

Plasticity

1. An incompressible elastic-perfect-plastic cylindrical rod, of Young's modulus E , yield stress $\sigma_Y \ll E$, length L and cross section A is compressed/pulled under uniaxial stress along its axis until its length is multiplied by a factor λ . How much work did the external loading perform? How much of it was dissipated? Work in the regime that $|\lambda - 1| \ll 1$, but plastic deformation does occur.
2. Consider the setting shown in Fig 1a: three elastic-perfect-plastic rods with cross sectional area A are connected with pins that can transfer only axial forces but no torques, and a vertical force F is exerted on them. The top pins are held at fixed positions to the ceiling (but not at a fixed angle). All rods have Young's modulus E and yield stress $\sigma_Y \ll E$. When $F = 0$ the system is stress-free. Assume small deformations.

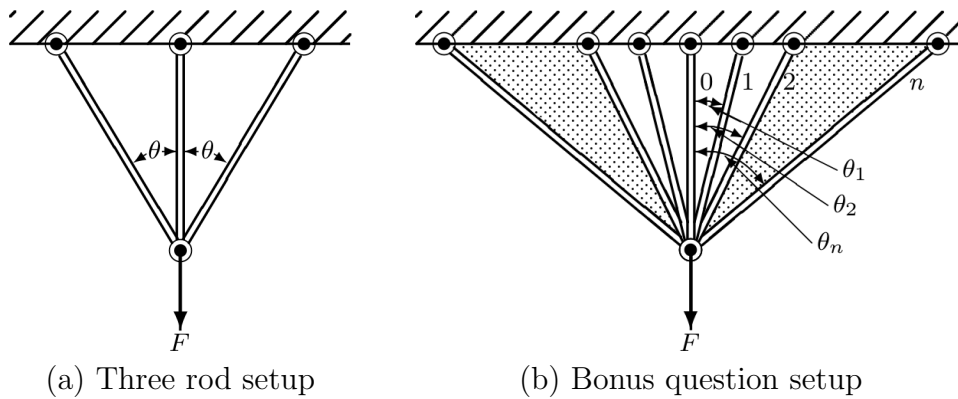


Figure 1: n -rods setup.

- (a) Denote the vertical displacement of the loading point by Δ . Calculate and plot $\Delta(F)$ (choose some values for the parameters you need). What is the maximal force F_E for which the response is elastic? What is the maximal force F_U that can be applied?
 - (b) Calculate the residual strains and stresses if the force is removed after the displacement was Δ .
 - (c) Suppose no force is applied, but the temperature is increased (or decreased) by ΔT . Calculate the minimal temperature difference ΔT_E that causes plastic deformation (assume α_T, σ_Y, E are T -independent).
 - (d) Bonus: repeat (a) for the case where there are 5 bars, or better yet, $2n + 1$. The setup is shown in Figure 1b. Assume the system is symmetric with respect to horizontal reflection.
3. In class, we've found the elasto-plastic solution for a spherical shell. We now look at some interesting aspects of the results.
 - (a) Examine numerically Eq. (11.38) from the lecture notes. For the case that $b = 10a$, plot c as a function of p . Can you analytically explain what happens when $p \rightarrow p_U$? (hint: yes you can).

- (b) For the case that $p = \sigma_Y$, plot c/a as a function of b/a . What is the asymptotic value of c when $b/a \rightarrow \infty$?
- (c) Find the displacement field $u_r(r)$ (from symmetry, \vec{u} is a function of r only and other components vanish). Is the stress/strain/displacement field continuous/differentiable across the elasto-plastic boundary?

Guidance: In the elastic region, there's a particularly simple relation between u_r and some of the strain components. In the plastic region, the volumetric part of the deformation is still elastic - we still have $\text{tr } \boldsymbol{\sigma} = K \text{tr } \boldsymbol{\epsilon}$, where K is the bulk modulus.

4. Continuing our TA session, consider an elastic-perfect-plastic 2D annulus with internal and external radii a, b , subject to internal pressure p and zero outer pressure, under *plane-stress* conditions. Use the Tresca yield criterion, and perform the analysis that was done in class:

- (a) Find the stress field $\sigma_{ij}(r)$, the minimal internal pressure that induces plastic flow (p_E), the ultimate pressure for which the entire annulus is plastic p_U , and give an equation that determines the radius of the elasto-plastic boundary c . Try and solve this in a different method than the one showed in the TA session.
- (b) Show that your solution is valid only if

$$1 + \frac{c^2}{b^2} - \log \frac{c^2}{a^2} \geq 0 . \quad (1)$$

What happens if this criterion is not satisfied? Why is this problem not present in plane strain conditions?

- (c) Considering this, what is the condition on a/b that ensures that p_U exists? Give an equation that describes, for a given value of a/b , the maximal possible value of c/a . What is this value when $b/a \rightarrow \infty$?