Drag Reduction by Microbubbles in Turbulent Flows: The Limit of Minute Bubbles

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Drag reduction by microbubbles is a promising engineering method for improving ship performance. A fundamental theory of the phenomenon is lacking, however, making actual design quite haphazard. We offer here a theory of drag reduction by microbubbles in the limit of very small bubbles, when the effect of the bubbles is mainly to normalize the density and the viscosity of the carrier fluid. The theory culminates with a prediction of the degree of drag reduction given the concentration profile of the bubbles. Comparisons with experiments are discussed and the road ahead is sketched.

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The idea of reducing drag friction by placing a thin layer of air between a ship and its water boundary was patented already in the 19th century [1]. Drag reduction by the injection of microbubbles into the turbulent boundary layer has been the subject of intensive research since the first experimental observation of this phenomenon in [2]; see the comprehensive review [3]. The reduction of skin-friction drag by microbubbles has important technological and engineering advantages, especially for marine transportation by huge and relatively slow ships like tankers, but also for many other applications, such as hydrofoils, in-pipe transportation, etc. The voluminous literature on the engineering aspects of the problem cannot be referenced in full. It suffices to mention impressive results such as the microbubble drag reduction by about 80% on a flat plane [4] and up to 32% on a 50 m long flat plane ship [5]. Some steps in understanding the phenomenon have been made. The authors of Ref. [6] found that the drag reduction correlates with the maximum void fraction in the boundary layer. It was understood that the "local distribution and shape [of the microbubble void fraction $C(\boldsymbol{r})$] in the boundary layer have paramount influence in the drag reduction" [7]. Many researchers (see, e.g., [8]) found that the effect of microbubbles decreases downstream and that the bubble size is another important factor influencing frictional resistance.

Legner [9] stated that the "decrease of the medium density as the gas concentration increases provides the primary drag reduction mechanism." Unfortunately, the analysis of Ref. [9] does not contain any spatial dependencies, taking the distribution of bubble void fraction to be homogeneous. In addition, Legner [9] concluded that the increase of the dynamic fluid viscosity, caused by the bubbles, leads to an *increase of frictional drag*. In contradiction, other studies (see, e.g., Ref. [10]) lead to the opposite conclusion that *the increase* of the viscosity, caused by microbubbles, *decreases* the friction drag. To date this confusion has not been resolved theoretically.

The aim of this Letter is to offer a theory of drag reduction by microbubbles in the limit that their diameter d is small enough to neglect the gravity force ($\propto d^3$) compared to the Stokes force ($\propto d$), and is small compared to the Kolmogorov viscous scale η ($d/\eta < 1$). The void fraction $C(\boldsymbol{r})$ is fixed and not too large [$C(\boldsymbol{r}) \leq 0.05$)]. In addition, we assume that the scale of variation $\ell_C \equiv C(\boldsymbol{r})/|\boldsymbol{\nabla} C(\boldsymbol{r})| \ll z$ where z is the distance from the wall. In this limit we can demonstrate explicitly a mechanism for drag reduction which stems from the decrease of the fluid density and the increase in the fluid viscosity. This is not to say that there are no additional possible mechanisms of drag reduction by larger bubbles due to their influence on the structure of turbulence, including near wall coherent structures [11–13]. The theoretical description of such effects is, however, very difficult; they stem entirely from finite bubble-size effects, and they should be taken only as a further step in the development of the theory.

As a starting point for the theoretical development, we take the two-fluid description of turbulent flows with bubbles which is presented in Ref. [14]. In this description the bubbles are of diameter d which is very small. We do not consider individual bubbles, but rather describe them by a field of void fraction $C(\boldsymbol{r}, t) \ll 1$ and velocity $\boldsymbol{w}(\boldsymbol{r}, t)$. The carrier fluid has density ρ_0, viscosity μ_0, and velocity $\boldsymbol{U}(\boldsymbol{r}, t)$. We take the air density of the bubbles to be zero and the acceleration due to gravity, $\boldsymbol{g}$, to act in the $\hat{z}$ direction which is normal to the wall. Disregarding terms of the order of d^2 one writes the equation of motion

$$(1 - C)\rho_0 \frac{D\boldsymbol{U}}{Dt} = \frac{18C\mu_0}{d^2}(\boldsymbol{w} - \boldsymbol{U}) - (1 - C)\boldsymbol{\nabla} p + (1 - C)\rho_0\boldsymbol{g} + 2(1 - C)\boldsymbol{\nabla} \cdot (\mu \boldsymbol{E}_m) - \frac{3}{4}\mu_0 C\nabla^2\boldsymbol{U}, \quad (1)$$

$$2\mu_0\boldsymbol{\nabla} \cdot \boldsymbol{E}_f - \boldsymbol{\nabla} p - \frac{18\mu_0}{d^2}(\boldsymbol{w} - \boldsymbol{U}) + \frac{3}{4}\mu_0\nabla^2\boldsymbol{U} = 0. \quad (2)$$

In these equations

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{U} \cdot \nabla, \qquad \boldsymbol{E}_m \equiv \tfrac{1}{2}[(\boldsymbol{\nabla U}_m) + (\boldsymbol{\nabla U}_m)^T],$$

$$\boldsymbol{U}m \equiv (1 - C)\boldsymbol{U} + C\boldsymbol{w}, \qquad \boldsymbol{E}_f \equiv \tfrac{1}{2}[(\boldsymbol{\nabla U}) + (\boldsymbol{\nabla U})^T].$$

The effective viscosity which appears in these equations is determined by the bubble concentration [15],

$$\mu \equiv (1 + 5C/2)\mu_0. \tag{3}$$

These equation should be supplemented with

$$\partial(1 - C)/\partial t + \nabla \cdot [(1 - C)\boldsymbol{U}] = 0, \tag{4}$$

$$\partial C/\partial t + \nabla \cdot (C\boldsymbol{w}) = 0. \tag{5}$$

We now simplify the equations further in the limit $d \to 0$ by evaluating the term proportional to d^{-2} in Eq. (1) using the same term in Eq. (2). We find

$$(1 - C)\rho_0 D\boldsymbol{U}/Dt = (1 - C)[-\nabla p + \rho_0 \boldsymbol{g} + 2\nabla \cdot (\mu \boldsymbol{E}_m)] + C(-\nabla p + 2\mu_0 \nabla \cdot \boldsymbol{E}_f). \tag{6}$$

In the same limit $\boldsymbol{U}_m = \boldsymbol{U}$ and $\boldsymbol{E}_m = \boldsymbol{E}_f \equiv \boldsymbol{E}$.

After some further simplifications in which we retain only terms linear in $C(\boldsymbol{r})$, one gets (see, for example, [12])

$$\rho D\boldsymbol{U}/Dt = -\nabla p + \rho \boldsymbol{g} + 2\nabla \cdot (\mu \boldsymbol{E}), \tag{7}$$

$$\nabla \cdot \boldsymbol{U} = 0, \qquad DC/Dt = 0, \tag{8}$$

where the effective density of the suspension is

$$\rho \equiv (1 - C)\rho_0. \tag{9}$$

The important conclusion is that dilute ($C \ll 1$) solutions of minute microbubbles ($d \to 0$) can be described by a one-fluid model with modified density ρ and viscosity μ. This also implies that no clustering of bubbles is taken into account. Such clustering is expected to occur only when the diameter of the bubbles exceeds a critical length of the order of Kolmogorov scale [16]. Note that, consistent with this, the velocity field remains incompressible; this result is valid for minute microbubbles $d \to 0$ for arbitrary concentrations C. Having these results at hand we are poised to offer a theory of drag reduction that is quite similar to the theory by the same authors for drag reduction by flexible polymers [17].

Consider a flow in channel geometry (with half channel width L); the mean flow is in the x direction, the wall normal direction is z, and the spanwise direction is y. We take the bubble concentration $C(\boldsymbol{r})$ to be given and time independent. The fluid velocity $\boldsymbol{U}(\boldsymbol{r})$ is a sum of its average (over time) and a fluctuating part:

$$\boldsymbol{U}(\boldsymbol{r}, t) = \boldsymbol{V}(z) + \boldsymbol{u}(\boldsymbol{r}, t), \qquad \boldsymbol{V}(z) \equiv \langle \boldsymbol{U}(\boldsymbol{r}, t)\rangle. \tag{10}$$

For channel flows all the averages and, in particular, $\boldsymbol{V}(z) \Rightarrow V(z)$ are functions of z only. The objects that enter the theory are the mean shear $S(z)$, the Reynolds stress $W(z)$, and the kinetic energy $K(z)$; these are defined, respectively, as

$$S(z) \equiv \frac{dV(z)}{dz}, \quad W(z) \equiv -\rho(z)\langle u_x u_z\rangle, \quad K(z) = \frac{\rho(z)}{2}\langle |\boldsymbol{u}|^2\rangle.$$

Under the assumption $\ell_c \ll y$ we derive a pointwise balance equation for the flux of mechanical momentum, relating these objects [17]. Near the wall (for $z \ll L$) it reads

$$\mu(z)S(z) + W(z) = p'L. \tag{11}$$

On the right-hand side of this equation we see the production of momentum due to the pressure gradient; on the left-hand side (LHS) we have the Reynolds stress and the viscous contribution to the momentum flux, with the latter usually being negligible (in Newtonian turbulence $\mu = \mu_0$) everywhere except in the viscous boundary layer.

A second relation between $S(z)$, $W(z)$, and $K(z)$ is obtained from the energy balance. The energy is created by the large scale motions at a rate of $W(z)S(z)$. It is cascaded down the scales by a flux of energy, and is finally dissipated at a rate ϵ, where $\epsilon = \mu(z)\langle |\nabla u|^2\rangle$. We cannot calculate ϵ exactly, but we can estimate it rather well at a point z away from the wall. When viscous effects are dominant, this term is estimated as $[\mu(z)/\rho(z)] \times (a/z)^2 K(z)$ (the velocity is then rather smooth; the gradient exists and can be estimated by the typical velocity at z over the distance from the wall). Here a is a constant of the order of unity. When the Reynolds number is large, the viscous dissipation is the same as the turbulent energy flux down the scales, which can be estimated as $K(z)/\tau(z)$ where $\tau(z)$ is the typical eddy turnover time at z. The latter is estimated as $\sqrt{\rho(z)}z/b\sqrt{K(z)}$ where b is another constant of the order of unity. We can thus write the energy balance equation at point z as

$$\left[\frac{\mu(z)}{\rho(z)}\left(\frac{a}{z}\right)^2 + \frac{b\sqrt{K(z)}}{\sqrt{\rho(z)}z}\right]K(z) = W(z)S(z), \tag{12}$$

where the bigger of the two terms on the LHS should prevail. We note that contrary to Eq. (11) which is exact, Eq. (12) is not exact. It was shown, however, to give excellent order of magnitude estimates as far as drag reduction is concerned [17,18]. Finally, we quote the experimental fact [19,20] that outside the viscous boundary layer

$$W(z) = c^2 K(z), \tag{13}$$

with a coefficient $c \leq 1$ [17].

We can change variables now in favor of wall units according to

$$S^+ \equiv [\mu(z)/p'L]S, \qquad z^+ \equiv z\sqrt{\rho(z)p'L}/\mu(z), \qquad W^+ \equiv W/p'L, \qquad K^+ \equiv K/p'L. \tag{14}$$

In these units our balance equations read

$$S^+ + W^+ = 1, \qquad K^+ = c^2 W^+, \tag{15}$$

$$\left[\left(\frac{a}{z^+}\right)^2 + \frac{b}{z^+}\sqrt{K^+}\right]K^+ = W^+ S^+. \tag{16}$$

This set of equations is readily solved, giving

$$S^+(z^+) = 1, \qquad \text{for } z^+ \le z_v^+, \tag{17}$$

$$S^+(z^+) = \frac{2\kappa^2(zv^+)^2 - 1 + \sqrt{4\kappa^2[(z^+)^2 - (zv^+)^2] + 1}}{2\kappa^2(z^+)^2}, \qquad \text{for } z^+ \ge z_v^+. \tag{18}$$

In these equations we defined $\kappa \equiv c^3/b$, $z_v^+ \equiv a/c$. The mean velocity at point z can be obtained by integrating,

$$V(z) = \int_0^z S(z')dz' = \int_0^z \frac{p'L}{\mu(z')} S^+(z^+(z'))dz'. \tag{19}$$

A measure of drag reduction is the relative increase in the mean center-line velocity in the bubbly flow with respect to the neat Newtonian fluid:

$$\Delta V \equiv V_{\rm bub}(L) - V_{\rm N}(L). \tag{20}$$

Clearly, $\Delta V > 0$ corresponds to the drag *reduction,* while $\Delta V < 0$ corresponds to the drag *enhancement.* We obtain an expression for ΔV from Eq. (19) by expanding to linear order in $C(z)$ (where our equations are valid anyway):

$$\Delta V^+ \equiv \Delta V\sqrt{\rho_0/p'L} = \int_0^\infty \chi(z^+)C(z^+)dz^+. \tag{21}$$

Here the response function χ consist of two parts, one due to the density variation χ_ρ and the other due to the viscosity variation χ_μ:

$$\chi(z^+) = \chi_\rho(z^+) + \chi_\mu(z^+), \tag{22}$$

$$\chi_\rho(z^+) = -z^+\partial S^+(z^+)/2\partial z^+, \tag{23}$$

$$\chi_\mu(z^+) = -5\partial[S^+(z^+)z^+]/2\partial z^+. \tag{24}$$

In writing Eq. (21) we have used the fact that in experiments the bubbles tend to be localized in a finite region near the wall (see, for example, [10]), i.e., $C(z) \to 0$ sufficiently fast as $z \to \infty$, and we extended the integration range to infinity.

Equations (21)–(24) are the main theoretical predictions of this Letter. To complement the theory we now present estimates of the numerical value of the expected drag reduction, and compare it with a relevant experiment. The simplest model takes the parameters in agreement with the classical von Kármán boundary layer theory, i.e., $\kappa \approx 0.436$ and $z_v^+ \approx 5.6$. Evaluating the response function χ with these parameters results in the findings presented in Fig. 1. We see that in the viscous layer, where Eq. (17) is relevant, $\chi_\rho \approx 0$ while χ_μ is negative. This means that having a bubble concentration in this region does not buy us drag reduction due to the density variation, but it leads to drag enhancement due to the viscosity increase. This is far from being surprising, since in this region the momentum flux is dominated by the viscous term μS. For a fixed momentum flux any increase in viscosity must decrease S and correspondingly lead to drag enhancement. The most efficient drag reduction can be obtained by placing the bubble concentration out of the viscous layer, but not too far from the wall, say, at $6 \le z^+ \le 30$. In this region both the decrease in density and the increase in viscosity lead to drag reduction. The momentum in this region is transported mainly by the Reynolds stress $-\rho\langle u_x u_z\rangle$. The effect of density reduction is absolutely clear: it leads to the reduction in momentum flux. For a given momentum generation $p'L$ this has to result in the increase of the mean momentum of the flow. More interesting and counterintuitive is the effect of increasing viscosity. In order to understand it, we remind the reader that for intermediate values of z^+ there is no well-developed turbulent cascade, and outer and inner scales of turbulence are of the same order of magnitude. Therefore the increase of viscosity reduces the turbulent energy, in contrast to fully developed turbulence where changes of viscosity simply modify the Kolmogorov scale without any effect on the turbulent energy, that is dominated by the outer scale. The decrease in turbulent energy here reduces the Reynolds stress; see Eq. (13). It is interesting to note that this effect of increasing viscosity is essentially the same as the mechanism for drag reduction in the case of elastic polymers [17,18]. For polymers, however, the increase in viscosity can be very significant and the linear approximation that is used here is not applicable.

In comparing with experiments we need to consider low bubble concentrations. An interesting experiment was reported in [10], where both $C(z)$ and $V^+(z^+)$ are shown. We note that this experiment deals with a developing boundary layer rather than a steady channel geometry, but near the wall the Reynolds number can be considered rather time independent. Digitizing the published profiles $C(z)$ and integrating them numerically against our function $\chi(z)$, we obtain results for ΔV^+, which appear in reasonable agreement with the data of [10], as long as $C(z)$ is small, $C \le 0.1$. For the three lowest values of bubble concentration, roughly estimated in section 2 of the experimental apparatus as 0.02, 0.05, and 0.08, we find ΔV^+ values of 0.4, 1.0, and 1.6 as compared with experimental values

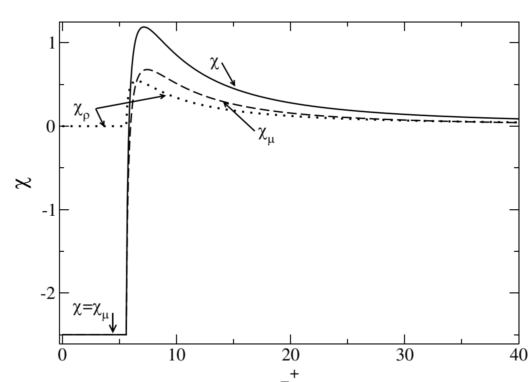

FIG. 1. Plots of the model response function χ and its contributions χ_ρ and χ_μ due to the density and viscosity variations. These functions are computed from our model shear function $S^+(z^+)$ as shown in Eqs. (17) and (18) plugged into Eqs. (22) and (23).

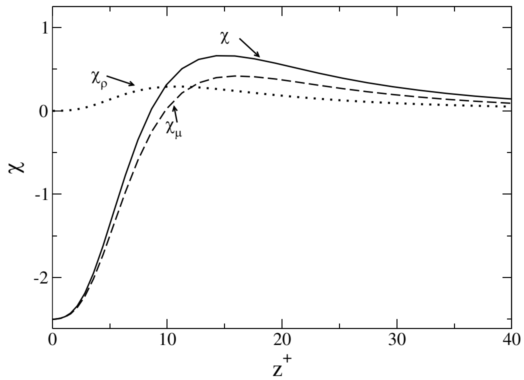

FIG. 2. Plots of the response function χ and its contributions χ_ρ and χ_μ due to the density and viscosity variations computed from the simulation data of [21].

of 0, 1.1, and 2.5 (with an accuracy of about 0.5). For higher values of $C(z)$ the results of the experiment become sensitive to nonlinear effects; in particular, ΔV^+ is no longer linear in C .

It appears extremely worthwhile to test the theory presented here by numerical simulations that would be designed to do so. We should stress that a careful measurement of $S^+(z^+)$ in either experiments or simulations, in addition to a determination of ΔV, can provide a very good test of our theory. Equation (21) is more general than our model (18), and it can be tested directly if $S^+(z^+)$ and its z^+ derivative are known. Since the response function χ is a property of the reference (Newtonian) flow, we can take it from Newtonian data. As an example of such a calculation we have considered the results of numerical simulations for a Newtonian channel flow available in [21], where the profile $S^+(z^+)$ is provided. We have used it to compute the response function χ and its two contributions according to Eqs. (22) and (23). The results are presented in Fig. 2. We see that the qualitative predictions of our model for χ are excellently reproduced by the numerical data, even though the smoother crossover between the viscous and logarithmic layers translates to smoother functions χ_ρ and χ_μ. A similar comparison for channel flow with bubbles will shed important additional light on our approach.

We reiterate that additional nonlinear contributions to drag reduction are expected to come in when the concentration increases, and especially when the bubble diameter d increases. In particular, for larger bubbles the gravity force becomes important. One should definitely examine theoretically the nonlinear and finite size effects and incorporate them into a more complete theory of drag reduction by microbubbles. It is the proposition of this Letter, however, that the limit $C(z) \ll 1$ and $d \to 0$ is a relevant limit where the theory simplifies considerably and where experiments, and especially numerical simulations, can give valuable support for the present theory. It is important to exhaust the linear effects of drag reduction by minute microbubbles before landing on the much more involved nonlinear theory.

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