RAMAN SCATTERING OF LIGHT AND THE FARADAY EFFECT IN MAGNETICALLY ORDERED DIELECTRICS

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Submitted December 12, 1966

We construct a phenomenological theory of Raman scattering of light in magnetically ordered dielectrics with two magnetic sublattices, where the scattering is caused both by a direct interaction of the magnetic field of the wave with the spin system and by an indirect interaction of the electric field of the wave with the spins through spin-orbit interaction. These scattering mechanisms are closely connected with the electrical and magnetic gyrotropy of the medium which is responsible for the Faraday effect. We show that taking into account the rotation of the polarization plane of the incident and the scattered radiation leads to a dependence of the differential extinction coefficient on the size and shape of the sample.

RAMAN scattering (RS) is caused by the interaction of the electromagnetic waves with the natural vibrations of the scattering object. In magnetically ordered crystals the degrees of freedom connected with the spin system are important and they lead to the appearance of additional lines in the spectrum of the scattered radiation. From a phenomenological point of view these lines of the spectrum are connected with the modulation of the optical characteristics of the medium by spin waves and therefore the frequencies of the incident and scattered light will differ by the magnitudes of the resonance frequencies of the spin system. The modulation of the permittivity and permeability tensors \( \varepsilon \) and \( \mu \) of the medium is caused by their dependence on the magnetizations \( \vec{M} \) of the sublattices of the crystal.

In the present paper we restrict our considerations to merely the linear terms in the expansion of \( \varepsilon \) and \( \mu \) in powers of \( \vec{M} \). In a transparent crystal only the off-diagonal, purely imaginary components of these tensors \( \varepsilon \) and \( \mu \) will be proportional to \( \vec{M} \); these lead respectively to the electrical and magnetic gyrotropy of the medium. The electrical gyrotropy \( (\varepsilon_{XY}, \varepsilon_{XZ}, \varepsilon_{YZ}) \) arises from the spin-orbit interaction; here the angle of rotation of the polarization plane of the light over a distance equal to the wavelength 2\( \lambda \) turns out to be of the order of the ratio of the spin-orbit interaction \( \varepsilon_{SO} \) to the energy splitting of the levels in the crystal field \( \varepsilon_{cr} \), where \( \varepsilon_{SO}/\varepsilon_{cr} \approx 3 \times 10^{-5} \) to \( 3 \times 10^{-3} \). The magnetic gyrotropy of the medium is caused by the direct interaction of the magnetic field of the incident wave with the magnetization of the crystal; here the corresponding angle \( G_m \approx 2\gamma M/\omega \), where \( \gamma \approx e\hbar/mc \) is the gyromagnetic ratio and \( \omega \) the frequency of the incident light (in the order of \( 3 \times 10^{-5} \)).

The mechanism of the Raman scattering of light by a spin system considered in the present paper consists thus in the modulation of the off-diagonal components of the tensors \( \varepsilon \) and \( \mu \) by the thermal vibrations of the magnetization \( \vec{M} \) of the sublattices. We elucidate here the close connection between the Raman scattering of light and the "electrical" \( G_e \) and "magnetic" \( G_m \) Faraday effect. The extinction coefficient \( h_m \) of the "magnetic" Raman scattering (MRS) \( h_m \) is determined by the magnitude of the "magnetic" rotation of the polarization plane of the light:

\[
h_m \approx -\frac{\omega}{c} \left( \frac{a}{\lambda} \right)^3 \text{ch} \frac{h\Omega}{2kT},
\]

where \( \omega \) and \( \lambda \) are the frequency and wavelength of the incident light, \( a \) is the lattice constant (\( a \approx 5 \times 10^{-8} \) cm) and \( \Omega \) the frequency of the uniform precession of the magnetization. For the "electrical" Raman scattering (ERS) we have similarly

\[
h_e \approx -\frac{\omega}{c} \left( \frac{a}{\lambda} \right)^3 \text{ch} \frac{h\Omega}{2kT}.
\]

The scattering intensity characterizes the differential extinction coefficient \( dh \) which is equal to the ratio of the number of photons scattered into the solid angle \( d\theta \) per unit time and unit volume of the medium to the photon current density in the incident light: \( h = f(dh/d\theta) d\theta \).
We shall give estimates for the case of a ferro-
dielectric. Bearing in mind that $G \approx 3 \times 10^{-5}$ to
$3 \times 10^{-3}$ and putting $kT/|\mu| \approx 50$ we find in the optical
band of frequencies $\hbar \approx 10^{-8}$ to $10^{-12}$. This is
only a few orders of magnitude less than the Bril-
louin scattering in crystals and when lasers are
present it can certainly be observed.\(^{[5]}\)

When one considers the scattering of light one usually
does not take the gyrotropy of a crystal into
account, assuming it to be small. In our case the
very existence of the scattering is connected with
the gyrotropy, and neglect of the rotation of the pola-
ration plane of the incident and the scattered
light is admissible only in the case of a sufficiently
small sample so that the angle of rotation of the pola-
rization plane $\psi$ over its length is much less
than $\pi$. When the characteristic dimensions of the
 crystal are such that $\psi \approx \pi$, the angular characteris-
tics $f = h^{-1} d\psi / d\theta$ and the polarization proper-
ties of the scattered light depend on its shape and size.
From a physical point of view the occurrence of such "shape effects" is connected with the exist-
ence of a preferential direction in the crystal: the direc-
tion of the magnetization $\mathbf{S} = \mathbf{M}/M$. The prob-
ability for the scattering of light in a well-defined solid angle $d\theta$ by a small region of the crystal de-
PENDs on the angle between $\theta$ and the direc-
tion of the polarization of the light in it. This angle is dif-
ferrerent in different points of the sample and the an-
gular characteristics $f = h^{-1} d\psi / d\theta$ when linearly polarized light will scattered will depend on the size
and shape of the sample. This dependence can be
neglected for small ($\psi \ll \pi$) and large ($\psi \gg \pi$) sam-
ple (see Eq. (20d)). In the latter case $f$ does for
linearly polarized light not depend on the direction of the polarization and is the same as $f$ for natural light (Eqs. (16b, c)). For natural and circularly polarized light only the polarization properties of the scattered light will depend on the shape of the sample.

We must note the close analogy between mag-
netic RS and ERS. The position and shape of the MRS and ERS spectral lines turn out to be the
same. It may happen that in one frequency range $\omega \approx \omega_1$ the Faraday effect is caused by magnetic
gyrotropy, and in another range, when $\omega \approx \omega_2$, by the elec-
tric one. In that case MRS dominates for $\omega \approx \omega_1$ and ERS dominates for $\omega \approx \omega_2$. Also

$$h(\omega_1)/v_{S}G^2(\omega_1) \sim h(\omega_2)/v_{S}G^2(\omega_2),$$

and in ferrodielectrics these quantities are exactly
the same. Moreover, when natural or circularly polarized light is scattered $f(\omega_1) = f(\omega_2)$. When lin-
early polarized light is scattered some differences in the properties of MRS and ERS will occur.

In the present paper we consider RS in crystals
with two magnetic sublattices. This is connected
with the fact that the majority of the crystals which
are transparent in the visible region have just such
a structure (MnF$_2$, FeF$_2$, RbNiF$_3$, and so on\(^{[4]}\)).

In conclusion we emphasize that we limit our-
selves to the linear terms in the expansion of $\epsilon$ in
terms of the sublattice magnetizations. We thereby
disregard RS connected with the participation of
two magnons. In accordance with the conservation
laws magnons with any momentum up to the limiting
momentum can take part in such a process. A mi-
croscopic theory is therefore essentially necessary
to describe it which takes, for instance, the spin
wave dispersion into account. Two-magnon Raman
scattering can occur in first order perturbation
theory in the parameter $\epsilon_{ex}/\epsilon_{er}$ where $\epsilon_{ex}$ is
the energy splitting of the levels due to exchange inter-
action. This mechanism of two-magnon RS will not
contain additional small parameters as compared to
the one-magnon RS which is considered in the
present paper from a macroscopic point of view. A
microscopic analysis of two-magnon scattering will
be the subject of a separate communication.

1. THE WAVE EQUATION IN A GYROTROPIC MEDIUM

Landau and Lifshitz\(^{[5]}\) have shown that the ma-
croscopic Maxwell equations in a medium for optical
frequencies do not have the usual form

$$\text{rot} \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0, \quad \text{rot} \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{D}}{\partial t} = 0.$$

We shall therefore start directly from the Lorentz
equation for the microscopic electromagnetic field $\mathbf{e}$, $\mathbf{h}$. After averaging them we get

$$\text{rot} \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} = 0,$$  \hspace{1cm} (2a)

$$\text{rot} \mathbf{B} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c} (\rho v + j_\|). \hspace{1cm} (2b)$$

Here $\mathbf{E} = \mathbf{\hat{e}}$, $\mathbf{B} = \mathbf{\hat{b}}$, $\rho$ is in the case of dielectrics
the density of bound charges, and $v$ their velocity.
We have here on the right-hand side of (2b) added
the current $j_{\|}$ caused by the spin magnetization
$M_8$ of the ferroelectric.\(^{[2]}\) In the case of a small
spin-orbit interaction such a splitting-up of the
current in an orbital $\rho \mathbf{v}$ and a spin $j_{\|}$ part is un-
ambiguous for frequencies for which the macro-
scopic Maxwell equations make sense, and then\(^{[6]}\)

\(^{2}\)For the sake of simplicity we first of all consider a ferro-
dielectric. The final results will be equally valid also for crys-
tals with two magnetic sublattices.
We must add to the Maxwell equations (2) the equation of motion of the magnetic moment. Bearing in mind that $\Omega \ll \omega$ and $M \approx M_s$ we have for vibrations of frequency $\omega$

$$\frac{\partial M}{\partial t} = \gamma [MH(\omega)] + \gamma [\mathcal{M}(\Omega)H^x(\omega^x)],$$

where $\gamma$ is the gyromagnetic ratio. From (2)–(8) we get the Maxwell equations for frequency $\omega$. Eliminating $H$ from them we get a wave equation for $\mathcal{D}$:

$$\nabla \mathcal{D} + k_d^2 \mathcal{D} = 2i(\mathbf{G} \nabla) \mathbf{rot} \mathcal{D} - 4\pi \{ \mathbf{rot} \mathbf{[AD^x]} + \mathbf{rot} \mathbf{[CrotD^x]} \},$$

where $k_d^2 = \epsilon(\omega/c)^2$, $A(Q) = \frac{g}{\omega} \mathbf{M}(Q)$, $C(Q) = \frac{y}{\omega} \mathbf{M}(Q)$.

In a dielectric with two sublattices $^1M$ and $^2M$ the final Eq. (9) has the same form. Here

$$\omega G = -2\pi \omega^{-1}(g + \gamma)\mathbf{M}$$

and $q$ are real even functions of the frequency $\omega$.

2. THE FARADAY EFFECT

When there is no external electromagnetic field Eq. (9) becomes homogeneous and describes the propagation of plane waves $\mathbf{D} \sim e^{iK \cdot \mathbf{r}}$ in the medium. It has a non-trivial solution for two values of $k$:

$$k^\pm = k_0 [1 \mp 2aG]^{-1/2} = k_0 [1 \pm aG], \ a = \frac{k}{k_0},$$

corresponding to the propagation of waves which are right-hand and left-hand circularly polarized. The rotation of the plane of polarization per unit length of the sample is $k_0 \alpha \mathbf{G}$ and the dimensionless parameter $2\gamma^\pm \ll 1$ is the angle of rotation of the polarization plane for $\mathbf{G} \parallel \alpha$ over a distance equal to the wavelength.

In antiferromagnetics which are odd in Turov's terminology we have $\mathbf{L} \rightarrow -\mathbf{L}$ under an inversion $\mathbf{M} \rightarrow -\mathbf{M}$. Then $g$ is a tensor and $q$ a pseudo-tensor of second rank $(\mathbf{q} = 0)$. Moreover, $\gamma_1 = \gamma_2$, i.e., $\gamma^\pm = 0$ and $\mathbf{M} = 0$ without an external magnetic field. Hence in pure antiferromagnetics $\mathbf{G} = 0$.
(see (10a)) and there is no Faraday effect without an external magnetic field. In even antiferromagnetics under an inversion M → M and L → L and weak ferromagnetism, q ≠ 0, is possible and there is a Faraday effect caused both by ferromagnetic and by antiferromagnetic moments.

3. THE GREEN FUNCTION OF THE WAVE EQUATION IN A GYROTROPIC MEDIUM

We shall solve the inhomogeneous Eq. (9) by a Green function method. To do this we consider first the corresponding equation with point sources:

\[ \nabla^2 D + k_0^2 D + 2i(GV) \nabla D = -4\pi \delta(r-r'), \quad |n| = 1, \]

in the k-representation it becomes algebraic and can easily be solved. Taking the inverse Fourier transformation\(^3\) we get the Green function of the d'Alembert equation:

\[ D(r-r') = \exp{ik_0 l(r-r')} \]

The expressions for \( K_x \) and \( K_y \) which are of interest to us can be obtained from (12b):

\[ K_x = \mathcal{N}^+ \exp{(ik_0 + r)} + \mathcal{N}^- \exp{(ik_0 - r)}, \]

\[ -iK_y = \mathcal{N}^+ \exp{(ik_0 + r)} - \mathcal{N}^- \exp{(ik_0 - r)}. \]

\( \mathcal{N}^\pm \) has a very simple form if the incident light is circularly polarized:

\[ (D^\pm) = D^\pm \exp{(ik_0 \pm r)} \]

In contrast to (13) and (14) here the z-axis is \( \pm \beta \), the x-axis \( \pm (\alpha \times \beta) \), and \( k_0 = k_0(1 + \beta \cdot G) \). Then

\[ 2\mathcal{N}^+(D^z) = \int d^3r \exp{(ik_0 \pm r)} - ik_0a + \exp{(ia \pm \beta)}, \]

\[ 2\mathcal{N}^-(D^z) = \int d^3r \exp{(ik_0 \pm r)} - ik_0a - \exp{(ia \pm \beta)}. \]

From this we easily obtain expressions for the correlators we need. E.g.,

\[ 4 \langle \mathcal{N}^+ (D^z) \mathcal{N}^+ (D^z) \rangle_{|a, q} = V (D^z)^2 \langle T^z S^z \rangle_{|a, \omega, q}, \]

\[ \times \int d^3r \exp{(2ik_0(aG) \cdot \alpha)}. \]

Here \( V \) is the volume of the scattering medium, the integral is taken over that volume, \( q = k_0(\alpha - \beta) \), and

\[ \langle S^z S^z \rangle_{|a, q} = \int d^3r \langle S^z S^z \rangle_{|a} \exp{(iqr)}. \]

For the sake of simplicity we shall assume in the following that one kind of scattering, e.g., magnetic scattering \( (A \neq 0, C = 0) \) dominates.

The differential extinction coefficient \( dh/d\theta \) is by definition equal to

\[ \frac{dh(\omega)}{d\theta} = \frac{\varepsilon^2 R^2 S \rho I(\omega)}{V |D^z|^2}. \]

If the incident light is circularly polarized \( D^\pm \neq 0 \), then
\[
\frac{dh^z(\omega)}{d\theta} = \frac{e^2\omega^4}{4\varepsilon^4} \left\{ \sum_{j=1}^{3} (A_j A_{\bar{j}})_{a, q} \pm 2 \text{Im} (A_3 A_{\bar{3}})_{a, q} \right\}, \tag{16a}
\]

Here \( \omega = \omega^\text{ex} + \Omega \) and

\[ A_1 = A [a\beta], \quad A_2 = A\beta, \quad A_3 = Aa. \]

In natural light right-hand and left-hand polarizer-occurrence with equal probability. Therefore

\[
\frac{dh(\omega)}{d\theta} = \frac{1}{2} \left( \frac{dh^+}{d\theta} + \frac{dh^-}{d\theta} \right) = \frac{e^2\omega^4}{4\varepsilon^4} \sum_{j=1}^{3} (A_j A_{\bar{j}})_{a, q}, \tag{16b}
\]

Light which is linearly polarized with an angle \( \varphi \) to the direction of \([a\beta]\) at the origin is a superposition of left-hand and right-hand polarized waves, and \( D^\pm = D e^{\pm i\varphi}. \) Using (15) and (16a) we get for linearly polarized light

\[
\frac{dh_\pm(\omega)}{d\theta} = \frac{e^2\omega^4}{4\varepsilon^4} \sum_{j=1}^{3} (A_j A_{\bar{j}})_{a, q} \pm \frac{2}{V} \left( (A_1 A_{\bar{1}})_{a, q} + (A_2 A_{\bar{2}})_{a, q} + (A_3 A_{\bar{3}})_{a, q} \right). \tag{16c}
\]

The extinction coefficient for the scattering of linearly polarized light depends thus on the size and shape of the sample. When the polarization plane of the incident light undergoes many rotations along the length of the sample, one can neglect the last terms in (16c) and the extinction coefficients of linearly polarized (16c) and natural light (16b) are the same. By averaging (16c) over the angle \( \varphi \) we obtain again (16b). We obtain similar results in the case when not the magnetic but the electrical Raman scattering \( (A = 0), \) \( C \neq 0) \) dominates: to obtain \( dh^z/d\theta \) and \( dh^\varphi/d\theta \) we must in Eqs. (16) replace \( A \) by \( C \) and change the sign to its opposite in front of the integrals in (16c).

The extinction coefficients of MRS and ERS can in magnetically ordered dielectrics thus be expressed in terms of the correlators of the ferromagnetic and antiferromagnetic moments:

\[ \langle M_i M_{\bar{i}} \rangle_{a, q}, \quad \langle M_i L_{\bar{i}} \rangle_{a, q}, \quad \langle L_{\bar{i}} (L_{\bar{i}}) \rangle_{a, q}. \]

To evaluate them it is necessary to make well-defined assumptions about the kind of the magnetic structure of the ferrite.

5. CALCULATION OF THE CORRELATORS

Using the theory of non-thermodynamic fluctuations of several quantities one can show that

\[
\langle \delta M_i \delta M_{\bar{i}} \rangle_{a, q} = -\frac{i}{4\varepsilon} (\mp \chi^{a\bar{g}} (\Omega, q) - \chi^{a\bar{g}} (\Omega, q)) cth \frac{\hbar \Omega}{2T}. \tag{17}
\]

Here \( \chi^{a\bar{g}} (\Omega, q) \) is the magnetic susceptibility tensor of the \( a \)-sublattice in the case where the magnetic field with frequency \( \Omega \) and wavevector \( q \) acts only upon the \( b \)-sublattice. When there is no interaction between the sublattices, \( \chi^{a\bar{g}} = \delta^{a\bar{g}} \)

and we are led to an obvious result: there is no correlation between the sublattices.

We shall evaluate \( \chi^{a\bar{g}} \) in the case when there are no losses. We shall assume that \( aM \rightarrow aM + a m, \quad aM \rightarrow e^{q \cdot r - \Omega t} \)

\( a \) and the internal anisotropy fields \( H_A \) and \( A \). We shall take into account the static demagnetizing field. However, we shall take into account the demagnetizing field of the spin wave \( H_p \)

which is important for evaluating the correlators for frequencies comparable to \( 4\pi\gamma M. \)

The equations of motion have the form

\[ -i\Omega m = \gamma (aM + a m), \quad (aH + a h) \]

where

\[ H = H_0 + H_\alpha + \Omega I_\lambda + \lambda^2 M + \gamma m, \quad H = H + H_\alpha + \Omega I_\lambda + \lambda M \]

\[ H_p = -4\pi \left( q (m^2 + q^2 m) \right) \]

Here \( \alpha \) is a high-frequency field acting upon the \( a \)-sublattice. In the usual consideration of ferromagnetic resonance there is no necessity to separate \( h \) and \( h \) as we are interested in the quantity \( \chi^{a\bar{g}} \) determined by the equation

\[ \langle m_i + m_{\bar{i}} \rangle = \chi^{a\bar{g}} (h_0 + h). \]

However, we are interested in the susceptibility \( \chi^{a\bar{g}} : m^2 = \chi^{a\bar{g}} \beta h_k \) (there is here no summation over \( k \) or \( \beta \)). In the initial equations of motion we have not taken the energy losses into account and, of course, when solving them we get the Hermitian part of the tensor \( \chi^{a\bar{g}} \). We find its anti-Hermitian part from dispersion relations. Substituting this expression into (17) we get the correlators

\[ \langle M_i \beta M_{\bar{k}} \rangle \quad \text{through which in an obvious way we can express the correlators} \quad \langle M_i M_{\bar{k}} \rangle, \quad (M_i \bar{L}_{\bar{k}}), \quad \langle L_i \bar{L}_{\bar{k}} \rangle. \]

It is convenient to write down separately the final expressions for them for a ferromagnetic when we can neglect the external field \( H_0 \) and the anisotropy field \( H_A \) compared with the difference between the exchange fields for the sublattices and for an antiferromagnetic when it is impossible to do so.
The correlators for a ferromagnetic far from the compensation points have the form:

\[ \langle \mathcal{M}_n \mathcal{M}_m \rangle_{q \to q} = \frac{\pi \hbar v_{+}}{M} \left\{ \delta M \Omega_{h} \delta (\Omega^{2} - \Omega^{2}) \right\}, \]

where

\[ \Omega_{h} = \gamma (\Omega_{+}^{M} + \gamma L). \]

For an antiferromagnetic ("easy axis") type

\[ \langle \mathcal{M}_n \mathcal{M}_m \rangle_{q \to q} = \langle \mathcal{M}_n \mathcal{M}_m \rangle_{q \to q} = \frac{\pi \hbar v_{+}}{M} \left\{ \delta M \Omega_{h} \delta (\Omega^{2} - \Omega^{2}) \right\}, \]

where

\[ \Omega_{h} = \gamma (\Omega_{+}^{M} + \gamma L). \]

6. Angular Characteristics and Spectrum of Raman Scattering

We shall assume for the sake of simplicity that the ferromagnetic resonance frequency \( \Omega_{f} > 4 \pi v_{+} M \).

\[ \frac{\partial h}{\partial \Omega} = \left[ 1 - \cos \Omega \cos \beta \cos \gamma \right] \frac{2 \pi \hbar v_{+}}{c^{3}} \langle A^{2} \rangle_{|\Omega|, 0}. \]

The intensity of the Stokes and the anti-Stokes components can thus be essentially different (by a factor three when \( \cos \beta = 1 \)).

\[ \frac{\partial h}{\partial \Omega} = \left[ 1 - \cos \Omega \cos \beta \cos \gamma \right] \frac{2 \pi \hbar v_{+}}{c^{3}} \langle A^{2} \rangle_{|\Omega|, 0}. \]

In the case of scattering of circularly polarized light we have instead of (20b)

\[ h(\omega_{\pm} + \Omega) = \left[ 1 - \cos \beta \right] \frac{2 \pi \hbar v_{+}}{c^{3}} \langle A^{2} \rangle_{|\Omega|, 0}. \]

The intensity of the Stokes and the anti-Stokes components can thus be essentially different (by a factor three when \( \cos \beta = 1 \)).

\[ \frac{\partial h_{+}}{\partial \Omega} = \left[ 1 - \cos \Omega \cos \beta \cos \gamma \right] \frac{2 \pi \hbar v_{+}}{c^{3}} \langle A^{2} \rangle_{|\Omega|, 0}. \]

The intensity of the Stokes and the anti-Stokes components can thus be essentially different (by a factor three when \( \cos \beta = 1 \)).

\[ \frac{\partial h_{-}}{\partial \Omega} = \left[ 1 - \cos \Omega \cos \beta \cos \gamma \right] \frac{2 \pi \hbar v_{+}}{c^{3}} \langle A^{2} \rangle_{|\Omega|, 0}. \]
where

\[ J = \frac{1}{V} \int \cos \left[ 2k_0 \langle \mathbf{G} \cdot \mathbf{r} \rangle \right] d^3 \mathbf{r} \]

Here \( \varphi \) is the angle between the plane of polarization in the center of the sample and the direction \( [\mathbf{a} \times \mathbf{b}] \). For a spherical sample of radius \( R \)

\[ J = \frac{3}{4} \frac{\sin \psi}{\varphi - \cos \psi} \left[ \frac{1}{\varphi} - \cos \psi \right] \]

Here \( \psi = 2k_0 \langle \mathbf{G} \cdot \mathbf{r} \rangle \) is the angle of rotation of the polarization plane over a length equal to the diameter of the sample. When \( \psi \ll \pi \) this expression is equal to unity and vanishes for the first time when \( \psi = \pi/6 \).

For a parallelepiped with edges \( l_x, l_y, l_z \)

\[ J = \sin \psi_x \sin \psi_y \sin \psi_z \frac{\sin \psi_x \sin \psi_y \sin \psi_z}{\psi_x \psi_y \psi_z} \]

where \( \psi_x = k_0 \langle \mathbf{G} \cdot \mathbf{r} \rangle \) is the angle between the polarization planes of the light at opposite faces (along the direction \( l_x \)) of the parallelepiped.

We note that Eqs. (20a, c) refer equally well to ERS; in (20d) it is only necessary to change the sign in front of \( J \).

In conclusion we estimate the intensity of the one-magnon Raman scattering of light. From (10), (18a), and (19) we get for a ferromagnetic:

\[ \langle A^2 \rangle_{|q|, \phi} = \left( \frac{G}{2 \pi} \right)^2 \frac{n_{Hy} +}{2M} \sinh \frac{h \Omega}{2T} \delta(\Omega - \Omega_f) \]  \( \text{(21)} \)

Substituting this into (20b) and bearing in mind that \( \gamma_{Hy}/2M \approx a^3 \) where \( a \) is the lattice constant, we get easily the estimates (1a) and (1b).

From (18a) it is clear that an estimate of the quantity \( \langle A^2 \rangle_{|q|, \phi} \) for an exchange line of a ferromagnetic will contain an additional small parameter \( \gamma_{Hy}/2 \) \( \approx 10^{-3} \). In an antiferromagnetic an estimate of \( \langle A^2 \rangle_{|q|, \phi} \) also will contain the additional parameter \( \sqrt{H_A/\lambda L} \) which can be small:

\[ \sqrt{H_A/H_F} = \sqrt{H_A/\lambda L} \approx 0.5 \text{ to } 10^{-2} \] Similar estimates can also be made for ERS.

I express my deep gratitude to V. L. Gurevich for posing the problem and supervising my work. It is my pleasant duty to thank A. I. Ainsel'm, G. M. Nedlin, A. G. Gurevich, O. V. Konstantinov, and B. I. Shklovskii for fruitful discussions of this paper and useful advice.