Nonlinear theory of parametric excitation of spin waves in antiferromagnets

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In the present paper we show that these experiments can be explained in natural fashion within the framework of a nonlinear theory that takes into account the interaction of parametrically-excited waves with one another.[11–12] The main factor of the theory is the concept of "pairing", or the pump-induced phase correlation of the waves in ±k pairs. The pairing leads to a strong increase of the pair interaction in comparison with the interaction of the individual waves. This makes it possible to simplify the Hamiltonian of the problem to the form

$$\mathcal{H}=\sum_{\mathbf{k}}\Big\{\big[\omega_{\mathbf{k}}+\sum_{\mathbf{k}'}T_{\mathbf{k}\mathbf{k}'}|a_{\mathbf{k}'}|^2\big]a_{\mathbf{k}}a_{\mathbf{k}}^{*}+\frac{1}{2}\Big[\Big(hV_{\mathbf{k}}e^{-i\omega_p t}+\frac{1}{2}\sum_{\mathbf{k}'}S_{\mathbf{k}\mathbf{k}'}a_{\mathbf{k}'}a_{-\mathbf{k}'}\Big)a_{\mathbf{k}}^{*}a_{-\mathbf{k}}^{*}+\text{c.c.}\Big]\Big\}. \quad (1)$$

The canonical variables are here the complex amplitudes $a_{\mathbf{k}}$ of the traveling waves, their dispersion law $\omega_{\mathbf{k}}$, and the amplitude h of the spatially-homogeneous microwave pump field.

Within the framework of the described approximation, we were able to obtain good qualitative and quantitative agreement with a large number of experimental facts on parametric excitation of SW in ferromagnets.[13–15] The distinguishing feature of EP-AFM is the need for taking into account the interaction of the SW of the low-frequency branch of the spectrum and of the pump field with the oscillations (homogeneous precession, HP) of the high-frequency branch. However, in a typical experimental situation, when the HP oscillations occur at a frequency ω_p that is far from its resonance Ω_0, this complication is of no fundamental importance: we can confine ourselves in the equations of motion to the linear approximation of the HP in the amplitude, solve these equations, and eliminate the HP from the Hamiltonian of the problem. This procedure leads to a renormalization of the coefficients of the Hamiltonian (1), namely $T_{\mathbf{k}\mathbf{k}'}\to\tilde{T}_{\mathbf{k}\mathbf{k}}$, $V_{\mathbf{k}}\to\tilde{V}_{\mathbf{k}}$, and $S_{\mathbf{k}\mathbf{k}'}\to\tilde{S}_{\mathbf{k}\mathbf{k}'}$.

When account is taken of the exchange Zeeman interaction and of the Dzyaloshinskii interaction, these coefficients take the following form for EP-AFM:

$$\begin{aligned}\tilde{V}_{\mathbf{k}}&=\tilde{V}=\frac{g^2}{2\omega_p}(2H_o+H_D)\\ \tilde{T}_{\mathbf{k}\mathbf{k}'}&=\tilde{T}=\tilde{S}_{\mathbf{k}\mathbf{k}'}=\tilde{S}=-\frac{g^2H_E}{2\omega_pM_o}H_o(4H_o+H_D)\end{aligned} \quad (2)$$

Here H_0 is the external field, H_D is the Dzyaloshinskiĭ field, H_E is the exchange field, M_0 is the sublattice magnetization, and g is the gyromagnetic ratio ($g \approx 2\pi \cdot 2.8$ MHz/Oe). As is customary, we have disregarded the small dipole-dipole interaction and found that the coefficients of the Hamiltonian (2) do not depend on the wave vectors.

As shown in[12], the integral characteristics of the system of parametric spin waves are not very sensitive to the manner in which the SW damping decrement γ_k depends on k, so that to obtain approximate results we shall assume $\gamma k = \gamma$. Then the equations of motion for the parametric SW

$$i\left(\frac{\partial a_{\mathbf{k}}}{\partial t} + \gamma_{\mathbf{k}} a_{\mathbf{k}}\right) = \frac{\delta \mathcal{H}}{\delta a_{\mathbf{k}}^*} \tag{3}$$

do not contain angular dependences and can be easily solved.[11,12] In the stationary state, the SW are isotropically distributed over the "resonant surface" (sphere): $\omega_{\mathbf{k}} + 2\tilde{T}\Sigma_{\mathbf{k}'} |a_{\mathbf{k}'}|^2 = \omega_p/2$, all the phases of the pairs are equal,

$$\phi_{\mathbf{k}} + \phi_{-\mathbf{k}} = \arcsin(\gamma/h\tilde{V}) \tag{4}$$

and the integral amplitude of the pairs depends in simple fashion on the supercriticality

$$\sum_{\mathbf{k}} |a_{\mathbf{k}}|^2 = \sqrt{(h\tilde{V})^2 - \gamma^2}/|\tilde{S}|. \tag{5}$$

In the traditional experiments[1-7] one investigates the nonlinear susceptibility χ of the AFM, defined by the equation $m_x(\omega_p) = \chi \cdot h_x(\omega_p)$, where m_x is the longitudinal component of the magnetization at the pump frequency ω_p.

Using (2), (4), and (5) we can show that

$$\begin{aligned} \chi = \chi' + i\chi'' &= \chi_0 + 2\sum_{\mathbf{k}} \tilde{V}_{\mathbf{k}} a_{\mathbf{k}} a_{-\mathbf{k}}/h\omega_p \\ &= \chi_0 + 2\tilde{V}^2(-p + 1 + i\sqrt{p-1})/|\tilde{S}|_p \\ 2\tilde{V}^2/|\tilde{S}| &= \chi_0 (H_D + 2H_0)^2/H_0(4H_0 + H_D), \end{aligned} \tag{6}$$

where χ_0 is the linear susceptibility of the EP-EFM and coincides far from resonance with the static susceptibility M_0/H_{ex}; $p = h^2/h_{th}^2$ is the supercriticality.

We emphasize that the obtained expression does not contain explicitly the damping γ_k, the nature of which has not yet been explained, and therefore admits of a direct comparison of experiment.

V.V. Kveder and L.A. Prozorova have recently performed detailed experiments on the dependence of χ' and χ'' on p (see the next article in this issue[16]). Many of the results obtained in their paper are directly described by formula (6), namely, the susceptibilities ($\chi_0 = \chi'$) and χ'' are of the same order of magnitude as the static susceptibility $\chi_0 \approx 10^{-3}$–10^{-4}, and the difference $(\chi' - \chi_0)$, which describes the nonlinear detuning of the resonator frequency, is negative in all fields H_0.

In accordance with (6), practically no dependence of χ on H_0 is observed in $CsMnF_3$ crystals ($H_D = 0$), and χ'' decreases with increasing H_0 in $MnCo_3$ ($H_D = 4.4$ kOe). The $\chi''(p)$ curves are similar in shape, and the maximum of the relative absorption χ'' in the experiment occurs at an excess $p_m \approx 3$–5 dB ($p_m = 3$ dB in accordance with formula (6)). Some quantitative discrepancy between the theoretical and experimental values of χ''_m is apparently due to the fact that we disregarded the dipole-dipole interaction, which leads to a dependence of γ_k and $S_{kk'}$ on the wave vectors. In addition, an important role should be played by the interaction of the parametrically excited SW with the thermal waves, which leads to a dependence of the damping of the parametric SW on their amplitude, as observed in experiment.[5,6]

The present results show that the concept of pairing of excited spin waves should serve as a basis for the understanding of phenomena that occur during the nonlinear stage of development of parametric instability in EP-AFM.

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