Freak Waves in the Ocean

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Ships are disappearing all over the world's oceans at a rate of about one every week. These drownings often happen in mysterious circumstances. With little evidence researchers usually put the blame on human errors or poor maintenance. But an alarming series of drownings and near drownings including world class vessels has pushed the search for better reasons than the regular ones: Freak (or Rogue, Monster, Killing) Waves.

The Freak Wave in the Ocean is a catastrophic event when energy and momentum of the wave field spontaneously concentrate in a localized area of space generating of short wave train consisting of several waves with energy and momentum density in order of magnitude exceeding the background level. Freak waves could be disastrous for ships, drilling platforms, lighthouses and other coastal constructions.

I will present observations of Freak Waves, their effect on ships and discuss possible mechanisms of their creation and evolutions.

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Lake near Grenoble: Wind ≈ 10 nots, Waves Hight ≈ 30 cm

- ***Freak (or Rogue, Monster, Killing) Waves observations***

Freak waves up to 30 meters high that rise up from calm seas to destroy ships do exist

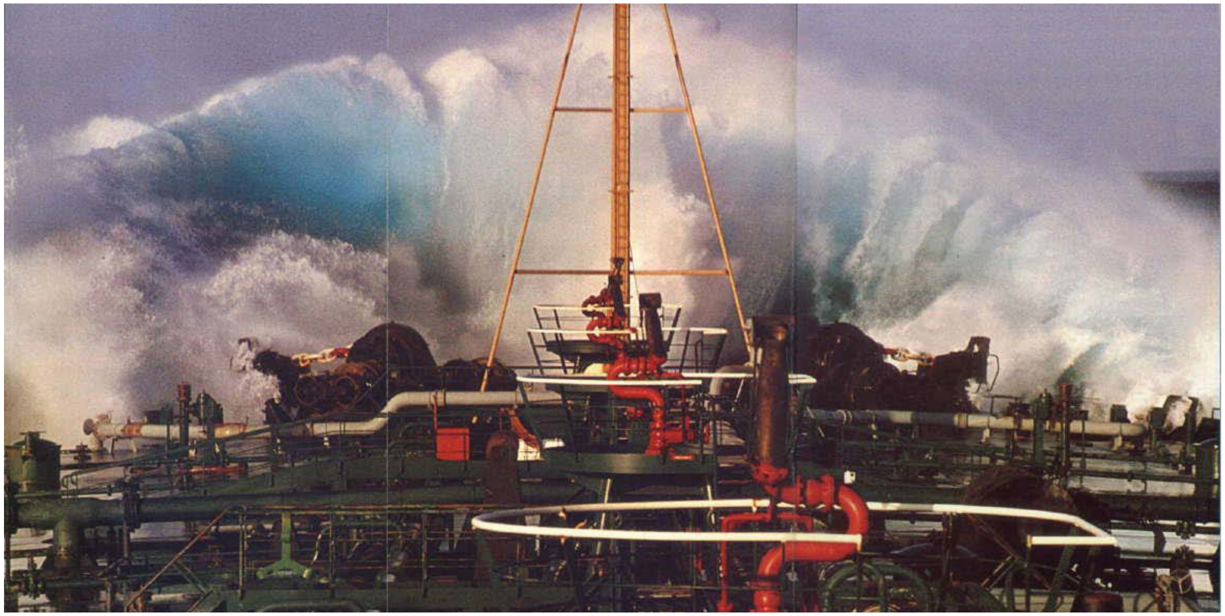
21 July 2004: the supertanker Esso Languedoc, during a storm off Durban in South Africa in 1980. The mast seen starboard in the photo stands 25 metres above mean sea level. The mean wave height at the time was between 5-10 meters.

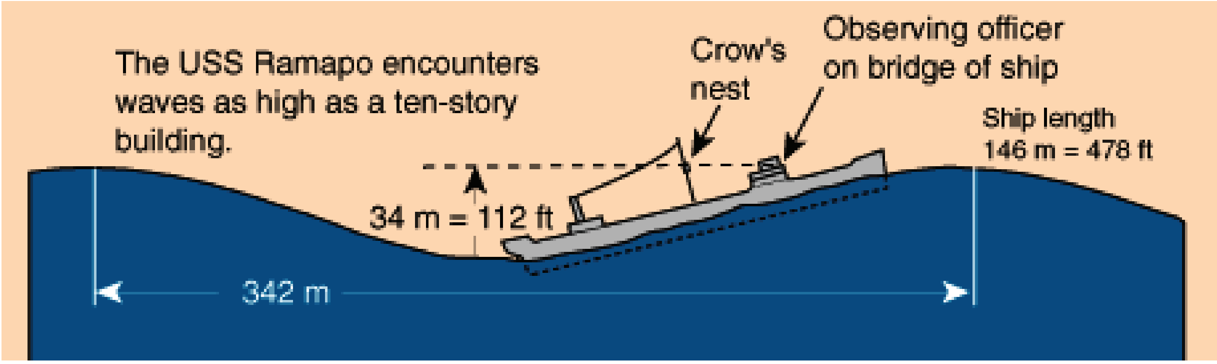
 

September 5, 2001. A bulk carrier Ikan Tanda ran aground off Scarborough on the southern Cape Peninsula's Atlantic coast during the worst storm for 50 years:

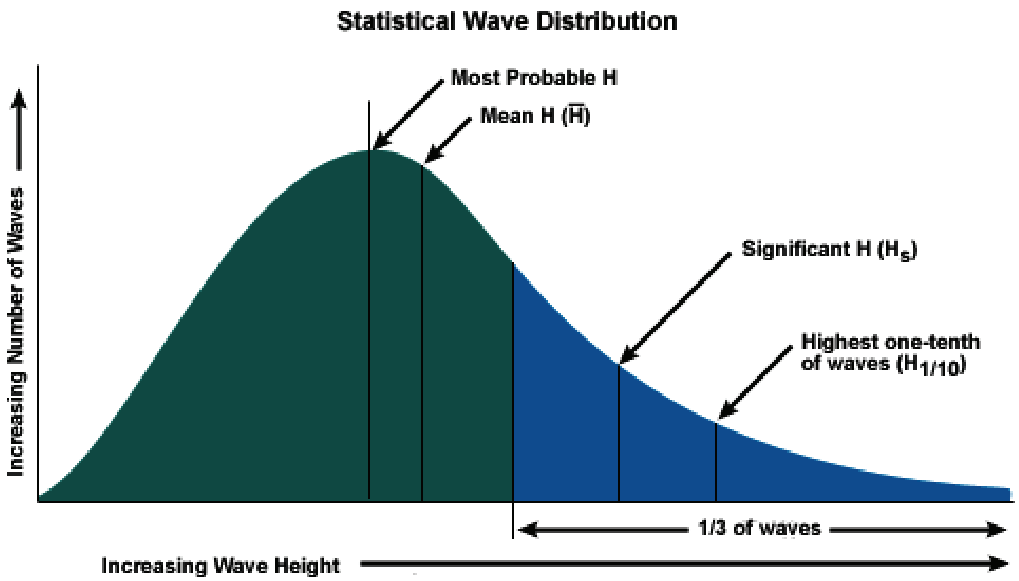
Wind: $48 \div 55$ knots. Very high waves ($9 \div 12.5$ m.) with long overhanging crests, the resulting foam, in great patches, is blown in dense white streaks along the wind direction


WILSTAR




The USS Ramapo encounters waves as high as a ten-story building.
Crow's nest
Observing officer on bridge of ship
Ship length 146 m = 478 ft
34 m = 112 ft
342 m

• *Ocean wave statistics (≈ Rayleigh Distribution) and Freak Waves*



Significant Wave Height H_{sig} is the average wave height (trough to crest) of the one-third largest waves.

If, e.g. $H_{\text{sig}} = 10$ m:

•1 in 10 will be > 10.7 m

•1 in 100 will be > 15.1 m

•1 in 1000 will > 18.6 m.

– *Freak Waves (FWs)* are defined as waves whose height $\boldsymbol{H > H_{\text{sig}}}$,

– *FWs* are not necessarily the biggest waves at sea;

– *FWs* are, rather, surprisingly large waves for a given sea state.

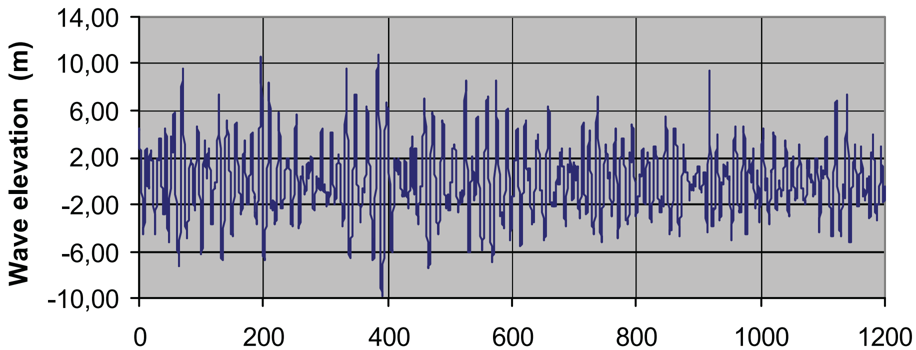
– *FWs* are not tsunamis, which are set in motion by earthquakes [and] travel at high speed, building up as they approach the shore.

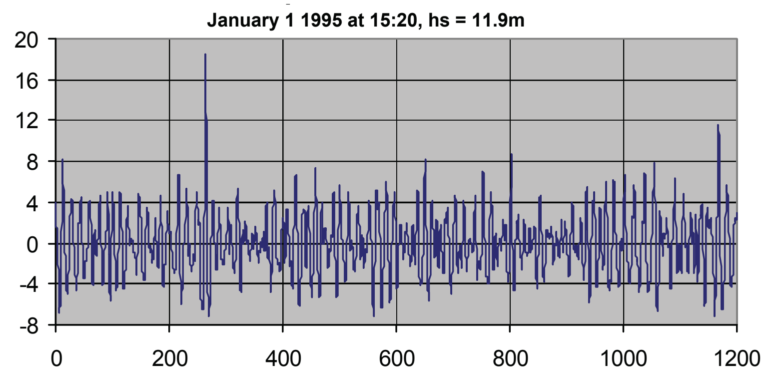
– *FWs* seem to occur in deep water or where a number of physical factors such as strong winds and fast currents converge.

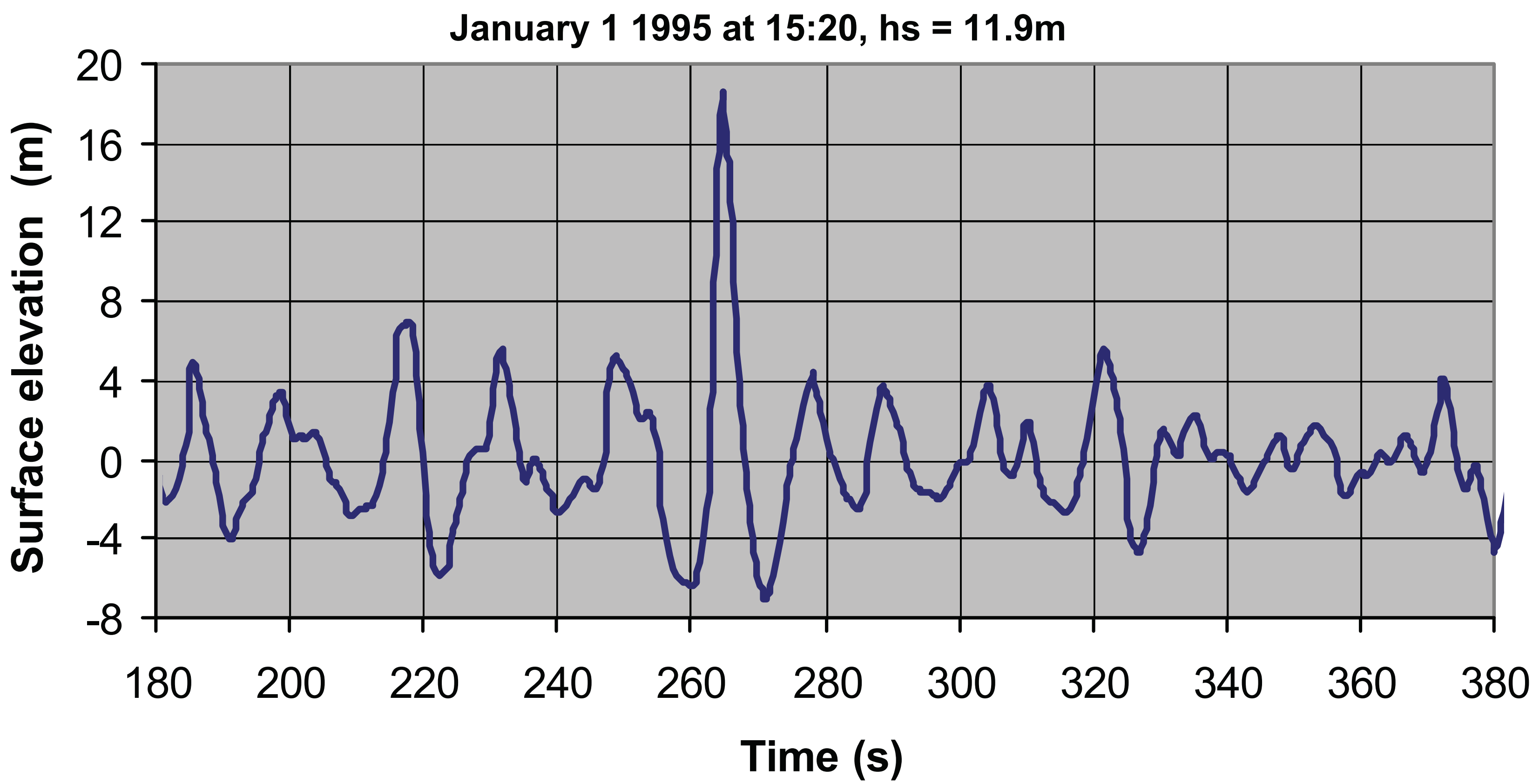
- ***Freak wave registrations, since 1995***

Draupner Nord Sea Platform

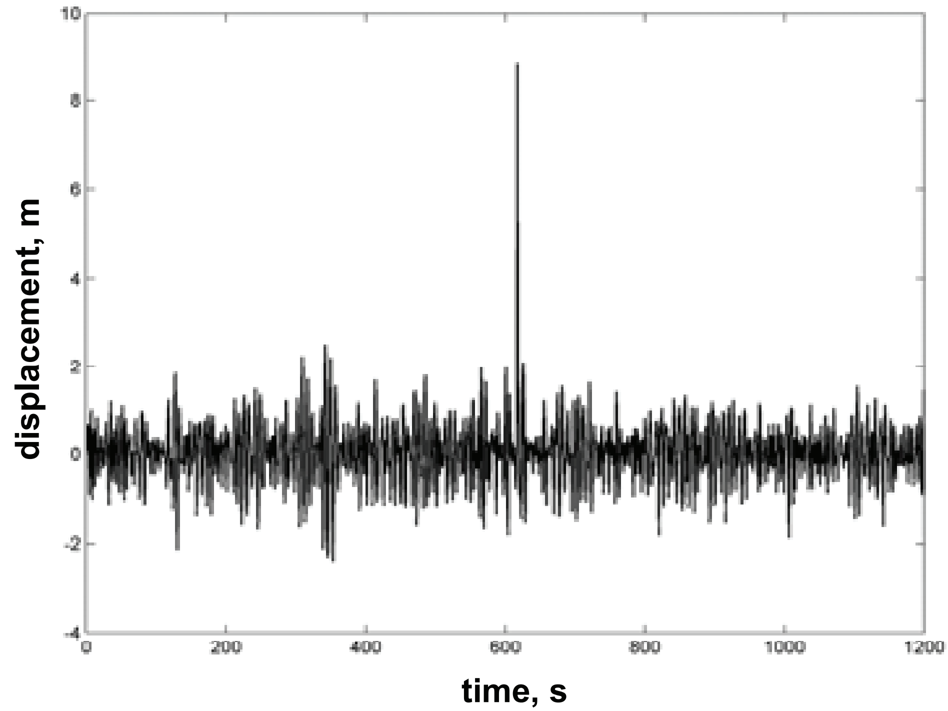
●First registration of FWs: Draupner wave records, Jan. 1, 1995





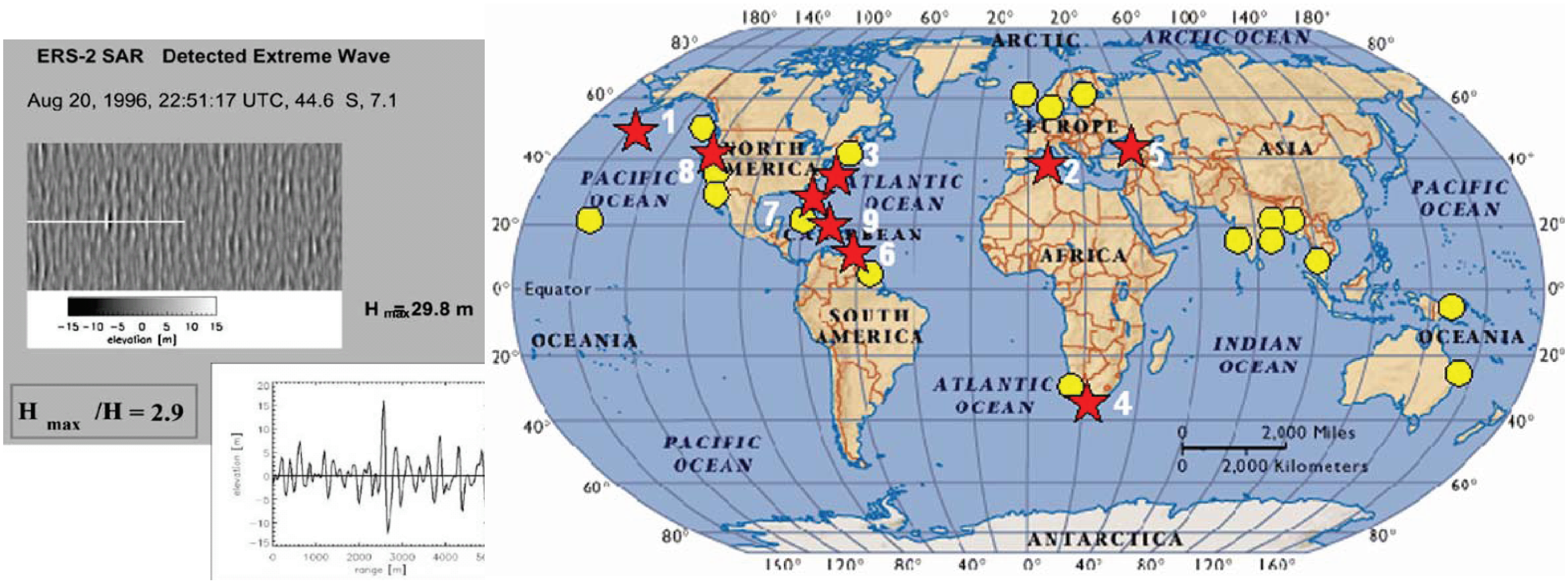


•Black Sea FW record, Gelenndzhik Buoy, Nov. 22, 2001



when $H_{\text{sig}} \approx 2.6\,\text{m}$. Observed FW hight $\boldsymbol{H_{\text{fw}} = 10.32\,\text{m} \approx 3.9 H_{\text{sig}}}$

• *FW detection by suttellite radar and FWs, detected in 2005*



"True FWs" detected in 2005 (9 events) are marked by **full red stars**, full YELLOW circles mark all other reported abnormally large waves: **1.** 27 Jan, cruiser "Explorer", damaged; **2.** 14 Feb. cruiser "Grant Voyager", damaged; **3.** 16 Apr. cruiser "Norwegian Dawn"; **4.** 26 Aug., Kalk Bay, South Africa; **5.** 14 Sept Blue Bay, Black sea; **6.** 16 Oct., Maracas Beach, Trinidad Island; **7.** 24 Oct., Grand Baham, the Bahams; **8.** 11 Nov., Port Orford, USA; **9.** 10 Dec., Petit Havre.

PRELIMINARIES:

• *Gravity waves (GWs) on a deep water*

Wave amplitude: $\eta(\vec{r},t) \propto \exp\left[i\vec{k}\cdot\vec{r} - i\omega(\vec{k})t - \gamma(k)\,t\right]$

Relevant physical parameters: the gravity acceleration $g \approx 980{,}665$ cm/s^2 and kinematic viscosity $\nu \simeq 0.01\,\text{cm}^2/\text{s}$ in water for $T \simeq 20$ C.

(no fluid density: inertial and gravity masses are the same).

Dimensional reasoning with $[\omega] = [\nu] =$s^{-1} gives:

$\omega(k) \simeq \sqrt{gk}\,,$ exact result: $\omega(k) = \sqrt{gk}\,,$ $\gamma(k) \simeq \nu k^2\,,$ exact result: $\gamma(k) = \nu k^2\,.$

FW length $\lambda \sim 50 \div 300\,$m. Taking $\lambda = 2\pi/k = 100\,$m one estimates:

wave period $T = 2\pi/\omega \approx 8.0\,$s,

viscous damping time $\tau = 1/\gamma \approx 2.5\cdot 10^8\,\text{s} \approx 8\,\text{years}$

group velocity $v = d\omega/dk = \frac{1}{2}\sqrt{g/k} \approx 6.25\,$m/s , and

viscous mean free pass $\ell = v(k)\tau \approx 1.6\cdot 10^6$ Km ≈ 40 Earth equator lenth

Conclusion: viscous damping can be ignored and one can describe FWs by the Euler equations for potential fluid motions with the gravity force, constant water density

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} + \vec{\nabla}p + \vec{g} = 0\,, \quad \rho = 1 \quad \vec{v} = \vec{\nabla}\Phi\,, \tag{1}$$

and proper dynamical boundary conditions at the water surface $z = \eta(x,y,t)$ and without bottom, $\lim_{z\to-\infty}\vec{v}(\vec{r},t) = 0$.

- ***Hamiltonian description of the Gravity waves***

Following Zakharov, (1968) 3d-Eq. (1) for the GWs can be presented in the 2d-Hamiltonian form for the surface shape $\eta(x, y, t)$ and velocity potential $\psi(x, y, t) = \Phi(x, y, z = \eta, t)$ on the surface (Hamiltonian $\mathcal{H}[\eta, \psi]$ is the system energy):

$$\frac{\partial \eta}{\partial t} = \frac{\delta \mathcal{H}}{\delta \psi}, \quad \frac{\partial \psi}{\partial t} = -\frac{\delta \mathcal{H}}{\delta \eta} \, . \tag{2a}$$

Introducing new complex variables $a(x, y, t) = (\eta + i\psi)/\sqrt{2}$ and denoting its (discrete) Fourier transform as $c_{\vec{k}}(t)$ one gets canonical form of Hamiltonian Eq. (2a):

$$i \frac{dc_{\vec{k}}}{dt} = \frac{\partial \mathcal{H}}{\partial c^*_{\vec{k}}}, \quad \mathcal{H} = \sum_{\vec{k}} \omega(k) \, c_{\vec{k}} c^*_{\vec{k}} + \mathcal{H}_{\text{int}} \, , \quad \mathcal{H}_{\text{int}} \propto c^3 \, , \; c^4 \, , \ldots \tag{2b}$$

Here we assumed space homogeneity (deep sea) and smallness of wave amplitudes with respect of the wave-length (steepness $\eta k \ll 1$). For gravity waves 3-wave processes $1 \Leftrightarrow 2$ are forbidden and the leading interaction is the 4-wave scattering of $2 \Leftrightarrow 2$:

$$\omega(\vec{k}_1) + \omega(\vec{k}_2) = \omega(\vec{k}_3) + \omega(\vec{k}_4) \, , \quad \vec{k}_1 + \vec{k}_2 = \vec{k}_3 + \vec{k}_4 \, . \tag{2c}$$

In this case Eq. (2b) takes the form (with 4-wave amplitude of interaction, $T_{\vec{k},\vec{k}_1;\vec{k}_2,\vec{k}_3}$):

$$\frac{\partial c_{\vec{k}}}{\partial t} + i\omega_{\vec{k}} c_{\vec{k}} = -\frac{i}{2} \sum_{\vec{k}+\vec{k}_1=\vec{k}_2+\vec{k}_3} T_{\vec{k},\vec{k}_1;\vec{k}_2,\vec{k}_3} c^*_{\vec{k}_1} c_{\vec{k}_2} c_{\vec{k}_3} \, . \tag{2d}$$

• *Nonlinear Shrödinger Equation (NLSE)*

As a rule, FWs appear, when intensive long GWs are almost monochromatic, having $\vec{k}$-vectors in the vicinity of some $\vec{k}_0$ (see radar observations). In this case its wave packet can be considered as a plane wave with "slow" modulation $C(\vec{r},t)$ of its amplitude, $\boldsymbol{c(\vec{r},t) = C(\vec{r},t)\mathbf{exp}\{i[\vec{k}_0\cdot\vec{r} - i\omega(\vec{k}_0)t]\}}$.

Using Eq. (2d) one derives the following NLSE for the slow "envelope" $C(\vec{r},t)$:

$$\left[i\frac{\partial}{\partial t} + i\,v\frac{\partial}{\partial x} + \frac{v}{2k}\left(\frac{\partial^2}{\partial y^2} - \frac{1}{2}\frac{\partial^2}{\partial x^2}\right) + T|C|^2\right]C(r,t) = 0\,, \tag{3}$$

that contains essential part of the truth about creation and evolution of FWs. Here

– cyan term describes wave propagation with the group velocity $\boldsymbol{v}$ in the x-direction,

– green terms describe diffraction and dispersion of the wave package, and

– nonlinear red term ($\boldsymbol{T \equiv T_{k_0,k_0;k_0,k_0} < 0}$ for gravity wave) can be considered as the self-consistent (attractive for $\boldsymbol{T < 0}$) potential energy, proportional to the local density of "quasi-particles" $\boldsymbol{N(\vec{r},t) = |C(\vec{r},t)|^2}$.

– NLSE (3) conserves total number of particles $N = \int |C|^2 d^d r$, and the Hamiltonian

$$\mathcal{H} = \frac{1}{2}\int\left[i\vec{v}\cdot\left(C^*\vec{\nabla}C - C\vec{\nabla}C^*\right) + \omega''\left(\frac{\partial C}{\partial z}\right)^2 + \frac{v}{k}|\Delta_\perp C|^2 + 2T|C|^4\right]d^d r\ .$$

PHYSICS of FWs: •*Temporal and spatial focusing of GW energy*

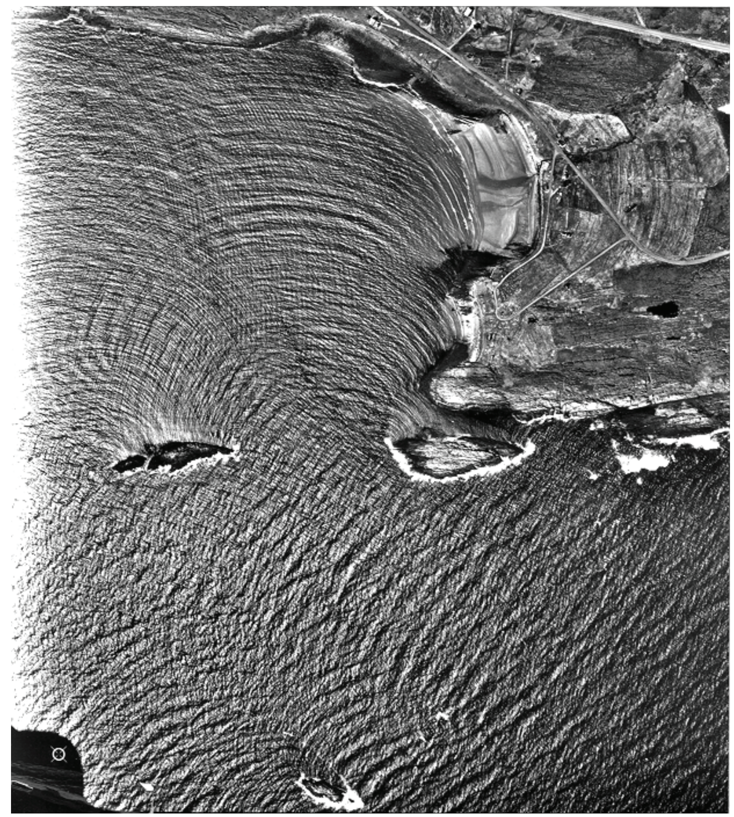
– Group velocity of GWs depends on λ: $\boldsymbol{v(k) = \sqrt{g/4\,k}}$. By producing a train of waves with increasing λ, one can create very large waves. If H of the chirped wave train is below the rms value of the background random waves it will remain invisible until it focuses. The ordering of such a train can be created by a strengthening wind in a storm area that will produce waves with increasing velocities that will focus far away from the production area and will seem appearing out of nowhere.

– Spatial focusing by current and wind refraction:

Fornberg (1998) have shown that even small random current fluctuations with RMS values of the order 10 cm/s and typical scales of the order of 10 km can give formation of caustics provided the incoming wave field is unidirectional and narrow banded.

Similar effect can be caused by the spacially dependent wind that also affect the GW group velocity.

The same is true in coastal areas due to the $\omega(k)$ dependence on the sea depth, see photo $\Rightarrow$



- ***Wave braking $\Rightarrow$ creation of "white horses" $\Rightarrow$***

effective mechanism of the GW energy dissipation, limiting their hight H at $\approx 0.1\,\lambda$

•***Modulation instability of GWs and Decuman (tenth) Wave***
(Whitham, Lighthill, Benjamin, Feir and independently by Zakharov, Ostrovskii) Effective self-attraction of GWs in the NLSE leads to longitudinal and lateral modulation with long period Λ ($\lambda/\Lambda \simeq \sqrt{\langle\eta^2\rangle}/\lambda$) of the sea-wave amplitudes η. Non-linear estimate, that accounts for "white horses" gives the nonlinearity $\sqrt{\langle\eta^2\rangle}/\lambda \simeq 0.1$. This results in $\Lambda \simeq 10\lambda$, nonlinear energy accumulation in each $10^{\rm th}$ wave, see Ivan Aivazovskii painting "Decumen wave" (1850) in Russian Museum, St. Petersburg:

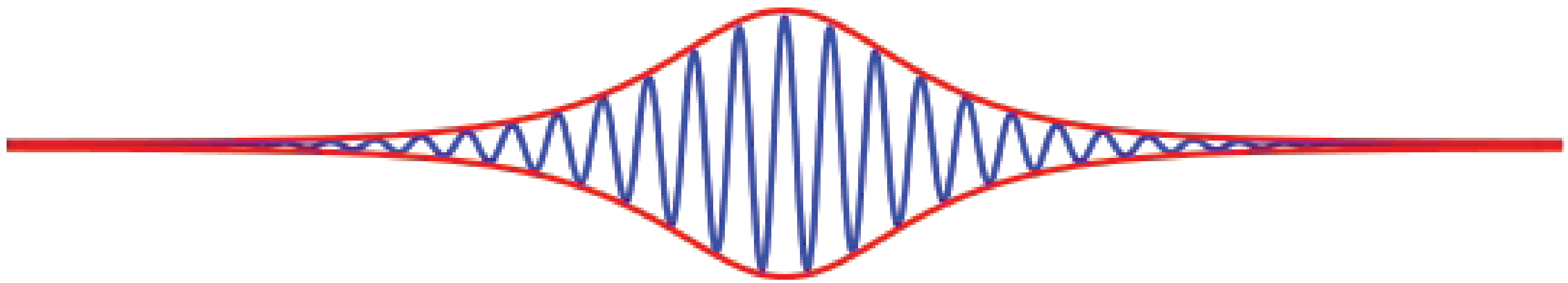
•Nonlinear stage of the modulation instability in the NLSE

leads to creation of solitons of the type:

$$C(z,t) = \sqrt{\frac{2}{|T|}} \frac{\lambda \exp(i\lambda^2 t)}{\cosh(\sqrt{2}\lambda z/\sqrt{\omega''})} \qquad (4)$$

– Blue line below shows wave amplitude $\eta(x, t_0)$,

– Red line shows its envelope: NLSE soliton (4):



We should notice that NLSE (3) is fully-integrable and has infinite number of integrals of motion. As a result, all solitons are globally stable, they do not exchange their energy in their collisions, therefore nonlinear accumulation of the NLSE-soliton energy is prohibited. This makes an expanation of the FWs creation in the NLSE approximation to be problematic.

A way out was suggested by Dyachenko and Zakharov (DZ),: one has to relax weak-nonlinearity approximation, leading to the NLSE (3) and to study

$\Downarrow$ the wave dynamics in framework of full Euler Equation (1) $\Downarrow$

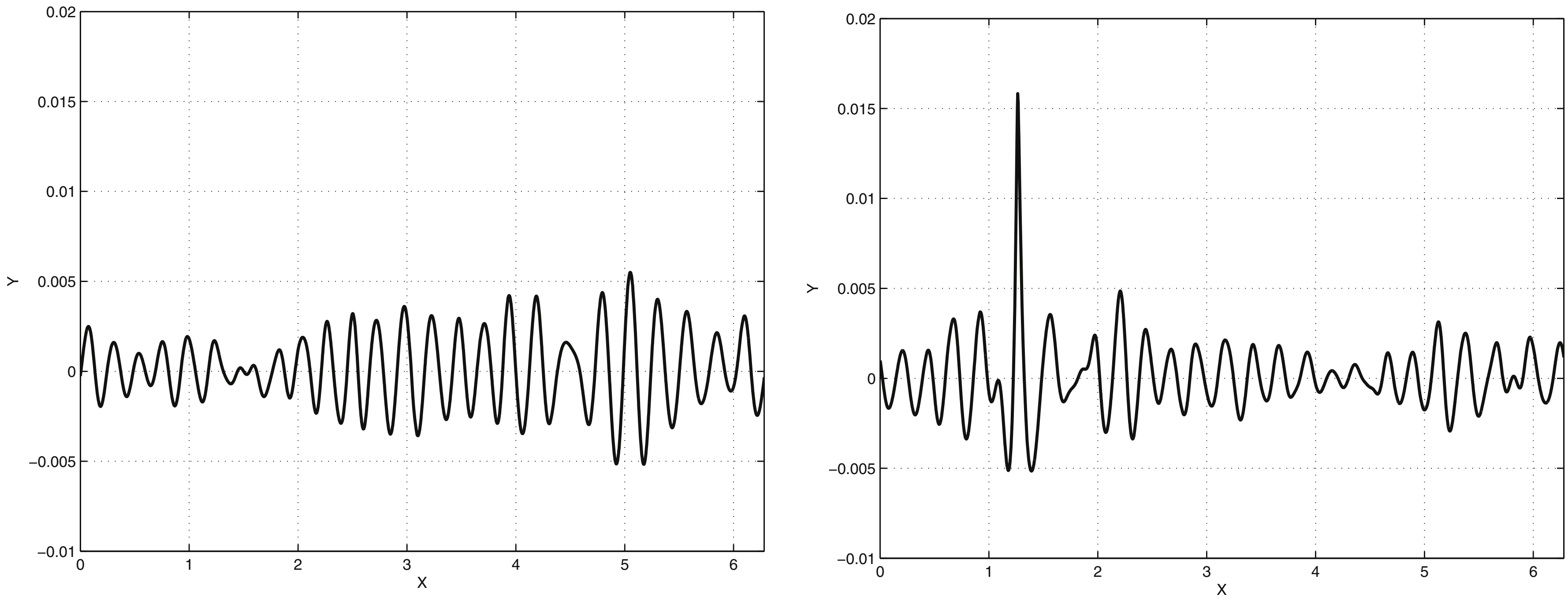
- ***Gravity-wave dynamics in framework of Euler Equation (EE)***

Notice that NLSE is valid for the wave steepness $\boldsymbol{\mu \equiv \sqrt{\langle|\nabla\eta|^2\rangle} \leq 0.05}$, while in the windy sea $\boldsymbol{\mu \approx 0.1}$ and in the FWs $\boldsymbol{\mu > 0.2}$.

DZS [1] solved numerically EE (1) for potential 2d $(x \rightarrow, z \downarrow)$ and show that:

– From (random) initial conditions with $\mu \approx 0.045$ freak waves often appears after a short time $\approx 500T$;

– Maximal probability of FW generation correspond to $\mu \simeq 0.045$



Left: Initial profile, $\mu \approx 0.056$; Right: FW profile, $\max|\nabla\eta| = 0.558$

[1] 1. A. **D**yachenko and V. **Z**akharov, ZhETF Letts, **88**, 356 (2008), **ZD** & R. **S**havin Eur. Phys. J. Special Topics **185**, 113124 (2010)

• *Conclusion and Perspectives*

– Freak Waves (FWs) are very common and natural physical phenomenon.

– Accumulation of wave energy starts from the modulation instability of narrow packet of intensive long gravity waves, which results in the phenomenon of "tenth wave", chain of propagating solitons, the stable solutions of the Nonlinear Schrödinger Equation (NLSE). Additional spatial and/or temporal focusing may increase level of nonlinearity above applicability level of NLSE and result in further nonlinear energy focusing that ends up with formation of FWs, breaking or nonbreaking.

– In a sense, the main question is not how FWs can be generated, but why they are not so often, as one expects from the presented model, or, in other words:
"Why traveling by sea is possible at all?

– The proposed answer is as follows:
"The real energy spectra of the sea and ocean waves have long power-like tail of short waves, that serves as efficient mechanism of the FW-energy dissipation."