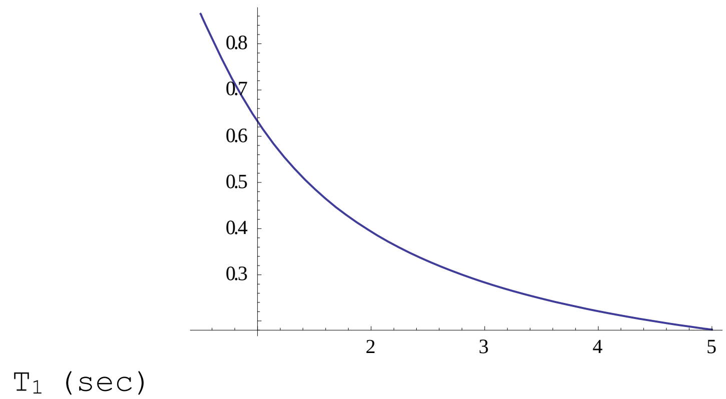
MRI Primer: Assignment #7 Solution

Imaging Edema & Cancer

1. The signal equation for a spoiled GRE is

$$S = M_0 \frac{\left(1 - e^{-TR/T_1}\right)\sin(\alpha)}{1 - \cos(\alpha)\, e^{-TR/T_1}} e^{-TE/T_2^*}$$

   Note the appearance of T_2* since this is a gradient echo. Since T_2* and M_0 are constants, the term $_0 \; {}^{-TE/T_2*}$ is a scaling constant independent of T_1 and therefore unimportant. For $\alpha = \;\;^\circ$, $S \propto 1 - e^{-TR/T_1}$ and looks like this as a function of T_1 for TR=1 sec:



   The curve clearly shows signal intensity decreases as T_1 increases, leading to lower (darker) signal intensities for edema which has higher T_1 than surrounding tissue.

2. Taking $M_0 e^{-TE/T_2*} = 1$ for simplicity, and noise with unit standard deviation, and using the definitions

$$\text{SNR} = \frac{\text{Signal}}{\text{SD}_{noise}} \qquad \text{CNR} = \frac{\text{Signal}_{healthy} - \text{Signal}_{edema}}{\text{SD}_{noise}}$$

   we have:

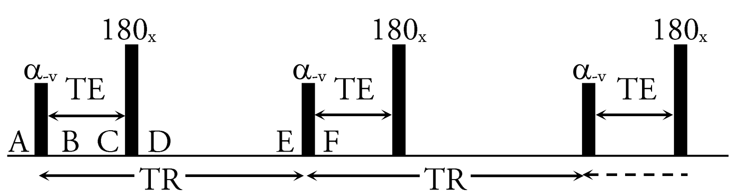
TR (sec)	α (deg)	$SNR_{healthy}$	SNR_{edema}	CNR	Scan Time
0.05	10	0.134	0.039	0.095	12.8 sec
0.05	90	0.049	0.025	0.024	12.8 sec
1	10	0.172	0.079	0.093	4:16 min
1	90	0.632	0.393	0.239	4:16 min

The scan time is calculated as TR·(number of phase encoding steps). Since the image is 256×256 it has 256 frequency encoding steps and 256 phase encoding steps and the time it requires is 256·TR. Looking at the table it is clear there is no clear cut winner. Maximum CNR is obtained at TR=1 sec, α=90°, but at the cost of very long scan times: 4:16 minutes for a single slice. In a realistic scenario we would have to scan multiple slices. A compromise can be had at TR=0.05 sec, α=10°, which sacrifices some CNR but achieves a drastically shorter scan time of 12.8 sec.
Note that assuming a different standard deviation for the noise would simply scale the numbers by a constant overall factor and would not alter our conclusions.

3. If a cancer is "invisible" (i.e. appears just like the healthy tissue around it) on a T_1-weighted image it simply tells us the T_1 of cancer is too similar to that of its surroundings – in our case, the edema that surrounds it.
4. As shown by the first graph, shortening T_1 increases the signal intensity, which is precisely what happens with the enhancing tumor. Physically what's happening is that for very short T_1s the magnetization quickly returns to thermal equilibrium prior to the next excitation pulse in the series, allowing us to generate more signal with each excitation.

For a spin echo sequence the signal intensity is proportional to $\propto \; {}^{-TE/T_2}$. Longer T_2s lead to higher signal intensities, implying that the T_2 of edema is **longer** than that of the surrounding tissue. Note that longer T_1s lead to a decrease in signal intensity, while longer T_2s lead to an increase in signal intensity, at least for the simple sequences we have discussed so far (this is almost a universal trait of all sequences, but some outliers exist).

Dynamic Equilibrium of a Spin Echo Sequence



The essence of dynamic equilibrium is that

$${}^{A} = {}^{E}$$

$${}^{B} = {}^{F}$$

Let's use the longitudinal second relation which implies:

$$M_z^A = M_z^E$$

Following excitation,

$$M_z^B = M_z^A \cos(\alpha).$$

We then have, after a time TE/2,

$$M_z^C = M_z^B e^{-\frac{TE}{2T_1}} + \left(1 - e^{-\frac{TE}{2T_1}}\right) M_0$$

$$= M_z^A \cos(\alpha) e^{-\frac{TE}{2T_1}} + \left(1 - e^{-\frac{TE}{2T_1}}\right) M_0 \qquad \left(\text{subs. } M_z^B\right)$$

.

Following the π-pulse two things happen: the z-component of the magnetization gets flipped, while the phase of the magnetization also gets inverted. However, the transverse and longitudinal magnetizations **do not get mixed.** Then:

$$M_z^D = -M_z^C$$

$$= -M_z^A \cos(\alpha) e^{-\frac{TE}{2T_1}} - \left(1 - e^{-\frac{TE}{2T_1}}\right) M_0 \qquad \left(\text{subs. } M_z^C\right)$$

At point E the xy magnetization has decayed (or has been spoiled), while M_z continues to relax for a duration $-\frac{\quad}{\quad}$:

$$M_z^E = M_z^D e^{-\frac{TR - TE/2}{T_1}} + \left(1 - e^{-\frac{TR - TE/2}{T_1}}\right) M_0$$

$$= \left[-M_z^A \cos(\alpha) e^{-\frac{TE}{2T_1}} - \left(1 - e^{-\frac{TE}{2T_1}}\right) M_0\right] e^{-\frac{TR - TE/2}{T_1}} + \left(1 - e^{-\frac{TR - TE/2}{T_1}}\right) M_0 \qquad \left(\text{subs. } M_z^D\right)$$

Using $M_z^E = M_z^A$, solving for $^{A}_{z}$ and simplifying, we obtain

$$M_z^A = M_0 \frac{1 - 2e^{-TR/T_1 + TE/2T_1} + e^{-TR/T_1}}{1 + \cos(\alpha) e^{-TR/T_1}}$$

whence

$$\boxed{M_{xy}^B = M_z^A \sin(\alpha) = M_0 \frac{1 - 2e^{-TR/T_1 + TE/2T_1} + e^{-TR/T_1}}{1 + \cos(\alpha) e^{-TR/T_1}} \sin(\alpha)}$$

For TE<<T_1,