INSTABILITY OF MONOCHROMATIC STANDING SPIN WAVES WITH PARALLEL PUMPING

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A study is made of the instability of standing spin waves in a ferromagnet with damping and with a pump h ~ exp $(i\omega_p t)$ involving four-wave processes having the conservation laws

$$2\omega_{\pm k} = \omega_{\pm k+\varkappa} + \omega_{\pm k-\varkappa},$$
$$\omega_k + \omega_{-k} = \omega_{k\pm\varkappa} + \omega_{-k\pm\varkappa}.$$

The pair instability threshold h_1 is nearly always small, $(h_1-h_c)/h_c \sim 10^{-4}$-$10^{-6}$, where h_c is the threshold for the appearance of pairs. The maximum instability increment $\gamma_m(\varkappa)$ lies in the region $\varkappa \ll k$; for large $\varkappa \simeq k$ the pairs are stable only with the unique choice $k = k_0$. A brief discussion is given of the nonlinear stage in the development of the instability.

At present there is great interest in the turbulence of nonlinear waves in states far from thermodynamic equilibrium. Some examples are weak turbulence in a plasma [1], amplification of acoustic noise under conditions of acoustic instability in piezoelectric semiconductors [2], and self-focusing of a laser beam in a nonlinear dielectric [3]. Turbulence in the parametric excitation of spin waves is this kind of problem.

Parametric excitation of spin waves in ferromagnetic materials was predicted and observed in the work of Anderson and Suhl [4], Suhl [5], and Schlöman [6] and has been widely studied theoretically, experimentally, and practically [7].

The detailed behavior of spin waves past the threshold for parametric excitation has been considered by Zakharov, Starobinets and the author [8]. The main approximation used there was in simplifying the interaction Hamiltonian H_i of the spin waves in a way reminiscent of the BCS approximation in the theory of superconductivity. In particular, instead of the exact Hamiltonian

$$H_i = \frac{1}{2}\sum_{12,34} T_{12,34} a_1^* a_2^* a_3 a_4 \Delta(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{k}_3 - \mathbf{k}_4) \quad (1)$$

we have used the reduced Hamiltonian

$$\tilde{H}_i = \sum_{\mathbf{kk}'}\Big[T_{\mathbf{kk}'} a_\mathbf{k} a_\mathbf{k}^* a_{\mathbf{k}'} a_{\mathbf{k}'}^* + \frac{1}{2} S_{\mathbf{kk}'} a_\mathbf{k}^* a_{-\mathbf{k}}^* a_{\mathbf{k}'} a_{-\mathbf{k}'}\Big] - \frac{1}{2}\sum_\mathbf{k}[T_{\mathbf{kk}} a_\mathbf{k}^2 a_\mathbf{k}^{*2} + 2S_{\mathbf{kk}} |a_\mathbf{k}|^2 |a_{-\mathbf{k}}|^2], \quad (2)$$

which includes only the part of H_i which is diagonal in the pairs of spin waves with wave vectors **k** and −**k**. Here the canonical variables $a_\mathbf{k}$ are the complex amplitudes of traveling spin waves related by a transformation of the Holstein-Primakoff type [9, 10] to the magnetization **M**(**k**) of the ferromagnet while $T_{\mathbf{kk}'} = T_{\mathbf{kk}',\mathbf{kk}'}$, and $S_{\mathbf{kk}'} = T_{\mathbf{k},-\mathbf{k};\mathbf{k}'-\mathbf{k}'}$. The approximation in which we use the diagonal Hamiltonian (2) also takes no account of thermal noise and will be called the "S model." One of the main results obtained in [8] is that in ferromagnets which do not have axial symmetry (for example with anisotropies of the "easy plane" type) when the threshold is exceeded by less than 2 to 4 dB monochromatic standing spin waves $\pm\mathbf{k}_0$ of amplitude N will be excited, whose wave vectors are given by Eqs. (17) and (3). Such states, coherent over the whole crystal, can be observed for example in experiments on light scattering and other experiments. In [11] Zakharov and the author showed that thermal noise does not destroy the coherence if the standing wave is stable within the framework of the exact Hamiltonian (1).

A preliminary study of the stability of arbitrary stationary states can be carried out already in the "S model." In this model we should distinguish "external" stability, i.e., stability with regard to creation of other pairs, from "internal stability with regard to changes of the amplitude and phase of the excited waves. The need for external stability can always be satisfied; in the "S model" it uniquely determines a surface in k space where the excited spin waves can be in a stationary state; for one pair this is

$$\omega_{\mathbf{k}}-\frac{\omega_p}{2}+\frac{T_{\mathbf{kk}}N}{2}=0, \tag{3}$$

where ω_k is the spectrum of waves, ω_p is the pump frequency, $N = 2|a_0|^2 = 2|a_{\bar{0}}|^2$, $a_0 = a_{k_0}$, $a_{\bar{0}} = a_{-k_0}$. Internal pair stability does not always occur and for this we require

$$S(2S+T)>0. \tag{4}$$

We will further restrict this not very rigid condition. When the exchange interaction is predominant, T = −S, and T = S [9, 10] for ferromagnets with one-ion anisotropy and a pair has internal stability.

Here we study the stability of standing spin waves in a ferromagnet with damping and a pump within the framework of the complete Hamiltonian (1), i.e., the stability with regard to decay processes with conservation laws[1]

$$2\omega_{\pm\mathbf{k}_0}=\omega_{\pm\mathbf{k}_0+\varkappa}+\omega_{\pm\mathbf{k}_0-\varkappa}, \tag{5}$$

$$\omega_{\mathbf{k}_0}+\omega_{-\mathbf{k}_0}=\omega_{\mathbf{k}_0+\varkappa}+\omega_{-\mathbf{k}_0-\varkappa}. \tag{6}$$

For large $\varkappa$ the waves $(\mathbf{k}_0 + \varkappa)$ and $(\mathbf{k}_0 - \varkappa)$ as a rule cannot simultaneously fall into the region of k space where the damping is compensated by the pump and the threshold for the modulation instability (5) turns out to be high. In this case there is only the process (6) in which the waves interact in pairs $\pm\mathbf{k}$, which corresponds to the approximation of a diagonal Hamiltonian (2). Analysis of the expression for the increment (26) shows that the contribution of process (5) to the interaction of the waves is small if

$$\frac{\varkappa}{k_0}\geqslant\left(\frac{hV-\gamma}{\omega_0}\right)^{1/3}, \tag{7}$$

where hV is the pump amplitude (12), γ is the wave damping (13), and $hV = \gamma$ corresponds to the threshold for parametric excitation of the initial pair. This inequality limits the minimum distance between pairs under dynamical conditions in which they can still be described by the "S model." It is substantially more rigorous than the corresponding inequality (4) of [8] found from obvious considerations.

When $\varkappa$ is less than (7) both processes (5) and (6) occur and we should treat pairs $\pm\mathbf{k}_0$ with regard to creation of four coupled spin waves $\pm(\mathbf{k}_0 + \varkappa)$ and $\pm(\mathbf{k}_0 - \varkappa)$ which correspond to the dispersion relations obtained and studied in Sec. 3. It turns out that the behavior of the increment depends substantially on the signs of the coefficients S, T, and (2S + T) in the Hamiltonian. In particular with $\varkappa = 0$ we find for the increment γ_{ef}

$$(\gamma+\gamma_{ef})^2-\gamma^2=-2S(2S+T)N^2 \text{ and } \gamma_{ef}=0. \tag{8}$$

The first branch corresponds to "intrinsic" instability in the "S model" and the second to a state of neutral equilibrium with regard to changes in the phase difference $(\varphi_0 - \varphi_{\bar{0}})$ of a pair. The origin of the second mode involves the dynamic nature of the stationary state — there is no randomization of the phase differences and neutral equilibrium occurs due to spatial uniformity since the difference $(\varphi_0 - \varphi_{\bar{0}})$ determines the spatial positions of the nodes and antinodes of the standing waves. For small $\varkappa$ these modes remain stable if

$$T>0,\ (2S+T)>0. \tag{9}$$

It is interesting that T > 0 is the condition for no self-focusing of a monochromatic wave in a conservative medium. The conditions (9) also suffice for "intrinsic" stability. However for stability for all $\varkappa$ we also must satisfy the rigid requirement

$$\frac{(hV-\gamma)}{\gamma}\leqslant\left(\frac{\gamma}{\omega}\right)^2\left(\frac{S}{T}\right)^4. \tag{10}$$

Estimating $(\gamma/\omega) \sim 10^{-2}$–$10^{-3}$ we find $(hV - \gamma)/\gamma \sim 10^{-4}$–$10^{-6}$ for the instability threshold. If the pair is to remain stable when threshold is exceeded by several decibels ($hV - \gamma \simeq \gamma$) it is necessary that $|T| \sim (0.1$–$0.01)S$.

[1]Instability of pairs of monochromatic waves in a conservative medium was briefly discussed by Zakharov [12] and instability of pairs in a medium with damping and pumping slightly over threshold when the limitation of the amplitude arises from nonlinear damping has been discussed by Zakharov, Starobinets, and the author [10].

Thus monochromatic standing spin waves are practically always unstable with parallel pumping. At the end of the paper we briefly discuss in what sense we can retain the results obtained in the "S" model."

1. BASIC EQUATIONS

As in previous work [8-11, 13, 14] we will describe the ferromagnet within the framework of a classical Hamiltonian formalism. The Hamiltonian function is

$$H=\sum_{\mathbf{k}}\omega_{\mathbf{k}}a_{\mathbf{k}}a_{\mathbf{k}}^{*}+H_p+H_i, \tag{11}$$

where H_p is the Hamiltonian for interaction with an alternating magnetic field $h(t) = h\exp(i\omega_p t)$ (the pump)

$$H_p=\frac{1}{2}\sum_{\mathbf{k}}[hV_{\mathbf{k}}a_{\mathbf{k}}^{*}a_{-\mathbf{k}}^{*}+\text{c.c.}], \tag{12}$$

where H_i is the exact Hamiltonian for the interaction of the waves (1). The equation of motion is

$$\frac{\partial a_{\mathbf{k}}}{\partial t}+\gamma_{\mathbf{k}}a_{\mathbf{k}}=i\frac{\delta H}{\delta a_{\mathbf{k}}^{*}}, \tag{13}$$

where γ is a phenomenological spin wave damping parameter [9].

2. STATIONARY STATE OF A PAIR

We write Eq. (13) for a pair:

$$\left[\frac{\partial}{\partial t}+\gamma-i(\omega_0+T_{00}|a_0|^2+2S_{00}|a_{\bar{0}}|^2)\right]a_0=ihV_0a_{\bar{0}}^{*}. \tag{14}$$

Here $\omega_0 = \omega_{k_0}$, $V_0 = V_{k_0}$, etc., while the damping γ_k in what follows will be assumed independent of k for simplicity.

We introduce the amplitude N and the phase ψ of a pair

$$a_0=\sqrt{\frac{N}{2}}e^{i(\omega t+\varphi_0)},\ a_{\bar{0}}=\sqrt{\frac{N}{2}}e^{i(\omega t+\varphi_{\bar{0}})},\ \psi=\varphi_0+\varphi_{\bar{0}}. \tag{15}$$

In the stationary state $\omega = \omega_p/2$ so that we find from (14)

$$i\gamma+hV_0e^{-i\psi}+\delta+N\left(S_{00}+\frac{1}{2}T_{00}\right)=0,\ \delta=\omega_0-\frac{1}{2}\omega_p. \tag{16}$$

Similar relations have been found by Petrakovskii [15] using the Landau-Lifshits equation. It was postulated there that beyond the threshold for parallel pumping there exists only one pair with a certain (unknown!) detuning and the dependence of N and ψ on δ was investigated based on (16).

It has been shown [8, 14] that for "internal" stability of a pair it is necessary and sufficient that

$$2\delta+NT_{00}=0.$$

Here the amplitude N and the phase ψ in the stationary state are

$$N=\frac{\sqrt{(hV_0)^2-\gamma^2}}{|S_{00}|},\quad hV_0\sin\psi=\gamma. \tag{17}$$

In the following section we study the stability of an arbitrary stationary state with

$$\delta+\frac{NT_{00}}{2}=N(B-S_{00}), \tag{18}$$

within the framework of the exact Hamiltonian, where B is an arbitrary function of N and ψ and we show in particular that of all the stationary states (18) the most stable one is that for which $B = S_{00}$.

3. DISPERSION RELATIONS

Using the exact Hamiltonian (1), (11), (12), and Eq. (13) we write the linearized equations of motion for the perturbation waves

$$\left.\begin{aligned}b_1&=a_{\mathbf{k}_0+\varkappa}\exp\left(-\frac{i}{2}\omega_pt\right),\ b_{\bar{1}}^{*}=a_{-\mathbf{k}_0-\varkappa}^{*}\exp\left(\frac{i}{2}\omega_pt\right),\\ b_{\bar{2}}&=a_{-\mathbf{k}_0+\varkappa}\exp\left(-\frac{i}{2}\omega_pt\right),\ b_2^{*}=a_{\mathbf{k}_0-\varkappa}^{*}\exp\left(\frac{i}{2}\omega_pt\right),\end{aligned}\right\} \tag{19}$$

$$\begin{vmatrix}\Omega_1, & P_1, & FN, & \frac{1}{2}G^{*}\Sigma^{*}\\ P_1^{*}, & \Omega_1^{*}, & \frac{1}{2}G\Sigma, & F^{*}N\\ F^{*}N, & \frac{1}{2}G\Sigma, & \Omega_2, & P_2\\ \frac{1}{2}G^{*}\Sigma^{*}, & FN, & P_2^{*}, & \Omega_2^{*}\end{vmatrix}\begin{vmatrix}b_1\\ b_{\bar{1}}^{*}\\ b_{\bar{2}}\\ b_2^{*}\end{vmatrix}=0, \tag{20}$$

where

$$\Omega_j=\tilde{\omega}_j+i\left(\frac{\partial}{\partial t}+\gamma\right),\ j=1,\ 2;$$
$$\tilde{\omega}_j=\omega_j-\frac{1}{2}\omega_p+N(T_{j0}+T_{j\bar{0}});$$
$$P_j=hV_j+S_{j0}\Sigma,\ \Sigma=N\exp(i\psi);$$
$$F=T_{10\bar{2}0},\ G=T_{12,00}.$$

We seek a solution of Eq. (20) in the form $b \sim \exp(\gamma_{ef}t)$, where γ_{ef} is the instability increment. The condition of zero determinant for the system leads to a biquadratic equation for $(\gamma + \gamma_{ef})$ whose solution is

$$2(\gamma+\gamma_{ef})^2=(\tilde\omega_1^2+\tilde\omega_2^2)+|P_1|^2+|P_2|^2+\tfrac{1}{2}N(|G|^2-4|F|^2)\pm[(\tilde\omega_1^2-\tilde\omega_2^2-|P_1|^2+|P_2|^2)-|G\Sigma(\tilde\omega_1-\tilde\omega_2)+2N(FP_2^*-F^*P_1)|^2+|2FN(\tilde\omega_1+\tilde\omega_2)-(G^*\Sigma^*P_1+G\Sigma P_2^*)|]^{1/2}. \quad (21)$$

Before studying this complex expression we mention that for large $\varkappa$ it reduces to the simple condition

$$(\gamma+\gamma_{ef})^2=\sim\tilde\omega_1^2+|P_1|^2, \quad (22)$$

which follows directly from the "S model." Actually for large $\varkappa$ it is not possible to choose the direction of $\varkappa$ such that $\tilde\omega_1$ and $\tilde\omega_2$ are simultaneously small. Here the region of maximum increment corresponds to $\varkappa$ such that either $\tilde\omega_1$ or $\tilde\omega_2$ is small. Assuming that $\omega_2 \gg \tilde\omega_1 \sim \gamma$, and expanding the radical in (21) we at once see that Eq. (22) for γ_{ef} is valid.

Consequently the instability of interest to us lies in the region of small $\varkappa$. For simplicity we replace the coefficients $T_{\alpha\beta\gamma\delta}$ in (21) by their limits as $\varkappa \to 0$ and use the expansion

$$\tilde\omega_{k_0+\varkappa}=\pm\varkappa\mathbf{v}+\tfrac{1}{2}\widehat{L\varkappa^2}+N(T+S)+\delta,$$

where

$$\mathbf{v}=\frac{\partial\omega}{\partial\mathbf{k}},\quad \widehat{L\varkappa^2}=\sum_{\alpha\beta}\varkappa_\alpha\varkappa_\beta\frac{\partial^2\omega}{\partial k_\alpha\partial k_\beta},$$

$$V_{k_0\pm\varkappa}=V\left(1+\tfrac{1}{2}\widehat{W\varkappa^2}\right),\quad \widehat{W\varkappa^2}=\frac{1}{V}\sum_{\alpha\beta}\frac{\partial^2 V}{\partial k_\alpha\partial k_\beta}.$$

Then taking (15) and (18) into account we put (21) in the form

$$(\gamma+\gamma_{ef})^2-\gamma^2=-(\varkappa\mathbf{v})^2-\gamma^2W\varkappa^2-\tfrac{1}{4}(L\varkappa^2)^2-\tfrac{1}{2}L\varkappa^2(2B+T)N-N^2B(2S+T)\pm\{(\varkappa\mathbf{v})^2[(L\varkappa^2)^2+2L\varkappa^2(2B+T)N+4N^2B(B+T)]+N^2[SL\varkappa^2+NB(2S+T)]^2\}^{1/2}. \quad (23)$$

It should be expected that the instability will develop mainly for wave vectors $\varkappa$ near the surface

$$\pm\varkappa\mathbf{v}+\tfrac{1}{2}L\varkappa^2=0. \quad (24)$$

This means that $\varkappa$ is almost perpendicular to v and the quantity which is most sensitive to changes in the direction of $\varkappa$ in (23) will be the scalar product $\varkappa\mathbf{v}$. Therefore we will seek the maximum increment by varying (23) only in $\varkappa\mathbf{v}$. As a result we find that the maximum increment γ_m occurs at the surface

$$4(\varkappa\mathbf{v})^2=[L\varkappa^2+(2B+T)N]^2-T^2N^2-\frac{4N^2[SL\varkappa^2+B(2S+T)N]^2}{[L\varkappa^2+(2B+T)N]^2-T^2N^2}, \quad (25)$$

which for large $L\varkappa^2 \gg SN$ differs little from (24) and is

$$(\gamma+\gamma_m)^2-\gamma^2=-\gamma^2W\varkappa^2+(B-S)^2N^2+\frac{B^2T^2N^4+2ST(S-B)N^3(L\varkappa^2-2BN)}{[L\varkappa^2+(2B+T)N]^2-T^2N^2}.$$

We see that of the multitude of stationary solutions (18) only the one with B = S has a region of instability localized in k space. In what follows we will study only this state, which satisfies the condition of "external" instability (3) in the "S model." For this state

$$(\gamma+\gamma_m)^2-\gamma^2=-\gamma^2W\varkappa^2+\frac{T^2S^2N^4}{[L\varkappa^2+(2S+T)N]^2-T^2N^2}. \quad (26)$$

The second term in (26) is absent in the "S model." Comparing it with the first term for large $\varkappa$ we find the estimate (7) of the validity of the "S model." The behavior of the increment with $L\varkappa^2 \sim \gamma$ depends on the relationships of the signs of S, T, and (2S + T).

We first consider the case T > 0, S > 0. For this case Eq. (26) is valid with $\varkappa \gtrsim \varkappa_0$ and $\varkappa_0/k_0 \sim SN/\omega$. With $\varkappa \simeq \varkappa_0$

$$(\gamma+\gamma_m)^2-\gamma^2=\frac{T^2S}{(T+S)}N^2. \quad (27)$$

With $\varkappa < \varkappa_0$ the increment begins to decrease sharply and with $\varkappa \ll \varkappa_0$ we have $\varkappa \ll \varkappa_0$ $(\gamma+\gamma_m)^2-\gamma^2=(\varkappa v)^2TSN^2$ with $\varkappa \| v$. The behavior of

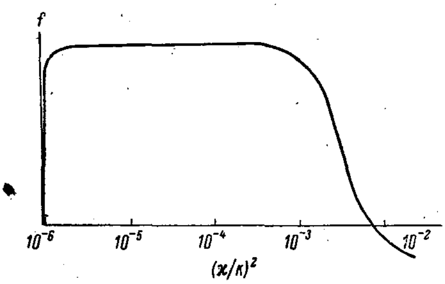

Fig. 1. Instability increment γ_m ($f = \gamma_m^2 + 2\gamma\gamma_m$) as a function of $\varkappa$ with S > 0 and T > 0 ($\gamma/\omega \simeq 10^{-3}$).

the increment for all $\varkappa$ and $T > 0$, $S > 0$ is shown schematically in Fig. 1.

If this condition is not satisfied then Eqs. (25) and (26) are only valid with $\varkappa > \varkappa_1$ where

$$L\varkappa_1^2 = [\sqrt{T^2+S^2} + |S| - (2S+T)]N.$$

For smaller $\varkappa$ the increment is greatest in the plane $\varkappa \perp v$ and is

$$(\gamma+\gamma_m)^2 - \gamma^2 = -\frac{1}{4}[L\varkappa^2 + (2S+T)N]^2 + \left(\frac{1}{4}T^2 - S^2\right)N^2 + |SN[L\varkappa^2 + (2S+T)N]| - \gamma^2 W\varkappa^2. \quad (28)$$

With $\varkappa = 0$ we find Eq. (8) from this and for small $\varkappa$ the behavior of (28) depends on the sign of $S(2S + T)$. With $S(2S + T) > 0$

$$(\gamma+\gamma_m)^2 - \gamma^2 = -\frac{1}{4}(L\varkappa^2)^2 - \frac{1}{2}L\varkappa^2 TN. \quad (29)$$

If $S(2S + T) < 0$, then

$$(\gamma+\gamma_m)^2 - \gamma^2 = -\frac{1}{4}(L\varkappa^2)^2 - \frac{1}{2}L\varkappa^2(4S+T)N - 2S(2S+T)N^2. \quad (30)$$

We give the most detailed treatment of the case of "intrinsic" pair stability where $S(2S + T) > 0$). Then for stability with small $\varkappa$ we need $T > 0$ as we see from (29). It necessarily follows from this that $S < 0$ (the unstable case $T > 0$, $S > 0$ occurs differently); therefore $(2S + T) > 0$. Here the increment is described by Eq. (29) with $\varkappa_2 > \varkappa > 0$, and $L\varkappa_2^2 = |2S + T|N$. If $\varkappa_1 > \varkappa > \varkappa_2$, the increment with $T > 0$, $S < 0$, and $(2S + T) < 0$ is described by Eq. (30). From this it follows that with $|S|N \sim \gamma$ the increment becomes positive already with $L\varkappa^2 = 2L\varkappa_2^2$. The schematic behavior of the increment is shown in Fig. 2. It achieves its greatest value

$$(\gamma+\gamma_m)^2 - \gamma^2 = -\gamma^2 W\varkappa_1^2 + \frac{|S|}{2}(\sqrt{T^2+S^2} - |S|)N^2 \quad (31)$$

with $\xi = \varkappa_1$. Assuming that $W \sim k_0^{-2}$ and $L \sim \omega k_0^{-2}$ we find the estimate (10) for the instability threshold.

As we see, the instability of the monochromatic standing waves is practically universal. The increment is usually positive with $(\varkappa/k)^3 < (hV - \gamma)/\omega$ and its behavior depends essentially on the relations among the signs of S, T, and $(2S + T)$, as schematically shown in Figs. 2 and 3.

There would be great interest in a qualitative study of the nonlinear stage in the development of the pair instability of interest. Generally speaking there are different possibilities. For example the total amplitude of the excited pairs will grow with $(S\Sigma n_k)^2 \gg (hV)^2 - \gamma^2$ until it is limited by a weaker mechanism, for example nonlinear damping. Evidently the excited part of k space will be quite wide in order that the phase differences within pairs become stochastic, and such a development can still be investigated within the framework of the "S model" for $S(S + 2T) < 0$. With $S(S + 2T) > 0$ the total amplitude $\sum_k n_k$ probably remains of the order of $[(hV)^2 - \gamma^2]^{1/2} S^{-1}$. The phase differences cannot become completely stochastic here since in the "S model" such a state is stable and should relax, collecting at one point in k space; on the other hand within the framework of the exact Hamiltonian it is stable and should expand in k space. We cannot exclude the possibility that the total amplitude will oscillate about some value and we can try to compare these oscillations with the nonlinear susceptibility χ'' of a ferromagnet.

Note that an important role in the nonlinear stage of development of the instability is played by the finiteness of the crystal size. The situation will be completely different in cases where there are many allowed points in k space or where

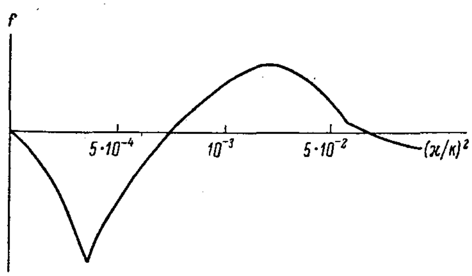

Fig. 2. Instability increment γ_m ($f = \gamma_m^2 + 2\gamma\gamma_m$) as a function of $\varkappa$ with $S < 0$, $(2S + T) > 0$.

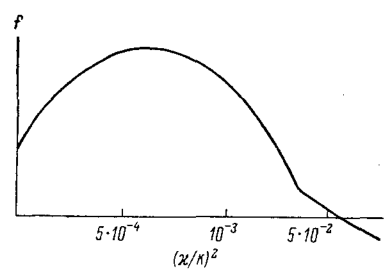

Fig. 3. Instability increment γ_m ($f = \gamma_m^2 + 2\gamma\gamma_m$) as a function of $\varkappa$ with $S < 0$, $T > 0$, and $2S + T < 0$.

there are only a few in the region of positive increments.

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