ISSN 0021-3640, JETP Letters, 2006, Vol. 84, No. 6, pp. 289–293. © Pleiades Publishing, Inc., 2006.

On the Problem of Catastrophic Relaxation in Superfluid ^{3}He-B¶

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Received August 15, 2006

In this Letter, we discuss the parametric instability of the texture of homogeneous (in time) spin precession and explain how the spatial inhomogeneity of the texture may change the threshold of the instability in comparison with the idealized spatial homogeneous case considered in our JETP Letter **83**, 530 (2006). This discussion is inspired by the critical comment of I.A. Fomin (JETP Lett., this issue) related to the above questions. In addition, we considered here the results of direct numerical solution of the full Leggett–Takagi equation of motion for magnetization in ^{3}He-B and experimental data for the magnetic field dependence of the catastrophic relaxation that provide solid support for the theory of this phenomenon presented in our 2006 JETP Letter.

PACS numbers: 67.57.Lm, 76.50.+g

DOI: 10.1134/S0021364006180032

INTRODUCTION

This Letter is inspired by the critical comment [1] on our JETP Letter "Solution of the Problem of Catastrophic Relaxation of Homogeneous Spin Precession in Superfluid ^{3}He-B" [2]. The self-sustained and long-lived spin precession with the coherent phase across the whole precessing domain—the homogeneously precessing domain (HPD)—is a unique feature of superfluid ^{3}He-B. This phenomenon bears all the ingredients of spin superfluidity [3]. Later, it was found that, at low temperatures below ~(0.4–0.5)T_c, the spin superfluidity experiences abrupt instability, which is called catastrophic relaxation [4].

After many attempts, it was finally recognized that the origin of the catastrophe is the parametric (Suhl) instability [5, 2]. Two competing contributions to the increment of the parametric instability were suggested:

$$V(\mathbf{r}) = V_{\rm SF}(\mathbf{r}) + V_{\rm BLV}(\mathbf{r}). \qquad (1)$$

The intrinsic contribution $V_{\rm SF}$, which was suggested by Sourovtsev–Fomin (SF) [5], is due to the anisotropy of the spin-wave velocity, and it comes from the region where the configuration of the spin and orbital vectors **S** and **L** is canonical—the so-called Brinkman–Smith (BS) mode of precession. As was suggested in our Bunkov–L'vov–Volovik (BLV) [2] contribution, $V_{\rm BLV}$ is due the oscillating spin-orbit energy and it only comes from the region where the precession deviates from the BS mode.

¶ The text was submitted by the authors in English.

Comment [1] clarifies aspects of the problem that may cause misunderstandings and thus require more detailed explanation and even further development. This is the subject of our Letter, which is organized as follows. Sections 1–3 are devoted to the questions that arose in [1], which are mainly related to the problem of parametric instability in the case of spatially inhomogeneous precession. In Section 4, we consider the extension of our theory to other precessing states. In Sections 5 and 6, we discuss the results of direct numerical solution of the full Leggett–Takagi equations for magnetization and the results of laboratory study of the magnetic field dependence of the catastrophic relaxation that provide additional support for our theory of this phenomena as presented in [2] and demonstrate that the SF mechanism of the instability [5] is in contradiction with the experiments.

1. STABILITY OF THE BASIC REFERENCE STATE

This is the first question of [1] that is useful to clarify. As stated in [1], the spin-orbit potential energy as a function of S_z has a concave or convex form depending on the orientations of **L** and **S**. Therefore, in some cases, the spatially homogeneous precession can be unstable [6]. For example, the conventional homogeneous precession is unstable in ^{3}He-A [7]. However, this is not applicable to a "texture" with some profile of the orientations of the orbital and spin moments, **L** and **S**, that realizes the minimum of the sum of the gradient and the spin-orbit energies (see [8] for the texture in slab geometry and [9] for the texture under discussion).

The texture is stable with respect to perturbations with the same rotation frequency ω_L. This texture, which is shown in Fig. 1 of [2], serves as the basic reference state whose stability with respect to parametric excitation of spin waves with the frequency $\omega_s(\pm\mathbf{k}) = \omega_L/2$ was studied in [2]. Notice again that the texture is stable with respect to perturbations with the frequency ω_L. This is the answer to the first question of [1]: "Why can the same mechanism that is responsible for the instability of the uniform precession in ^{3}He-A be disregarded?"

2. PARAMETRIC INSTABILITY IN SPATIALLY INHOMOGENEOUS MEDIA

The second statement of comment [1] is trivial: because the spatial variation of the parametric instability increment V "is not small …, two ways of averaging [$\exp(\langle V\rangle t)$ and $\langle\exp(Vt)\rangle$] can give very different results." The relevant question here is "What is an adequate way of averaging the instability increment (if it exists)?" To answer this question, one has to study parametric instability in spatially inhomogeneous media.

This problem has been discussed in various physical situations and is presented in many books, for example, in Section 6.5.2 of monograph [10], where explicit expressions for the instability increment for different types of spatial inhomogeneities are derived. Without going into details, we can say that the main physical message of this study is that, for weakly decaying parametric waves (which is our case), the threshold of the parametric instability can be estimated with good accuracy from the total energy balance in the sample (cell, in our case). Namely, at the threshold, the total energy influx into the system

$$W_+ \propto \int V[\mathbf{r}, \mathbf{k}(\mathbf{r})]d\mathbf{r}, \qquad (2)$$

has to be equal to the total rate of the energy dissipation W_-. With the same prefactor as in Eq. (2)

$$W_- \propto \int \gamma[\mathbf{k}(\mathbf{r})]d\mathbf{r}, \qquad (3)$$

where $\gamma(\mathbf{r})$ is the damping rate of the parametric waves.

The physical reason is that, under wave propagation in weakly inhomogeneous media, its frequency serves as the adiabatic invariant. This means that the wave frequency is independent of the position, while the wave-vector $\mathbf{k}(\mathbf{r})$ changes accordingly to the dispersion law $\omega(\mathbf{k}, \mathbf{r})$ = const. Because of that, even propagating in inhomogeneous media, the waves with

$$\omega[\mathbf{k}(\mathbf{r}), \mathbf{r}] = \omega_L/2, \qquad (4)$$

stay in parametric resonance with the pumping (homogeneous in frequency, but spatially inhomogeneous spin precession). In other words, if the mean free path of the wave exceeds the cell size, the entire cell can be considered as the resonator with some parametrical mode that is locally close to the planar spin wave. Under these conditions, the threshold can be well estimated from the condition

$$\int V[\mathbf{r}, \mathbf{k}(\mathbf{r})]d\mathbf{r} = \int \gamma[\mathbf{k}(\mathbf{r})]d\mathbf{r}. \qquad (5)$$

The accuracy of this estimate is about ±(10–30)% and related with ignoring the spatial variation of the actual profile of the parametric waves in Eq. (5) (for more details, see, i.e., [10]).

In experimental conditions (see Fig. 1 in [2]), one roughly says that the fraction of the cell volume (near the wall) with essential deviation from the planar geometry is about half of the total volume. This is the volume where the energy pumping is dominating [i.e., the integral in the LHS of Eq. (5)]. In this region, in agreement with the third comment of [1], the spin-wave vectors are relatively small; thus, one can neglect the wave damping. The wave damping [the integral in the RHS of Eq. (5)] is dominated by the central part of the cell, where the wave vectors are sufficiently large. Accordingly, the total energy balance in the cell is maintained by the spatial energy flux from the near-wall region to the central part of the cell. Under these conditions, the threshold Eq. (5) can be roughly rewritten in a simple manner

$$\mathcal{V}_V \max V[\mathbf{r}, \mathbf{k}(\mathbf{r})] \simeq \mathcal{V}_\gamma \max\gamma[\mathbf{k}(\mathbf{r})], \qquad (6)$$

in spite of the fact that the energy pumping and the energy damping are dominating in the different regions of the cell. Here, $\mathcal{V}_\gamma$ and $\mathcal{V}_V$ are the corresponding effective volumes that depend on the texture. As follows from the analytical considerations and numerical simulations, these volumes do not depend on the value of the magnetic field (see below), and, in the considered cell, they are roughly equal. This means that

$$\max V[\mathbf{r}, \mathbf{k}(\mathbf{r})] \simeq \max\gamma[\mathbf{k}(\mathbf{r})], \qquad (7)$$

which is the estimate used in [2].

3. WAVE DAMPING IN THE NEAR-WALL REGION

In the third complaint of [1], we were instructed how to estimate from Eq. (4) the wave vector of $\mathbf{k}(\mathbf{r})$. The estimate presented shows that wave vectors near the wall are small. Therefore, the wave damping is also small with respect to that the central region, which leads to the conclusion that we missed a factor of about 10 in our estimate of the parametric threshold. The misunderstanding of [1] regarding this point is related to the question "How does the value of the wave damping in the near-wall region effect the threshold of the parametric excitation in the cell?" We hope that our explanation in Section 2 is clear enough for one to realize that the energy balance in the system of weakly decaying waves has to be discussed rather globally for the entire cell, then, locally and point wise as mentioned in comment [1]. Therefore, the estimates of the spin-wave vectors in the near-wall region made in comment [1], being reasonable themselves, are irrelevant to the problem at hand. We can say the same about the statement made in [1] concerning the "additional factor ≈10 in the RHS of Eq. (28)" in [2]. As we explained in Section 2, the possible inaccuracy of our estimate of the threshold [2] does not exceed (20–30)%.

4. GENERALIZATION ON OTHER PRECESSING STATES

To conclude the subject of the spatial inhomogeneity, we mention that the precession under discussion—the HPD—is a very specific precessing state due to its unique symmetry. Only in the case of the Brinkman–Smith mode does the spin-orbit interaction not contribute to the amplitude of the parametric instability V; thus, the parametric excitation by the BLV mechanism requires the existence of the texture.

However, there are many other modes of spin precession in ^{3}He for which the spin-orbit potential energy as a function of S_z also has the concave form. These are the so-called HPD2 in ^{3}He-B [11], the special mode of precession in ^{3}He-A [12], the precession at half of the equilibrium magnetization and at almost zero magnetization observed in ^{3}He-B [13], etc. As distinct from the HPD based on the Brinkman–Smith mode, in all these modes, the spin-orbit energy is oscillating; thus, it produces the non-zero contribution to V even in the case of the spatially homogeneous precession. This means that, for all these modes of spatially homogeneous precession, the BLV mechanism based on the spin-orbit energy will compete with the SF mechanism even at moderate magnetic fields.

Notice that some of the predicted HPD modes still have not been observed or identified. Now, with our knowledge of the parametric instability mechanisms and their dependence on different parameters, we are able to find the region of parameters (magnetic field, temperature, angles of precession, superfluid velocity, etc.) where all the conditions for the stability of precession are satisfied.

Now, we are coming to the philosophical discussion that was opened in comment [1] concerning the "validity of an idealization for a particular experimental set up" and of acceptance of the "generalization of the theory in a way that makes possible its application for a wider class of experimental conditions." Obviously, the success of an idealization or generalization that is used in a theory depends on the experience, physical intuition, judgment, and courage of its authors, which are different for different investigators. We hope that we made it clear in Sections 1–3 that what seems to the author of comment [1] as "ambiguous assumptions and unjustified approximations" have indeed a clear sense and a well established physical background.

Nevertheless, we are happy to use the discussion with the author of [1] to provide our theory of catastrophic relaxation [2] with additional support from the direct numerical solution of full Leggett–Takagi equations [14] (in Section 5) and, especially, from the laboratory study of the magnetic field dependence of the temperature of catastrophic relaxation in ^{3}He (Section 6). Note that most of the complications related to the spatially inhomogeneous precession disappear at high magnetic fields, where the wave vector **k** is almost constant across the cell.

5. DISCUSSION OF NUMERICAL EXPERIMENT

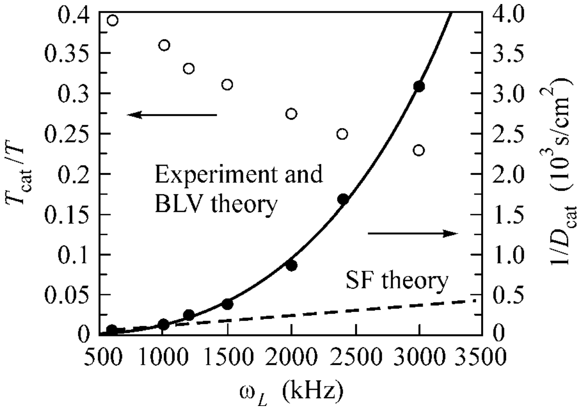
One-dimensional numerical simulations of the Leggett–Takagi equations were performed in [14]. It was found that, in the viscous limit, i.e., at sufficiently high temperature, the Brinkman–Smith configuration takes place in the central part of the cell being disturbed by the periodic perturbations in the angle θ coming from the peripheral region where the vector **L** considerably deviates from the Brinkman–Smith configuration. This observation confirms our statement made in Section 1 that the spatial inhomogeneous texture is stable, thus, providing a minimum of the spin-orbit and gradient energies at given boundary conditions and can not be destroyed by any relaxation processes.

At further cooling of the cell, the BS configuration is more and more disturbed, and, at some threshold temperature T_{th}, the catastrophe occurs: the exponential growth of standing spin waves [with the frequency $\omega_L/2$] starts in the peripheral region and propagates to the central part, taking energy from the homogeneous precession and finally destroying it. This picture is in full agreement with the parametric instability of the inhomogeneous precession studied in [2, 5].

Let us remember that there are two competing contributions to the increment of the parametric instability in Eq. (1). The "intrinsic" contribution V_{SF} comes from the whole sample, and the contribution V_{BLV}, which is due to the oscillating spin-orbit energy and it comes only from the regions where the configuration deviates from the Brinkman–Smith mode. In the one-dimensional geometry used in the numerics [14], the SF contribution is suppressed in the central region because the increment is zero for the waves radiating in the direction perpendicular to **L** (cf., Eq. (26) of [5] and Eq. (25) in [2]). This means that, while these particular numerical simulations support the process discussed by the BLV theory [2], they cannot either support or disregard the relevance of the SF contribution at given parameters of the simulations.

6. MAGNETIC FIELD DEPENDENCE: SF vs. BLV COMPETITION

To find out which contribution is dominating in real experimental situations, we will compare both contributions to the increment (1) [2, 5] directly with laboratory experiments. The BLV arguments [2] demonstrate that, in moderate magnetic fields, the V_{BLV} contribution



The experimental data for the temperature of catastrophic relaxation as a function of the NMR frequency at 31 bar (○), left scale, and the estimated value of the inverse spin diffusion coefficient, (●) right scale. Solid line—BVL theoretical estimation [2], dashed line—SF results [5].

is dominating. Both groups [1, 2] agree that, according to the calculations of [5], the SF contribution V_{SF} dominates at large magnetic fields when $\omega_L \gg \Omega_L$. To see whether this is really the case, we examine here experimental data on the magnetic field dependence that include the region of higher magnetic fields.

The data most relevant for this goal are provided by the Cornell [15] and Grenoble [16] experiments, where the temperature of catastrophic relaxation has been measured as a function of the magnetic field (see figure). The open circles in the figure demonstrate the temperature $T_{cat}(\omega_L)$ of the catastrophic relaxation as a function of the magnetic field (or Larmor frequency ω_L) at 31 bar [15]. The main effect of the temperature is to provide the dissipation that damps the parametric instability via the spin-diffusion mechanism [5]. That is why the relevant physical quantity is the spin-diffusion coefficient $D(T, \omega_L)$, rather than the temperature itself, and we must convert the temperature to diffusion. According to [17], in the ballistic regime considered here when $\omega_L\tau(T) \gg 1$, the spin diffusion $D(T, \omega_L) \sim 1/\omega_L\tau(T)$, where $\tau(T)$ is the quasiparticle scattering time, which grows exponentially with cooling. The experimental temperature dependence of the spin diffusion is obtained from measurements of diffusion through the HPD boundary [18].

Combining these results with $T_{cat}(\omega_L)$, from the figure one obtains the spin diffusion at which the catastrophic relaxation occurs as a function of ω_L; the solid circles in the figure show $D_{cat}^{-1}(\omega_L)$. The experimental points well fit the cubic dependence

$$\frac{1}{D_{cat}^{BLV}} = \frac{\omega_L^3}{4\bar{a}\Omega_L^2 c_\parallel^2} \propto \omega_L^3, \quad (8)$$

obtained within the BLV mechanism (see Eqs. (23) and (24) in [2]).

For the quantitative comparison of the experimental ω_L dependence of $1/D_{cat}$ with Eq. (8), we shall take the experimental values of the parameters Ω_L, $c_\parallel$ and use the parameter $\bar{a}$, which characterizes the **L**-texture, as a fitting parameter. In the considered region of temperature, all these parameters are slow functions of the temperature. We shall use the data at 1 MHz and $T = 0.35T_c$, where reliable experimental data exist. From [19], we can estimate $\Omega_L^2 = 10^{11}$ Hz^2, while, from measurements of different modes of HPD oscillations [20] with pressure scaling by the Fermi velocity, we find $c_\parallel^2 = 1.5 \times 10^6$ cm^2/s^2. By introducing these values into Eq. (8), one obtains $\bar{a} = 0.07$, which is in good agreement with the theoretical estimation of $\bar{a} \approx 0.1$ for 6 bar made in [2].

In the current consideration, it is important that the parameter $\bar{a}$ does not depend on the magnetic field. The reason for this is that, according to [9], the characteristic length scale of the near wall region is $\sim c/\sqrt{\omega(\omega - \omega_L)}$. It is typically about c/Ω_L and thus does not depend on ω_L.

The next step is to compare the experimental results with the SF contribution [5]. It reads

$$\frac{1}{D_{cat}^{SF}} = \frac{\omega_L^2}{2\lambda_{max} c_\parallel^2} \propto \omega_L. \quad (9)$$

Here, we accounted for the fact that λ_{max} near the HPD boundary is equal to $0.016\omega_L$; therefore, the field dependence of $1/D_{cat}^{SF}$ should be linear. As one can see in the figure, this clearly contradicts the experiment. Furthermore, if we plot the value of $1/D_{cat}^{SF}$, we find (figure) that the SF result agrees with the experiment only in the region of NMR of 500–1000 MHz. At higher fields, the theoretical value of diffusion at which catastrophic relaxation with the SF mechanism should occur definitely disagrees with the experiments.

DISCUSSION

According to our theoretical analysis, in moderate magnetic fields, i.e., at ω_L smaller than about 1 MHz, the BLV contribution to the parametric instability of HPD is dominating. At these fields, the effect of the spatial inhomogeneity on the BLV mechanism is essential, and we clarified in Sections 2 and 3 the corresponding points of comment [1]. Most of the complications related to the issue of spatial inhomogeneity do not arise at high magnetic fields where $\omega_L \gg \Omega_L$, and the wave vector **k** is (almost) homogeneous along the cell. However, we expected that, at such a high field, the BLV contribution is subleading, while the SF contribution dominates if $\omega_L^2 > 10\Omega_L^2$. The same opinion was expressed in comment [1], where it was stressed that stronger magnetic fields have to be used for the experimental investigation of the "intrinsic" SF mechanism of catastrophic relaxation.

On the contrary, the surprising experimental fact is that the magnetic field dependence of the catastrophic relaxation demonstrates that, even up to a rather high field when $\omega_L^2 \sim 100\Omega_L^2$, it is still quantitatively and qualitatively described by the BLV contribution to the parametric instability. Moreover, the magnetic field dependence is in striking disagreement with the SF contribution. The point is that the SF contribution does not show up in these experiments at all.

We do not think that the explanation of this fact is related to possible calculational mistakes in the analytics presented in [5]. We feel that the reason(s) for the obvious qualitative disagreement between the SF analysis and experiment is deeper and may be related to possible violations of the SF approach in the region of large **k** vectors.

To make a long story short, the BLV mechanism of the parametric instability gives a quantitative description of the present experiments for any values of ω_L used, while the SF contribution, which seems to be the leading one (for large ω_L), is absent in the experiments for an unknown reason. This is the main puzzle; its solution requires further theoretical, numerical, and experimental efforts, which are beyond the scope of this Letter.

We acknowledge the critical comments by I.A. Fomin [1] that inspired the further clarification and development of our theory of catastrophic relaxation in superfluid ^{3}He-B presented in this Letter. This work was performed as the result of collaboration within the frameworks of the ESF Program COSLAB, the Large Scale Installation Program ULTI of the European Union (contract number: RITA-CT-2003-505313), and the ULTIMA Project of the "Agense National de Recherche" in France. This work was also supported in part by the Russian Foundation for Basic Research and the United States–Israel Binational Science Foundation.

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