# MRI Primer: Assignment #4

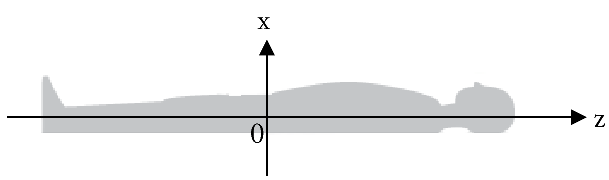
Due Tuesday, December 7, 2021

## Selective Excitation & $B_0$-Field Inhomogeneity

We've seen in class that any pulse is frequency-selective; that is, it has a bandwidth BW=$\gamma B_1$. Spins having an offset outside that bandwidth are mostly unaffected. With a gradient turned on, the offset becomes spatially dependent, and the frequency-selectivity translates into a physical slice. For a constant gradient **G** in the rotating frame:

$$\boldsymbol{B}_{eff} = \begin{pmatrix} B_1 \\ 0 \\ B_0 + \boldsymbol{G}\cdot\boldsymbol{r} - \frac{\omega_{rot}}{\gamma} \end{pmatrix}.$$

1. Assuming the excitation pulse is on resonance at the center of magnet, such that $\omega_{rot} = \omega_{RF} = \gamma B_0$, draw the excited slice for an excitation pulse with $G = 1\, mT/m$, $B_1 = 10\,\mu T$, and a flip angle of 90°. For simplicity, take **G** to be in the z-axis, and imaging your sample is two dimensional (in the x-z plane), so you can draw it on a piece of paper. Denote the slice's thickness, orientation and center in your coordinate system.




2. The main field $B_0$ is not *really* completely spatially homogeneous, and sometimes spatial inhomogeneity can become a sizable nuisance. Such spatial distortions of the field can come from many sources: engineering imperfections when building the magnet, or the fact that biological tissue (i.e. **you**!) is diamagnetic and in fact distorts the field around it. Let's explore what this means. Assume that, unbeknownst to the scientist obtaining the image, a **linear** spatial homogeneity $\Delta B(\boldsymbol{r}) = \eta z$ is added to the sample, such that $B_0$ becomes $B_0 \rightarrow B_0 + \Delta B(\boldsymbol{r}) = B_0 + \eta_1 z$ (and that your gradient is along z):

$$\boldsymbol{B}_{eff} = \begin{pmatrix} B_1 \\ 0 \\ B_0 + \eta_1 z + \boldsymbol{G}\cdot\boldsymbol{r} - \frac{\omega_{rot}}{\gamma} \end{pmatrix} = \begin{pmatrix} B_1 \\ 0 \\ \eta_1 z + Gz \end{pmatrix}.$$

(G=1 mT/m)

How would this affect the (i) width (ii) center and (iii) orientation of the slice (if at all)? Derive an analytical expression for the width of the slice. Draw the resulting slice for $\eta_1 = 1\,mT/m$.

3. How small should $\eta_1$ be to be "negligible"? That is, "when $\eta_1 \ll X$ the inhomogeneity is negligible" – what is X?
4. How would your answer to (2) change if the inhomogeneity also had a constant term, $\Delta B(\boldsymbol{r}) = \eta_0 + \eta_1 z$? Draw your answer. What are the expressions for the slice's center and thickness now? What is its orientation?
5. How would your answer to (2) change if the linear inhomogeneity was along the x-axis: $\Delta B(\boldsymbol{r}) = \eta_1 x$? Draw your answer, assuming $\eta_1 = 1\,mT/m$. Denote the slice's center, thickness and orientation.
6. Assume a **quadratic** spatial homogeneity along the **x-axis**, $\Delta B(\boldsymbol{r}) = \eta_2 x^2$. Draw, qualitatively, the shape of the excited "slice" (hint: what frequencies does the pulse excite? And to what points in the x-z plane would those frequencies correspond? That is, what shape does the set of points $\gamma G z + \gamma \eta_2 x^2 = const$ define in the x-z plane?)

## There Is More Than One Way of Exciting a Given Slice Thickness

In class we saw that applying an RF pulse with amplitude $B_1$ in the presence of a z-gradient of size G excites a slice of thickness:

$$\Delta z = \frac{B_1}{G} = \frac{BW}{\gamma G}$$

where $BW = \gamma B_1$ is the pulse's bandwidth. Thus it seems we can modify the slice thickness in two ways: change $B_1$ or change G. Let's examine the implications of each by first filling out the following table:

| *Slice thickness (mm)* | *Flip angle (Deg.)* | *$B_1$ ($\mu T$)* | *Bandwidth (For $^1H$) (Hz)* | *G (mT/m)* | *Duration (ms)* | *SAR ($\mu T^2 \cdot ms$)* |
|---|---|---|---|---|---|---|
| 10 | 90 | | | 0.58 | 1 | |
| | | 1.17 | | 0.12 | 5 | |
| 10 | 90 | 0.56 | | | 10 | |

Note: the specific absorption rate (SAR), which is the amount of energy deposited in the imaged tissue by the pulse, is proportional to

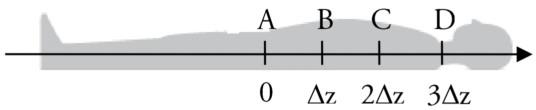
$$\int_0^T |B_1(t)|^2\, dt$$

where T is the pulse's duration. Use that quantity to evaluate the (relative) SAR in $\mu T^2 \cdot ms$.

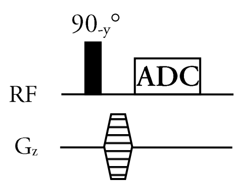
Based on the above table, this assignment and/or the lecture notes, give two possible reasons for wanting to use shorter excitation pulses to excite a given slice thickness (hint: one of them is in the previous problem … ), and two possible reasons for wanting to use longer excitation pulses.

## 1D Phase Encoding

In class we saw how phase encoding resolved two separate points. Here we're going to look in more depth into these notions by explicitly performing phase encoding for four separate spatial points:




For the purposes of this question we will only assume we have non-zero magnetization at points A, B, C and D. Also assume a perfectly homogeneous receiver coil, and neglect relaxation and any other magnet imperfections. The following phase encoding experiment is run four times, each time with a different gradient amplitude:




The gradient amplitude is varied such that

$$\gamma G t = 0, \frac{\pi}{2\Delta z}, \frac{2\pi}{2\Delta z}, \frac{3\pi}{2\Delta z},$$

and $t$ is the phase encoding gradient's duration (again, assumed very short).

1. Show that, starting from $s(t) = \int_{body} M_{xy}^{(rot)}(z,t)dz$, the signal acquired from each experiment can then be written (up to a constant) in the form

$$s(t) = \eta_A M_A + \eta_B M_B + \eta_C M_C + \eta_D M_D$$

where $\eta_i$ ($i$=A,B,C,D) is a complex number of the form $e^{i\alpha}$, depending on the value of $\gamma G t$, and $M_A$, $M_B$, $M_C$, $M_D$ are the thermal equilibrium magnetization vectors at points A, B, C and D, respectively.

2. Write down the signals from all four experiments using matrix notation; i.e.:

$$\begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = \begin{pmatrix} & & & \\ & 4\times 4 \quad \text{matrix} & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} M_A \\ M_B \\ M_C \\ M_D \end{pmatrix}$$

3. Use your favorite software package to invert the matrix, i.e. compute the matrix such that

$$\begin{pmatrix} & & & \\ & 4\times 4 \quad \text{matrix} & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = \begin{pmatrix} M_A \\ M_B \\ M_C \\ M_D \end{pmatrix}$$

   This is precisely how phase-encoded data is reconstructed.

4. Compare your reconstruction matrix to a $4^{th}$ order DFT matrix (if you're not sure what that is, look it up in Wikipedia). Are they the same? Same up to a constant? Similar?
5. Would your reconstruction matrix change if the points were spatially shifted by an amount $\delta z$? How would it change? In other words: how would you recover $M_A$, $M_B$, $M_C$ and $M_D$ from $s_1$, $s_2$, $s_3$ and $s_4$?