First Bifurcations in Circular Couette Flow: Laboratory and Numerical Experiments

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S.N.LUKASCHUK, V.S.L'VOV, A.A.PREDTECHENSKY, A.I.CHERNYKH

Institute of Automation and Electrometry,
Siberian Division of the USSR Academy of Sciences,
630090 Novosibirsk, USSR

Summary

The instabilities which precede the transition to chaotic flow in long Couette apparatus are investigated. The experimental data from three space points are analysed in terms of qualitative theory of differential equations. Within the experimental resolution the attractors were found which can be represented as a direct product of "fast" one-dimensional torus corresponding to azimuthal waves and "slow" two-dimensional torus corresponding to small interaction between the adjacent waves. Phase trajectories on the observed slow tori had rational values (13/31 and 3/7) of rotation number. The existence of these attractors has been predicted by simulation within the frame of phenomenological model for weakly interacting azimuthal waves.

In this report we present some results of study of laminar-turbulent transition in circular Couette flow with the outer cylinder at rest.

Experiment

Our experimental cell has radius ratio 1.57 (diameter of the inner cylinder Di=35 mm) and height-to-gap ratio 28. The Reynolds number (Re=W*Di*(Do-Di)/4V where W is an angular speed of inner cylinder, V is the viscosity of fluid, Do is the diameter of outer cylinder) was kept constant to within 0.015% during a few hours required for the flow to turn to steady state and for data acquisition. We have measured local hydrodynamic friction on the surface of outer cylinder by means of ion-current technique and then recorded files of experimental data in computer for later analysis.

With the increase in Reynolds number, the flow develops through the following dynamical regimes.

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bifurcation from a fixed point to a limit cycle (or a one-dimensional torus T1).

4. At R_3=1.02 the limit cycle looses its stability. According to Arnold's classification [3] the result of this bifurcation depends on where the eigenvalues L of Poincaré map linearisation intersect the unit circle at $R=R_3$. It may be simple splitting of limit cycle when L = 1, or Feigenbaum's cascade of doublings when L = -1, or origin of two-dimensional torus if L is a complex conjugated pair. In our case two-periodic motion originates resulting in slow (F2 = 0.009 Hz) modulation of traveling waves. The consideration of quasi-periodic case meets with additional difficulties connected with the possible origin of resonant trajectories on torus. Although the measure of resonant cases in parametrical space tends to zero at infinitely small supercriticality, at a finite supercriticality the resonances and associated heteroclinic processes can play a decisive role [4] . Turning to our experimental situation, it should be noted that a question about resonant properties of this torus looses its meaning due to a great ratio of fundamental frequencies: F1/F2=200. To simplify the study of the next bifurcations we have obtained the envelopes of main frequency of signals from the three probes. This procedure corresponds to the averaging of Poincaré maps along the fast limit cycle and gives new, "slow" variables. Such a transformation reduces our two-dimensional torus to a cycle with the frequency F2=0.009 Hz (Fig. 2). At R_4=1.027 this cycle looses its stability. During the time interval of the order of 1000s each signal taken separately seems to be chaotic. But the projection of phase trajectory on the plane in the space of measuring variables plotted during more than 10 000 s develops singularities which are typical of torus projection onto a plane (Fig. 3a). In particular, rotating this plane in three-dimensional space we may choose proper coordinates in which this attractor has a hole.

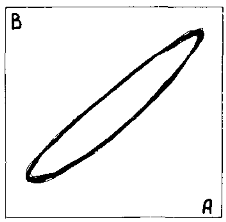


Fig.2. Limit cycle in "slow" variables

1. As it was shown before [1] , at Re=75 the Taylor vortices fill the gap from the end of annulus. Since the apparatus has fixed end plates, this process is not a bifurcation in the strict sense [2]. The primary flow contains 14 pairs of vortices. In order to investigate time-dependent regimes a column of 100 electro-chemical probes was used but only signals from the three probes placed near the outward jets of neighbouring pairs of vortices were analysed.

2. At Re_1=1030 the first bifurcation occurs developing the traveling azimuthal waves which produce the fast oscillations (F1=1.6 Hz) of hydrodynamic variables in laboratory frame of reference. The waves arise in all Taylor vortices simultaneously and their magnitude grows gradually in accordance with the Landau law.

3. The second instability at $R_2 = Re_2/Re_1 = 1.014$ leads to bifurcation of Reynolds-number dependence of wave amplitudes. Figure 1 presents the results of some experimental runs in neighbourhood of R_2.
This effect may be associated with existence of two stable space structures of different reflection symmetry about midplane. At $R<R_2$ the amplitude distribution is symmetrical relative to midplane but at $R>R_2$ the distribution has a strong antisymmetrical component.

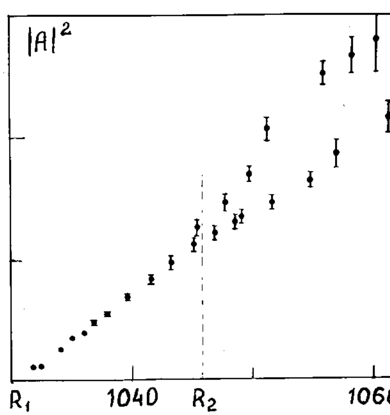
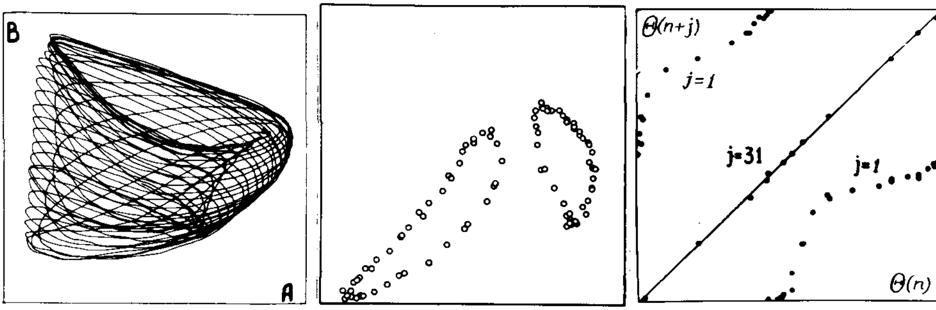


Fig.1. Bifurcation of wave amplitude dependence vs. Re

Further analysis of three-dimensional experimental information is based on the statement that any set of physically measured quantities gives a projection of hydrodynamical phase space onto the space of our measuring variables. This projection for experimental case is generic and therefore saves all properties of trajectories of real hydrodynamical flow. In terms of phase variables the first bifurcation corresponds to the Hopf



(a) (b) (c)

Fig.3. Projection (a), Poincaré section (b) and angle map (c) for torus with rotation number 13/31

Poincaré section of this attractor is shown in Fig. 3b that gives an evidence that the motion in slow variables occupies a surface of 2-torus. Consequently, with account of fast motion, the flow under study has an attractor on 3-torus. If we construct the period-one angle map $\theta(n+1)=f(\theta(n))$ (Fig. 3c), it becomes clear that the mapping of torus meridian into itself is a single-valued invertible function. Any integer power of this function exhibits the same character but there is a surprising feature that all experimental points occupy a diagonal of square if this power is equal to 31. It means that the phase trajectory is locked after 31 circuits of the slow 2-torus and repeats all its intricate bends. Consequently, the observed torus is a resonant one. At that time the meridian of torus is circuited 13 times yielding the rotation number 13/31.

Upon further increases of Re, the attractor in slow variables persists its toroidal form but the rotation number is varied passing into the region of stronger resonances. At $R \sim 1.030$ we have observed the resonance 3/7 (Fig. 4a) that differs from the previous value only by 2.5%. In figures 4b and 4c the data or Poincaré section, angle map and its seventh power are presented for this case. The finite thickness of trajectory rope is caused mainly by the transitional effect. The section of the rope has a form of line with a small curvature and its relative transverse size varies from 3 to 10 for different sections along the rope. If R exceeds 1.035, the resonant torus is destroyed.

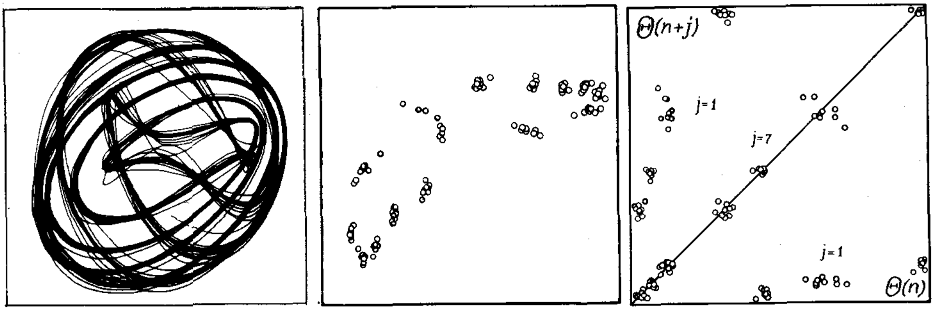


Fig.4. Torus with rotation number 3/7

Comparison with Theory

In our previous paper [1] the model of weakly interacted azimuthal waves was proposed to describe the complicated rearrangements of circular Couette flow during the transition to turbulence. In this model the pulsational component of velocity field has the form:

$$V_n(r,t) = \mathrm{Re}\{A_n(t)V(r)\exp(i\omega t)\}, \tag{1}$$

and the complex "slow" amplitudes A_n are governed by the following system of ordinary differential equations:

$$dA_n/dt = gA_n + (iT-q)|A|^2 \cdot A_n + (a+ib)(A_{n+1} + A_{n-1} - 2A_n)/4, \tag{2}$$

$$n = 1,\dots,N; \qquad A_0 = A_{N+1} = 0.$$

Here N is the number of vortex pairs, Q=T/q, B=b/a, G=g/a are some phenomenological coefficients. Only G has significant dependence on supercriticity $Re - Re_1$ but the others can be treated as constants in actual range of Re numbers. Under a proper choice of these coefficients (B=1.25, Q=10) the equation (2) gives a correct qualitative description of the observed sequence of bifurcations. Moreover, the bifurcation of amplitude distribution symmetry of azimuthal waves was founded firstly in computer simulation at G = 0.02 and only more attentive experiments showed a reality of this phenomenon. The existence of two-dimensional torus (which appears at G=0.025) also was predicted at first by model simulation. We also have verified the main model supposition that wavy modes in Taylor vortices are grouped into pairs with weak interaction between themselves and have founded that wavy motion in vortex pair is really strongly correlated when $R < 1.05$. At larger R the higher nonlinear terms must be taken into account in the model. These are only the arguments for the proposed model (2) but not a direct demonstration of its validity. The more reliable verification of this model is to restore its coefficients from experimental data by statistical method. This is the purpose of our future work.

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