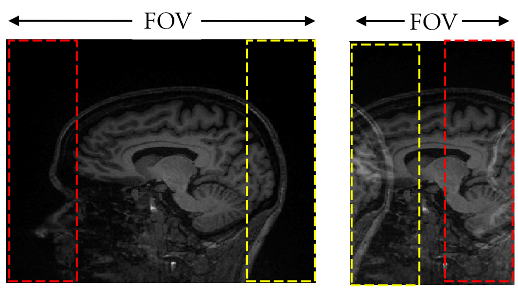
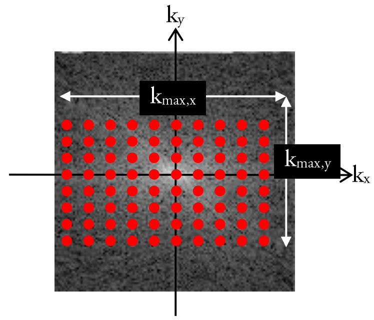
MRI Primer: Assignment #5 Solution

Setting Simple Imaging Parameters

1. The FOV should be at least as big as the object in question, which in this case is the "brain", contained within a 20×20 cm^2 region. Therefore, $FOV_x=FOV_y=20$ cm. You could select a larger FOV but that would be "wasteful" as it would place more stringent demands on the k-space sampling pattern (se #5 below).
2. Setting the FOV to be smaller than the object in question would lead to aliasing. This sort of phenomenon is shown in the following image:



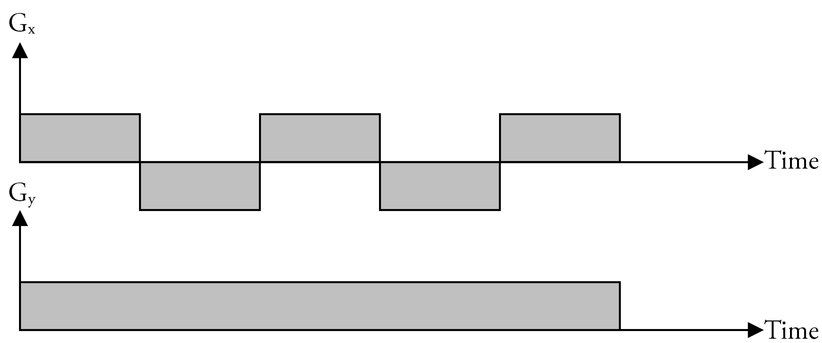
3. Our resolution along the readout axis comes for "free", meaning we can increase it without increasing the number of scans. We only "pay" for phase encoding steps, each of which requires an individual scan. Therefore, a 256×256 resolution image would require only 256 scans (and not 256×256=65,536 scans).
4. For the spins to return to thermal equilibrium we need to wait a time ~ $5T_1$ past excitation. If $T_1=1$ sec and we have 256 scans total, the entire imaging procedure will take $256\times5T_1\sim1280$ sec~21 minutes.
5. The FOV equals the inverse of the spacing between the points in k-space, so $\Delta k_x=\Delta k_y=1/(20\text{ cm})=1/(0.2\text{ m})=5\text{ m}^{-1}$. Of course you could make Δk_x, Δk_y even smaller than, which would correspond to a larger FOV, but this would be wasteful: for example, making Δk_y smaller would require us to carry out more phase encoding steps, i.e. more scans, meaning the imaging process would take longer. Since our resolution is 256×256 we need $k_{max,x}=k_{max,y}=256\times\Delta k_x=256\times\Delta k_y=1280\text{ m}^{-1}$. Note the geometrical meaning of $k_{max,x}$ in k-space:



So the "distance" in k-space from the origin to the farthest point in k-space is really $k_{max,x}/2$.

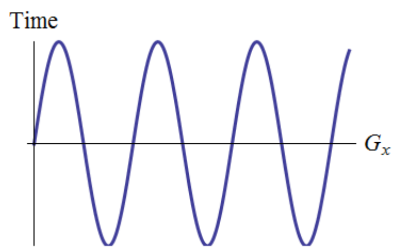
Deducing the Gradient Waveforms From The K-Space Trajectory

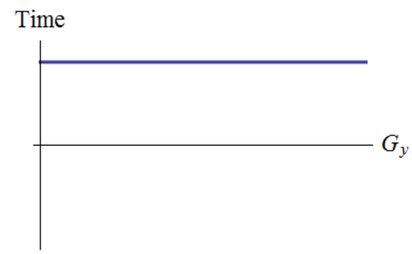
You can see by looking at the plot that k_y keeps advancing at a constant rate, meaning $k_y=\eta t$ for some t and therefore $G_y \sim dk_y/dt=\eta$ is a constant. k_x on the other hand increases and then decreases, meaning G_x is also a constant that changes sign intermittently (of course this would imply a discontinuity whenever it changes sign and an infinite slew rate which is unphysical, so the k-space trajectory shown is unphysical and in reality would have to be made smoother). Putting this together, we have



Deducing the k-Space Trajectory Given The Gradients

1. Schematically,



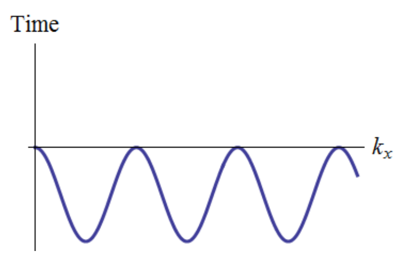


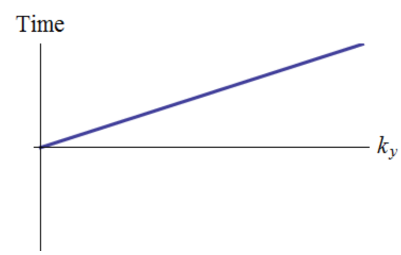
2. We move from gradients to **k**-space by integrating:

$$\mathbf{k}(t) = \bar{\gamma}\int_0^t \mathbf{G}(t')\,dt' = \left(-\tfrac{\bar{\gamma}G_0}{\omega}\left[\cos(\omega t)-1\right], \bar{\gamma}G_0 t\right)$$

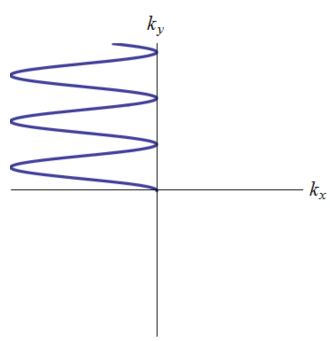
   Don't forget the integration has two bounds – upper (at t) and lower (at 0)!

3. Again, schematically,





4. This is called a parametric plot:



   So what we've done is revisited question (3) but from the analytical side, going from gradients to k-space trajectory. This trajectory is also realistic, compared to the previous one which has infinite slew rates (where dG/dt=∞).