Comment on "Superfluid Turbulence from Quantum Kelvin Wave to Classical Kolmogorov Cascades"

In [1], Yepez *et al.* report on a high-resolution numerical simulation of turbulence governed by the Gross-Pitaevskii equation (GPE). The results are generally very impressive, in particular, the obtained turbulent spectrum scalings are very convincing, and much cleaner than in any previous work. However, there are problems with interpreting these scalings.

First, the authors say that they observe the k^{-3} scaling at the scales less than the mean vortex core radius, and they call this range "Kelvin waves". But in all previous literature on quantum turbulence, including the papers cited by the authors, the name "Kelvin waves" was reserved for the motions of the quantized vortex lines with wavelengths greater than the vortex core size (and sometimes even greater than the mean distance between the vortex lines if the lines are organized in polarized bundles). Thus, to avoid confusion one should not attribute the k^{-3} scaling to the Kelvin wave turbulence.

On the other hand, from the very definition of the vortex core size (also called the healing length), it is clear that the linear terms in the GPE are greater than the nonlinear ones in the range of scales smaller than the mean vortex core radius (both inside and outside of the vortex core). This is a clear example of a weakly nonlinear wave system, and perhaps the weak turbulence theory should be employed to explain the scaling in this range [2–4].

Second, the authors say that they observe the $k^{-5/3}$ scaling at large scales, and they interpret it as Kolmogorov spectrum. But Kolmogorov theory is only relevant to incompressible (or nearly incompressible) turbulence, whereas, according to Figure 5 of the online supplement to this paper (and also the respective movie) the compressible energy is several times greater than the incompressible one. In classical fluids, such strongly compressible turbulence would be dominated by acoustic motions, shocks or random sound waves, and not vortices as in the Kolmogorov cascade. It therefore remains to be explained why a spectrum with an exponent close to the Kolmogorov $-5/3$ scaling is observed for the energy spectrum in the present work. There are two reference theories that might help to resolve this mystery. The first one, developed by Zakharov and Sagdeev (ZS) [5], deals with turbulence of random weakly nonlinear acoustic waves and it predicted a spectrum with exponent $-3/2$, a value very close to the Kolmogorov $-5/3$ scaling (the difference is only $1/6$). The second theory, developed by Kadomtsev and Petviashvill (KP) [6], considers a turbulence dominated by acoustic shocks, in which case the spectral exponent is -2. As we see, Kolmogorov $-5/3$ scaling is in between the ZS and the KP predictions for the acoustic dominated regimes, and its observation could possibly be due to coexistence of strong shocks and weak sound waves in the computed system. Clearly, much work remains to be done for finding the character of the turbulent motions (vortices vs shocks vs random sound waves), and one should refrain from premature interpretations until such work is done.

Victor L'vov[1] and Sergey Nazarenko[2]
[1]Department of Chemical Physics
The Weizmann Institute of Science
Rehovot 76100, Israel
[2]Mathematics Institute
Warwick University
Coventry, CV4 7AL, United Kingdom

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