi Kamiyama, Shiro Usui. A Retinal Ganglion Cell Model Based on Discrete Stochastic Ion Channels 134-137. [[Crossref](#)]
240. A. Kepecs, J. Lisman. 2003. Information encoding and computation with spikes and bursts. *Network: Computation in Neural Systems* **14**:1, 103-118. [[Crossref](#)]
241. J W Shuai, P Jung. 2003. Sub-threshold Ca²⁺ waves. *New Journal of Physics* **5**, 132-132. [[Crossref](#)]
242. Hugh P. C. Robinson, Annette Harsch. 2002. Stages of spike time variability during neuronal responses to transient inputs. *Physical Review E* **66**:6. . [[Crossref](#)]
243. Claude Meunier, Idan Segev. 2002. Playing the Devil's advocate: is the Hodgkin-Huxley model useful?. *Trends in Neurosciences* **25**:11, 558-563. [[Crossref](#)]

244. Julie S. Haas, John A. White. 2002. Frequency Selectivity of Layer II Stellate Cells in the Medial Entorhinal Cortex. *Journal of Neurophysiology* **88**:5, 2422-2429. [[Crossref](#)]
245. Naama Brenner, Oded Agam, William Bialek, Rob de Ruyter van Steveninck. 2002. Statistical properties of spike trains: Universal and stimulus-dependent aspects. *Physical Review E* **66**:3. . [[Crossref](#)]
246. J. W. SHUAI, S. ZENG, P. JUNG. 2002. COHERENCE RESONANCE: ON THE USE AND ABUSE OF THE FANO FACTOR. *Fluctuation and Noise Letters* **02**:03, L139-L146. [[Crossref](#)]
247. Boris M. Slepchenko, James C. Schaff, John H. Carson, Leslie M. Loew. 2002. Computational Cell Biology: Spatiotemporal Simulation of Cellular Events. *Annual Review of Biophysics and Biomolecular Structure* **31**:1, 423-441. [[Crossref](#)]
248. Jianfeng Feng, Guibin Li. 2002. Impact of Geometrical Structures on the Output of Neuronal Models: A Theoretical and Numerical Analysis. *Neural Computation* **14**:3, 621-640. [[Abstract](#)] [[PDF](#)] [[PDF Plus](#)]
249. Amit Manwani, Peter N. Steinmetz, Christof Koch. 2002. The Impact of Spike Timing Variability on the Signal-Encoding Performance of Neural Spiking Models. *Neural Computation* **14**:2, 347-367. [[Abstract](#)] [[PDF](#)] [[PDF Plus](#)]
250. J. W. Shuai, P. Jung. 2002. Optimal Intracellular Calcium Signaling. *Physical Review Letters* **88**:6. . [[Crossref](#)]
251. G.N. Borisyuk, R.M. Borisyuk, Yakov B. Kazanovich, Genrikh R. Ivanitskii. 2002. Models of neural dynamics in brain information processing the developments of 'the decade'. *Uspekhi Fizicheskib Nauk* **172**:10, 1189. [[Crossref](#)]
252. Leonid P. Savtchenko, Paul Gogan, Suzanne Tyč-Dumont. 2001. Dendritic spatial flicker of local membrane potential due to channel noise and probabilistic firing of hippocampal neurons in culture. *Neuroscience Research* **41**:2, 161-183. [[Crossref](#)]
253. P Jung, J. W Shuai. 2001. Optimal sizes of ion channel clusters. *Europhysics Letters (EPL)* **56**:1, 29-35. [[Crossref](#)]
254. Peter N. Steinmetz, Amit Manwani, Christof Koch. 2001. Variability and coding efficiency of noisy neural spike encoders. *Biosystems* **62**:1-3, 87-97. [[Crossref](#)]
255. Ira B Schwartz, Lora Billings, Joseph J Pancrazio, Joel M Schnur. 2001. Methods for short time series analysis of cell-based biosensor data. *Biosensors and Bioelectronics* **16**:7-8, 503-512. [[Crossref](#)]
256. Lora Billings, Ira B. Schwartz, Joseph J. Pancrazio, Joel M. Schnur. 2001. Dynamic and geometric analysis of short time series: a new comparative approach to cell-based biosensors. *Physics Letters A* **286**:2-3, 217-224. [[Crossref](#)]
257. Leonid Litvak, Bertrand Delgutte, Donald Eddington. 2001. Auditory nerve fiber responses to electric stimulation: Modulated and unmodulated pulse trains. *The Journal of the Acoustical Society of America* **110**:1, 368-379. [[Crossref](#)]
258. A.J. Matsuoka, J.T. Rubinstein, P.J. Abbas, C.A. Miller. 2001. The effects of interpulse interval on stochastic properties of electrical stimulation: models and measurements. *IEEE Transactions on Biomedical Engineering* **48**:4, 416-424. [[Crossref](#)]
259. J.-M. Fellous, A. R. Houweling, R. H. Modi, R.P.N. Rao, P.H.E. Tiesinga, T. J. Sejnowski. 2001. Frequency Dependence of Spike Timing Reliability in Cortical Pyramidal Cells and Interneurons. *Journal of Neurophysiology* **85**:4, 1782-1787. [[Crossref](#)]
260. Erik De Schutter. Computational Neuroscience: More Math Is Needed to Understand the Human Brain 381-391. [[Crossref](#)]
261. Rob de Ruyter van Steveninck, Alexander Borst, William Bialek. Real-Time Encoding of Motion: Answerable Questions and Questionable Answers from the Fly's Visual System 279-306. [[Crossref](#)]
262. J.A. White, J.S. Haas. Chapter 8 Intrinsic noise from voltage-gated ion channels: Effects on dynamics and reliability in intrinsically oscillatory neurons 257-278. [[Crossref](#)]
263. C. Meunier, I. Segev. Chapter 11 Neurones as physical objects: Structure, dynamics and function 353-467. [[Crossref](#)]
264. Satoshi Oode, Kazushi Murakoshi, Kiyohiko Nakamura. 2000. Analysis of the reliability and precision of spike timing by a single model neuron. *Systems and Computers in Japan* **31**:13, 1-10. [[Crossref](#)]
265. Desdemona Fricker, Richard Miles. 2000. EPSP Amplification and the Precision of Spike Timing in Hippocampal Neurons. *Neuron* **28**:2, 559-569. [[Crossref](#)]
266. Karunesh Ganguly, Laszlo Kiss, Mu-ming Poo. 2000. Enhancement of presynaptic neuronal excitability by correlated presynaptic and postsynaptic spiking. *Nature Neuroscience* **3**:10, 1018-1026. [[Crossref](#)]
267. John A. White, Jay T. Rubinstein, Alan R. Kay. 2000. Channel noise in neurons. *Trends in Neurosciences* **23**:3, 131-137. [[Crossref](#)]
268. Amit Manwani, Christof Koch. 1999. Detecting and Estimating Signals in Noisy Cable Structures, I: Neuronal Noise Sources. *Neural Computation* **11**:8, 1797-1829. [[Abstract](#)] [[PDF](#)] [[PDF Plus](#)]
269. Idan Segev, Elad Schneidman. 1999. Axons as computing devices: Basic insights gained from models. *Journal of Physiology-Paris* **93**:4, 263-270. [[Crossref](#)]
270. Desdemona Fricker, Jos A. H. Verheugen, Richard Miles. 1999. Cell-attached measurements of the firing threshold of rat hippocampal neurones. *The Journal of Physiology* **517**:3, 791-804. [[Crossref](#)]

271. W. Bialek. Thinking About The Brain 485-578. [[Crossref](#)]
272. Mahmut Ozer, Muhammet Uzuntarla. Synchronization between neuronal spiking activity and sub-threshold sinusoidal stimuli based on the FitzHugh-Nagumo model 415-421. [[Crossref](#)]
273. M. Gianni, F. Maggio, M. Liberti, A. Paffi, F. Apollonio, G. D'Inzeo. Modeling Biological Noise in Firing and Bursting Neurons in the Presence of an Electromagnetic Field 237-240. [[Crossref](#)]
274. G. Ashida, M. Kubo. Neuronal dynamics based on individual stochastic ion channels 1592-1596. [[Crossref](#)]
275. A.A. Faisal, S.B. Laughlin, J.A. White. How reliable is the connectivity in cortical neural networks? 1661-1666. [[Crossref](#)]
276. A. Patel, B. Kosko. Noise benefits in spiking retinal and sensory neuron models 410-415. [[Crossref](#)]
277. N.H. Ekmekci, M. Ozer. Effect of sub-threshold stimuli on neuronal dynamics 143-146. [[Crossref](#)]
278. N.H. Ekmekci, M. Ozer. Effect of DC Stimuli on Neuronal Dynamics 1-4. [[Crossref](#)]