# SPECTRUM EVOLUTION AT THE TRANSITION TO TURBULENCE IN A COUETTE FLOW

Z.B. KRUGLJAK, E.A. KUZNETSOV, V.S. LVOV, Yu.E. NESTERIKHIN,
A.A. PREDTECHENSKY, V.S. SOBOLEV, E.N. UTKIN and F.A. ZHURAVEL
*Institute of Automation and Electrometry, Siberian Branch of the Academy of Sciences, Novosibirsk, USSR*

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It is shown that the laminar–turbulent transition in a circular Couette flow combines characteristics of both Landau and stochastic attractor models. As the Reynolds number increases new modes are excited and the peaks of the spectrum are broadened smoothly.

*1. Introduction.* There exist two alternative viewpoints on the laminar turbulent transition problem. According to Landau's theory [1] this transition is represented as a sequence of bifurcations; each of them results in a new flow type with incommensurable frequency and consequently gives rise to a sharp peak in the power spectrum. With the increase of the number of peaks the spectrum practically does not differ from a continuous one. Thus, stochasticity of the motion is associated with excitation of a great number of degrees of freedom. The other point of view is connected with the idea of a stochastic "strange" attractor [2,3] explaining the stochastization of the motion of dynamic systems with a finite number of modes.

Here we present results on the transition to turbulence in a Couette flow. Unlike Gollub and Swinney [4], our picture of the onset of turbulence is shown to combine the features of both Landau and stochastic attractor models. As the Reynolds number increases new modes are excited and the peaks of the spectrum are broadened smoothly.

*2. Experimental technique.* The experimental installation consists of a hydrodynamic block, a laser Doppler velocimeter (LDV) and a computer system for data acquisition and processing. The hydrodynamic block incorporates two coaxial metal cylinders ($h$ = 33 mm, $d$ = 35 mm, $d$ = 55 mm) with radial beats of less than 5 $\mu$m. The fluid (water) temperature is kept constant to within 0.02°C. The stability of the rotation period is better than $10^{-4}$ in the range 0.1–10 s. The LDV worked by the reference beam direct-scatter method, its scattering volume being 600 × 100 × 100 $\mu m^3$. The device is used to measure the azimuthal velocity component $V_\varphi$. The computer system allows recording velocity counts in the computer memory and performing calculations of the power spectra $I_\omega$.

*3. Experimental results.* The Couette flow in our geometry loses its stability at Re = $Re_c$ = 76 ± 1. The number of Taylor vortices in our system (11–18) depends on the acceleration conditions [5]. The follow-

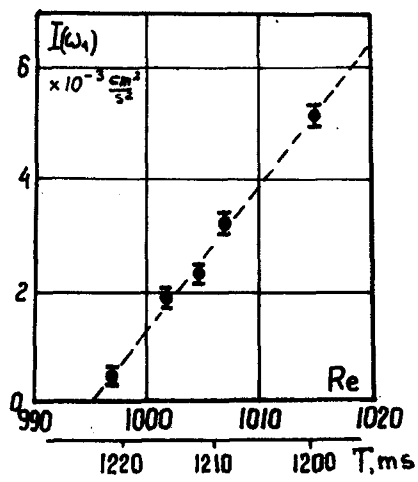


Fig. 1.

ing data correspond to a state with 15 vortex pairs arising at a very slow acceleration.

At Re -- $\text{Re}_1 = 995 \pm 1$ bent vortex oscillations arise, and a narrow peak at the frequency $\omega_1 = 1.93\omega_m$ ($\omega_m$ is the rotation frequency of the inner cylinder) is observed in the spectrum. Just above the critical volume, i.e. $\epsilon = (\text{Re} - \text{Re}_1)/\text{Re}_1$ small, the peak amplitude $A$ obeys the Landau law, i.e. $I = |A|^2 \sim \epsilon$ (fig. 1); the relative width of the spectral peak is 1.2 $\times$ $10^3$ at the $10^{-3}$ level. As Re is increased the second harmonic grows in importance (see fig. 2a). At $\epsilon$ = 0.025 a bifurcation occurs resulting in a slight peak

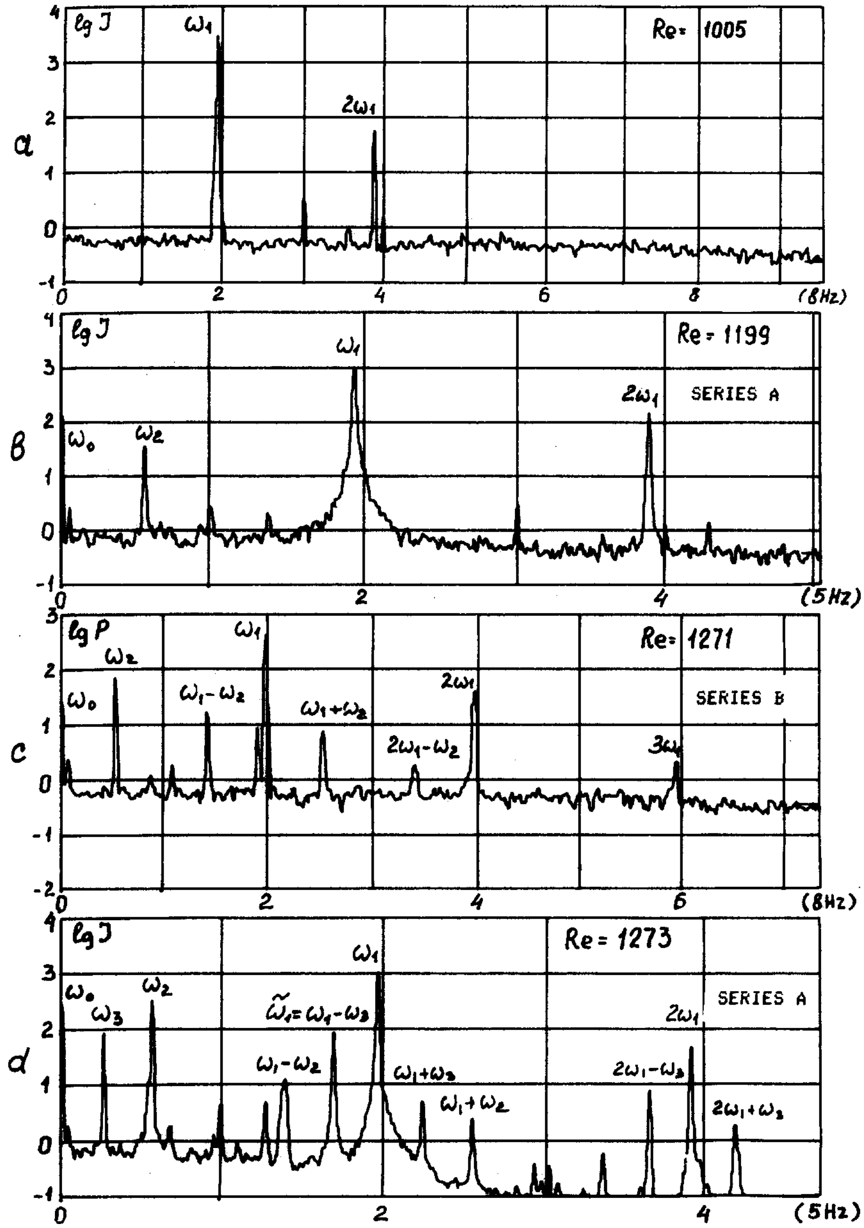


Fig. 2.

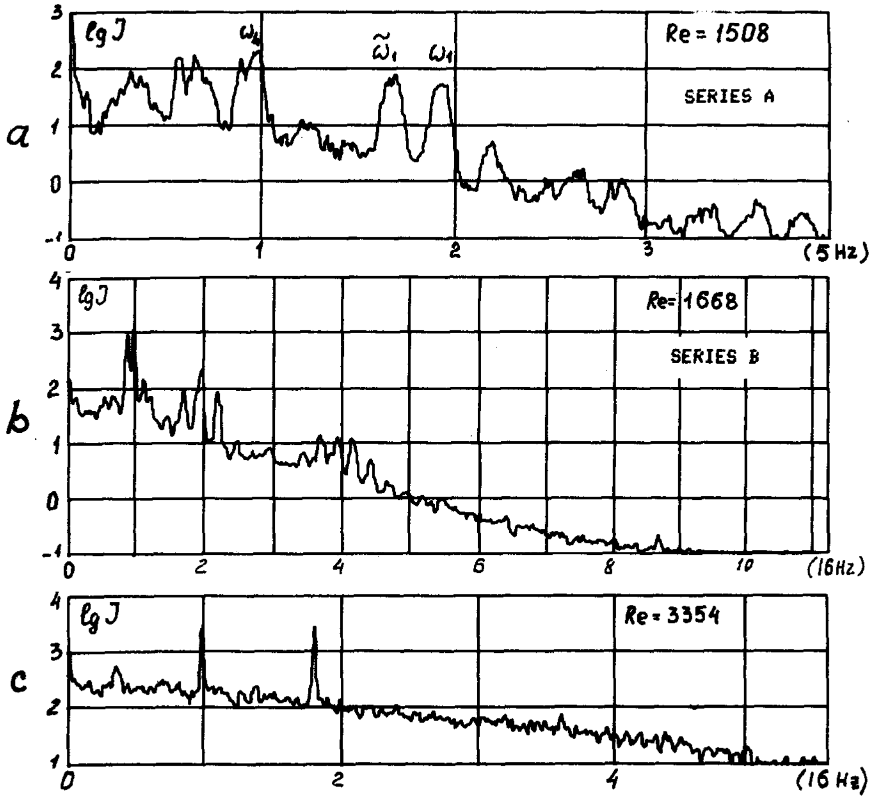


Fig. 3.

broadening, e.g. at $\epsilon = 0.07$, $\Delta\omega/\omega_1$ is about $2 \times 10^{-2}$. This results in a low-frequency motion with a characteristic time of variation of about $1/\Delta\omega$. Then a peak at frequency $\omega_2$ (incommensurable with the frequency $\omega_1$) as well as combinative harmonics appear in the spectrum. Further, at $\epsilon = 0.25$ in some runs (series A, fig. 2) another frequency mode ($\omega_3 = 0.36\ \omega_m$) arose, while in other runs (series B, fig. 2) such rearrangements are not observed. The difference between the initial flow types resulting in different bifurcations is not yet clear.

The intensities of the combinative motion increase in the region $\epsilon \approx 0.3$ and can even be compared with those of the initial motion, e.g. at Re = 1273 (see fig. 2d) the intensities of the $\omega_1$ and $\tilde{\omega}_1 = \omega_1 - \omega_3$ peaks are of the same order. In both cases of spectrum evolution a new peak appears at Re = 1300 at the frequency $\omega_4 = 0.95\ \omega_m$. It should be noted that with further increase of the rotation rate the distortion of the peak system occurs for both series in a similar way. It results in a smooth broadening of the peaks and at Re = 1600 (figs. 3a, b) the spectrum approaches a noise spectrum. Fig. 4 shows the Re dependence of the main peak frequencies and widths at the 0.1 level. At $\epsilon < 0.025$ the peak is seen to be comparatively narrow ($\Delta\omega/\omega = 10^{-3}$). At $\epsilon > 0.025$ we observe the specific

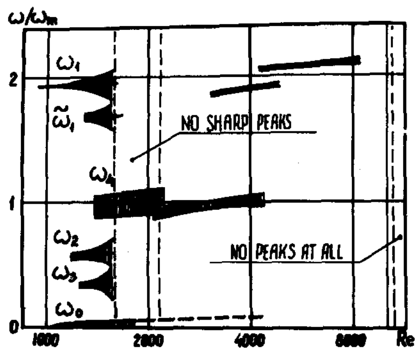


Fig. 4.

bifurcation connected with the considerable broadening of this peak. At still larger Re all peaks smoothly broaden to such an extent that one cannot observe pronounced maxima exceeding the continuous part of the spectrum by more than an order.

As Re is increased further, comparatively sharp peaks are observed again on the background of the continuous spectrum. For example, at Re = 3354 (fig. 3c) two narrow peaks with relative widths of about $10^{-2}$ are clearly seen. This system of secondary peaks is destroyed in qualitatively the same way as the initial one. At Re = 12 000 the turbulence spectrum is again continuous.

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