NONLINEAR THEORY OF FERROMAGNETIC RESONANCE

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Translated from Fizika Tverdogo Tela, Vol. 13, No. 2,
pp. 523-532, February, 1971
Original article submitted October 23, 1970

A consistent steady-state theory of nonlinear ferromagnetic resonance is constructed. It takes into account not only the interaction of the uniform precession with the spin waves but also the interaction of parametrically excited spin waves with one another. The role of the spin wave-spin wave interaction in determination of the behavior of the uniform precession and the spin waves above the threshold is different in the cases when the instability of uniform precession is due to a three-wave interaction $\omega_0 = \omega_k + \omega_{-k}$ or four-wave interaction $2\omega_0 = \omega_k + \omega_{-k}$. For the three-wave instability in the case when the uniform precession is excited at resonance ($\omega_p = \omega_0$) the amplitude of uniform precession is "frozen" at the threshold level right up to $h/h_c \sim 10^3$ (where h_c is the critical value of the alternating magnetic field at the threshold). In the nonresonance case ($\omega_p - \omega_0 \sim \omega_0$) the spin wave-spin wave interaction becomes important immediately above the threshold and leads to virtually complete "defreezing" of the amplitude of uniform precession, which continues to increase in accordance with a linear law. For the four-wave instability ($\omega_p \simeq \omega_0 = 2\omega_k$) the spin wave-spin wave interaction also leads to a linear increase in the uniform precession for $(h - h_c)/h_c > 1$. A preliminary comparison of the results of the theory with experimental results obtained by the authors on yttrium iron garnet single crystals confirms that it is necessary to take into account the interaction of parametric spin waves with one another in the theory of nonlinear ferromagnetic resonance.

The experiments of Bloembergen, Damon, and Wang [1, 2] showed that the phenomenon of ferromagnetic resonance cannot be explained by a simple theory that treats ferromagnetic resonance as oscillations of a "rigid" magnetization vector that is uniform over the crystal. The physical reason for the anomalous effects in ferromagnetic resonance at high amplitudes of the alternating field was established by Anderson and Suhl [3]; they showed that uniform precession of the magnetization may become unstable against spatially inhomogeneous perturbations of the magnetization (spin waves). At a certain threshold amplitude spin waves are excited parametrically by the uniform precession. They grow linearly with time and expend the energy of uniform precession until nonlinear effects set in which restrict the amplitude of the waves at a certain level that is appreciably higher than the thermal level.

As is well known [4], one must distinguish two principal instability mechanisms of ferromagnetic resonance: 1) The excitation of spin waves with frequency $\omega_k = \omega_p/2$ (ω_p is the pumping frequency); 2) excitation of spin waves with the frequency $\omega_k = \omega_p$. We shall refer to these mechanisms as the nonlinear processes of first and second order, respectively.

The existing theory of the state above threshold does not take into account the interaction of the spin waves with one another (the spin wave-spin wave interaction), although the latter is very strong at large amplitudes and cannot be neglected in the majority of cases.[1] At the same time, it is widely believed that a restriction of parametric excitation occurs basically because of a reaction of the spin waves to the uniform precession (the spin wave-uniform precession interaction). This opinion is based on the phenomenon of "freezing" of the uniform precession above the threshold predicted by the spin wave-uniform precession theory which has been applied in ferrite power limiters.

It is well known that a virtually complete cessation of growth of the uniform precession amplitude is observed in ferromagnetic resonance for first-order processes [6]. On the other hand, there are a number of facts that cannot be explained by the spin wave-uniform precession interaction. In particular, the behavior of the nonlinear susceptibilities χ' and χ'' far from resonance differs from the theoretical prediction when the pumping power greatly exceeds the threshold [7]. A particularly important fact is that the real part of the susceptibility χ' remains practically constant after the threshold whereas the spin wave-uniform precession interaction theory predicts a decrease which varies as h^{-2} (h is the amplitide of the alternating magnetic field). We would like to also mention the experiments on the saturation of ferromagnetic resonance (in the case of second-order processes), which indicate an increase in the uniform precession amplitude above the threshold [8].[2]

The aim of the present paper is to construct a consistent nonlinear theory of ferromagnetic resonance which takes into account both the spin wave-uniform precession and spin wave-spin wave interaction. From the theoretical point of view it is clear that as the spin waves grow above the threshold the energy of the spin wave-spin wave interaction will first become comparable with and then exceed the energy of the spin wave-uniform precession interaction. The pumping power at which the spin wave-spin wave interaction becomes important depends on the properties of the spin system and the type of nonlinear processes.

We shall restrict our treatment to the interaction of parametrically excited waves with one another and we shall not touch on the various mechanisms of nonlinear damping due to the interaction of parametric waves with the thermal bath.[3]

The main factor determining the character of the spin wave-spin wave interaction above the threshold is the establishment of a phase correlation of waves with equal wave vectors but opposite directions. The sum of the phase of the waves forming a pair, similar to a Cooper pair in superconductors, is in a definite relationship with the pumping phase, i.e., the phase within every pair is completely correlated whereas the individual phases of the waves are statistically independent. Above the threshold one should not therefore speak of the interaction of individual waves but of the interaction of pairs.

The basic nonlinear equations describing the "pairing" and interaction of pairs have been obtained by Zakharov, L'vov, and Starobinets [11]; in what follows we shall refer to the latter papers as I. The paper I treats the interaction of parametric waves due to processes of the type

$$\omega_k + \omega_{-k} = \omega_{k'} + \omega_{-k'} = \omega_p, \tag{1}$$

which are the dominant spin wave-spin wave processes for not too large excesses above the threshold ($\zeta \equiv h/h_c < (\omega/\gamma)^{1/2}$. The investigation of the interaction (1) showed that it can be represented by an additional pumping proportional to the total amplitude of the pairs. It is important that the phase of this pumping differs from that of the external pumping. The occurrence of additional pumping with a different phase results in a self-consistent change in the phases of the pairs and, hence, to a weakening of their coupling and a limitation of their amplitude.

The importance of the spin wave-spin wave interaction for processes occurring above the instability threshold of ferromagnetic resonance is different for nonlinear processes of the first and second orders. A characteristic feature of the first-order processes is the strong reaction which the spin waves exert on the uniform precession as a result of three-wave processes of the type

[1]It is interesting to cite Schlömann's opinion [5]. "This approximation (the elimination of the spin wave-spin wave interaction) is made to simplify the mathematical side of the problem and cannot, in general, be justified. In applying this approximation we probably lose a large part of the important physical information."

[2]We should like to mention the interesting investigations [9, 10] in which it is shown that allowance must be made for the spin wave-spin wave interaction if one wishes to explain the phenomenon of frequency doubling at saturation of the ferromagnetic resonance.

[3]The nonlinear damping due to to third-order processes may be important; to take it into account one must use Eq. (42) instead of Eq. (30) of [11]. We have not done this for simplicity, an omission that is also justified by the fact that there is a range of values of the constant magnetic field in which the three-wave processes are forbidden by the form of the spectrum.

$$\omega_{\mathbf{k}} + \omega_{-\mathbf{k}} = \omega_0. \tag{2}$$

At resonance ($\omega_0 = \omega_p$, $\omega_k = \omega_p/2$) the processes (2) are more important than those of (1) right up to very large excesses $\zeta > 10^3$ (this is proved in Sec. 2). In this region the uniform precession amplitude is virtually "frozen."

The investigation of the nonresonance case ($\omega_0 - \omega_p \sim \omega_0$, $\omega_k = \omega_p/2$) shows that at small excesses ($\zeta - 1 < 1$) the spin wave-uniform precession dominates, as before. However, if $\zeta - 1$ is not small compared with 1, the spin wave-spin wave interaction is as important as the spin wave-uniform precession interaction. It is interesting that for $\zeta \gg 1$ the spin wave-spin wave interaction may become predominant and the reaction can be neglected. The uniform precession amplitude is then completely "defrozen" and increases linearly.

For second-order processes ($\omega_0 \simeq \omega_p$, $\omega_k = \omega_p$) the spin wave-uniform precession interaction is due to four-wave processes of the type

$$\omega_{\mathbf{k}} + \omega_{-\mathbf{k}} = 2\omega_0, \tag{3}$$

which are the analog of the processes (1) with k' = 0. It follows that both the processes (1) and (3) must be taken into account except in the region of very small excesses ($\zeta - 1 \ll 1$), in which the processes (3) are predominant.

It should be noted that in crystals with magnetic inhomogeneities there is a more effective mechanism of spin wave-uniform precession interaction than the processes of the type (3). Inhomogeneities lead to a scattering of spin waves which occurs without alteration of frequency but with a change in the momentum, i.e., $\omega_k = \omega_0$ [5]. The theory of second-order processes considered in ~~this paper treats~~ a perfect crystal that does not contain inhomogeneities although even a few inhomogeneities exert an appreciable influence on the behavior of the uniform precession near threshold. It is to be expected that in a crystal with a small number of inhomogeneities there are three characteristic regions: 1) The region of small excesses ($\zeta - 1 \ll 1$), in which the inhomogeneity mechanism is predominant; 2) an intermediate region in which both mechanisms are important; 3) the region of large excesses, in which the processes (1) and (3) are predominant. In Sec. 4 we show that for $\zeta \gg 1$ the uniform precession amplitude increases linearly with the field although more slowly by a factor of approximately two than below the threshold.

1. BASIC EQUATIONS

We shall describe nonlinear phenomena in ferromagnetic resonance in the framework of the classical Hamiltonian formalism. The Hamiltonian of a system that includes uniform precession and spin waves has the form

$$\mathcal{H} = \omega_0 a_0 a_0^* + \sum_{\mathbf{k}} \omega_{\mathbf{k}} a_{\mathbf{k}} a_{\mathbf{k}}^* + \mathcal{H}_p + \mathcal{H}_{00} + \sum_{\mathbf{k}} \mathcal{H}_{0\mathbf{k}} + \sum_{\mathbf{kk'}} \mathcal{H}_{\mathbf{kk'}}. \tag{4}$$

Here

$$\mathcal{H}_p = h e^{i\omega_p t} U a_0^* + \text{c.c.} \tag{4a}$$

describes the interaction of uniform precession a_0 with the uniform pumping magnetic field with frequency ω_p (for simplicity we assume that this field is circularly polarized in a plane perpendicular to the constant magnetization);

$$\mathcal{H}_{00} = T_{00} |a_0|^4 \tag{4b}$$

describes the intrinsic nonlinear shift of the uniform precession frequency;

$$\mathcal{H}_{0\mathbf{k}} = \tfrac{1}{2}(V_{0\mathbf{k}}^* a_0^* a_{\mathbf{k}} a_{-\mathbf{k}} + \text{c.c.}) + (S_{0\mathbf{k}}^* a_0^* a_0^* a_{\mathbf{k}} a_{-\mathbf{k}} + \text{c.c.}) + 2T_{0\mathbf{k}} |a_0|^2 |a_{\mathbf{k}}|^2, \tag{4c}$$

describes the uniform precession-spin wave interaction. The first two terms determine the parametric interaction of the first and second order, respectively, and the last term the nonlinear frequency shift; finally,

$$\mathcal{H}_{\mathbf{kk'}} = S_{\mathbf{kk'}}^* a_{\mathbf{k}}^* a_{-\mathbf{k}}^* a_{\mathbf{k'}} a_{-\mathbf{k'}} (1 + 2\Delta_{\mathbf{kk'}}) + T_{\mathbf{kk'}} |a_{\mathbf{k}}|^2 |a_{\mathbf{k'}}|^2 (2 - \Delta_{\mathbf{kk'}})$$
$$(\Delta_{\mathbf{kk}} = 1, \quad \Delta_{\mathbf{kk'}} = 0 \ \text{ for } \mathbf{k} \neq \mathbf{k'}) \tag{4d}$$

describes the spin wave-spin wave interaction. The first term is the parametric interaction of pairs of spin waves and the second is the mutual nonlinear frequency shift.

For a system consisting of one or two pairs the Hamiltonian (4d) is exact. For a large number of pairs the terms that are nondiagonal in the pairs and are omitted in (4d) are averaged to zero because of the random distribution of the individual phases. The physical justification for the choice of $\mathcal{H}_{\mathbf{kk'}}$ in the form (4d) and the limits of applica-