Excitation of spin waves by a uniform precession of magnetization due to the two-magnon scattering

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The Raman scattering of light from spin waves in antiferromagnet $CoCO_3$, which are excited by a uniform precession of the magnetization (under the antiferromagnetic resonance conditions), was observed in recent experiments of A. Borovik-Romanov, V. Zhotikov, N. M. Kreines, and A. Pankov.[1] Therefore, it is of interest to determine theoretically the distribution function of spin waves n_k and its dependence on the ampitude of the uniform precession of the magnetization and on the parameters characterizing the scattering of spin waves and of the uniform precession from static inhomogeneities.

The Hamiltonian describing such scattering has the standard form (see Refs. 2 and 3)

$$\mathcal{H}_{imp}=\frac{1}{N}\sum_{\mathbf{k}\mathbf{k}'}g_{\mathbf{k}\mathbf{k}'}a_{\mathbf{k}}a^{*}_{\mathbf{k}'}b_{\mathbf{k}'-\mathbf{k}}. \quad (1)$$

Here, a_k are complex spin wave amplitudes; a_0 is the amplitude of the uniform precession; b_k is the Fourier transform of the static random field, whose scattering properties are characterized by a matrix $q_{kk'}$; N is the number of magnetic atoms in the crystal.

An equation of motion for

$$n_{\mathbf{k}}=\langle a^{*}_{\mathbf{k}}a_{\mathbf{k}}\rangle,\quad n_0=\langle a^{*}_0a_0\rangle,$$

averaged over the random field due to inhomogeneities can be obtained by the Wyld diagrammatic method as in Ref. 2. The homogeneous precession then satisfies the standard equation and the effect of inhomogeneities reduces to a two-magnon correction to the damping decrement γ_0 in a pure crystal (and, therefore, to the appropriate broadening of the antiferromagnetic resonance):

$$\gamma_0\to\Gamma_0=\gamma_0+\int\Gamma_{0\Omega}d\Omega,\quad \Gamma_{0\Omega}=\frac{\pi ca^3k^2_{\Omega}\,|g_{0\Omega}|^2}{(2\pi)^3v_{\Omega}}. \quad (2)$$

Here, C is the impurity concentration; a^3 is the volume of a unit cell; Ω is the solid angle; v_Ω is the projection of the group velocity of spin waves on the normal to the surface $\omega_k=\omega$; k_Ω is its radius.

Assuming that $n_k > n_k^0 = [\exp(\hbar\omega_k/T)-1]^{-1}$, we obtain the following distribution function of spin waves:

$$n_{\mathbf{k}}=\frac{c}{N[(\omega-\omega_{\mathbf{k}})^2+\Gamma^2_{\Omega}]}\left[\int|g_{\Omega\Omega'}|^2n_{\Omega'}d\Omega'+|g_{0\Omega}|^{-2}n_0\right], \quad (3)$$

where ω is the uniform precession frequency (not necessarily under resonance conditions) and Γ_Ω is the damping of spin waves taking into account the inhomogeneities:

$$\Gamma_{\Omega}=\gamma_{\Omega}+\int\Gamma_{\Omega\Omega'}d\Omega',\quad \Gamma_{\Omega\Omega'}=\frac{ca^3k^2_{\Omega'}\,|g_{\Omega\Omega'}|^2}{8\pi^2v_{\Omega'}}. \quad (4)$$

The distribution function governing the distribution of spin waves over the angles n_Ω can be expressed in terms of n_k as follows:

$$n_{\Omega}=\frac{Na^3k^2_{\Omega}}{8\pi^3v_{\Omega}}\int n_{\mathbf{k}}d\omega_{\mathbf{k}},\quad N_{\mathbf{k}}\equiv\sum_{\mathbf{k}}n_{\mathbf{k}}=\int n_{\Omega}d\Omega. \quad (5)$$

It follows from Eq. (3) that spin waves excited by the uniform precession are located in a layer of thickness Γ_Ω/v_Ω near the surface $\omega_k=\omega$. This can be easily explained since the uniform precession mode scattered from imperfections creates a stimulating force at a frequency ω acting on spin waves and the width of the resonance line for spin waves is determined by their total damping decrement, i.e., by Γ_Ω.

It is convenient to determine the angular distribution of spin waves n_Ω from Eq. (3). We shall first integrate Eq. (3) over ω_k and use Eqs. (4) and (5) to transform the latter equation into a "transport equation":

$$\left(\gamma_{\Omega}+\int\Gamma_{\Omega\Omega'}d\Omega'\right)n_{\Omega}=\Gamma_{\Omega 0}n_0+\int\Gamma_{\Omega\Omega'}n_{\Omega'}d\Omega'. \quad (6)$$

Integrating Eq. (6) over Ω, we obtain a simple expression representing the energy balance for spin waves:

$$\int\gamma_{\Omega}n_{\Omega}d\Omega=n_0\int\Gamma_{0\Omega}d\Omega=n_0(\Gamma_0-\gamma_0); \quad (7)$$

the term on the left-hand side represents the energy dissipation in a system of stimulated spin waves with $\mathbf{k}\neq 0$ due to the intrinsic relaxation mechanisms and the term on the right-hand side represents the energy dissipation of the uniform precession due to the two-magnon effects (such energy is transferred to spin waves with $\omega_k=\omega$). Naturally, the terms containing $\Gamma_{\Omega\Omega'}$ which describe the redistribution of the energy within a system of spin waves with $\omega_k=\omega$ due to the two-magnon effects do not appear in Eq. (7).

The actual form of the distribution function n_Ω depends on the dispersion law of spin waves ω_k and on the scattering matrix for impurities $g_{kk'}$. For example, for $\omega_k=\omega_{kx}$, $x=\cos\theta$, $\gamma_k=\gamma_k$, and $g_{kk'}=g_1$, $g_{0k}=g_0$, we obtain

$$\left.\begin{aligned}\Gamma_k&=\gamma_k+\frac{ca^3g_1^2}{2\pi}\int_0^1\frac{k_x^2dx}{v_x},\\ n(x)&=\frac{n_0}{4\pi}\,\frac{ca^3g_0^2}{2\pi}\,\frac{k_x^2}{v_x},\end{aligned}\right\} \quad (8)$$

where $\omega_{k_x,x}=\omega$.

Equation (7) then assumes the form

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$$N_k = 4\pi \int_0^1 n_x dx = n_0 \frac{\Gamma_0 - \gamma_0}{\gamma_k}. \tag{9}$$

It follows from Eqs. (9) and (7) that the total number of spin waves with $k \neq 0$, i.e., N_k can be much greater than the number of spin waves with $k = 0$ (i.e., n_0) provided the principal relaxation mechamism for the uniform precession is the two-magnon scattering. Therefore, we obtain

$$\Gamma_0 \gg \gamma_0 \approx \gamma_k,$$
$$N_k/n_0 \approx \Gamma_0/\gamma_k \gg 1.$$

Experiments on $CoCO_3$ yield $N_k/n_0 \approx 10^3$-10^4. To estimate the occupation numbers n_k, we shall use Eqs. (5), (7), and (9). As a result, we obtain

$$n_k \approx \frac{2\pi^2 n_0}{\hbar N} \frac{v_k}{k^2 a^3 \gamma_k}. \tag{10}$$

For antiferromagnets with an easy-plane anisotropy, we obtain $(n_0/\hbar N) \approx \psi^2$, where ψ is the angle of precession of the antiferromagnetic vector L [$\psi \approx (\Delta L_x/L)$, where x is the direction of **H**]. Setting also $\omega_k^2 = \omega_0^2 + \alpha^2 k^2$, we find that

$$n_k \approx \frac{2\pi^2 \psi^2 \alpha^2}{k a^3 \omega_k \gamma_k}.$$

Under experimental conditions, the following relations were satisfied: $(ka)^3 \approx 10^{-6}$, $(\omega_k/\gamma_k) > 10^3$, and $\psi^2 \lesssim 5 \cdot 10^{-7}$. Therefore, the situation when $n_k \gg 1$ ($n_k \approx 10^3$) can be easily achieved. It was reported in Ref. 1 that n_k was a factor of 20 greater than the background of thermal spin waves. For a more detailed comparison with the experimental results, the actual form of the function $g_{kk'}$ depending on the type of impurity is required.

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