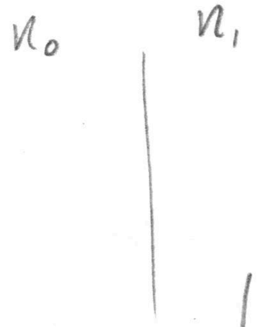
Free space and guided wave optics

Tutoriall 2

26.04.2017

Stratified Media

- A medium with stacked layers.
- The main question here is how can we design such a stack to get a specific trans. spectrum?
- You saw in lectures



$E_{\parallel}$ and $\frac{dE_{\parallel}}{dz}$ (*) are continous!

(*) $\vec{H} = \vec{\nabla} \times \vec{E} = \hat{k} \times \hat{E} \frac{\partial E}{\partial z}$

Therefore, we define:

$$\begin{pmatrix} E \\ F \end{pmatrix} = \begin{pmatrix} E \\ \frac{\partial E}{\partial z} \end{pmatrix}$$

In a 1D case:

$$E = E_+ e^{ikz} + E_- e^{-ikz}$$
$$F = ik E_+ e^{ikz} - ik E_- e^{-ikz}$$

How does free prop. look?

At $\underline{z=0}$: $E(0) = E_+ + E_-$ $\Rightarrow$ $2ikE_+ = ikE(0) + F(0)$

$F(0) = ikE_+ - ikE_-$ $\qquad$ $2ikE_- = ikE(0) - F(0)$

Use in z to get

$$E(z) = \left(\frac{1}{2}E(0) + \frac{F(0)}{2ik}\right)e^{ikz} + \left(\frac{1}{2}E(0) - \frac{F(0)}{2ik}\right)e^{-ikz}$$

$$= E(0)\cdot\cos(kz) + F_0\,\frac{\sin(kz)}{k}$$

Similarly for F:

$$F = E(0)\left(-k\sin(kz)\right) + F_0\cos(kz)$$

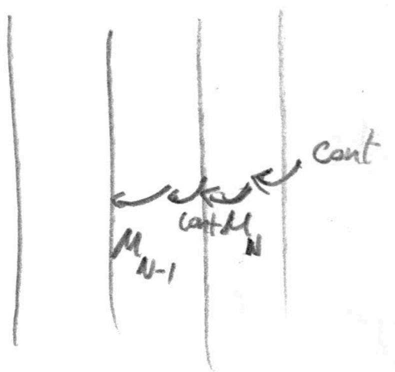
In matrix form:

$$\begin{pmatrix} E(z) \\ F(z) \end{pmatrix} = \underbrace{\begin{pmatrix} \cos(kz) & \frac{1}{k}\sin(kz) \\ -k\sin(kz) & \cos(kz) \end{pmatrix}}_{\text{Det}=1} \begin{pmatrix} E(0) \\ F(0) \end{pmatrix}$$

M^{-1}

$$\boxed{\begin{pmatrix} E(0) \\ F(0) \end{pmatrix} = \begin{pmatrix} \cos(kz) & -\frac{1}{k}\sin(kz) \\ k\sin(kz) & \cos(kz) \end{pmatrix} \begin{pmatrix} E(z) \\ F(z) \end{pmatrix}}$$

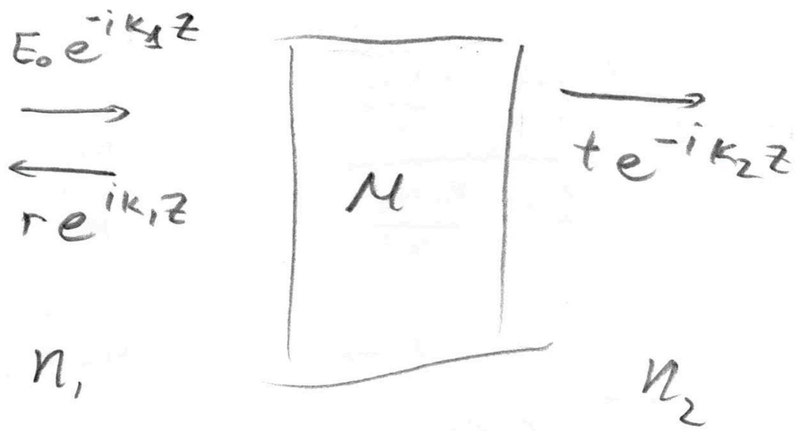
This is great since we can already propogate through the stack:



- The surf. are cont.
- Each film adds another prop. matrix

(*) Before we calculate such matrices, what can we do with it?

Let's take a general case



How do we get r?

On the left: (z=0) $E = (1+r)E_0$, $F = ik_1(r-1)E_0$

On the right $E = tE_0$, $F = -ik_2 t E_0$

$$\begin{pmatrix} 1+r \\ ik_1(1-r) \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} t \\ -ik_2 t \end{pmatrix}$$

And we can find expressions for r, t as a function of m_{ij}.
We will do it soon for a particular case.

(*) Going back to $\mathcal{M}$:

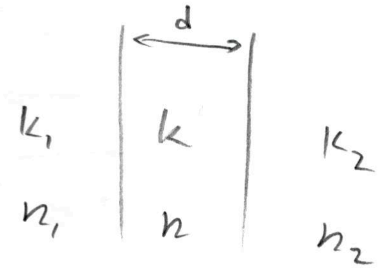
Example: single layer for anti-reflection

For destructive interference: $kz = \frac{\pi}{2}$ $\left(z = \frac{\lambda}{2}\right)$

$$\mathcal{M} = \begin{pmatrix} 0 & -\frac{1}{k} \\ k & 0 \end{pmatrix}$$

For this case (or every case where $m_{11} = m_{22} = 0$)

$$r = - \frac{1 - \frac{k_1 k_2}{k^2}}{1 + \frac{k_1 k_2}{k^2}}$$



$k_1 = n_1 k_0$ $\quad k = n k_0$ $\quad k_2 = n_2 k_0$

$$\boxed{r = - \frac{1 - \frac{n_1 n_2}{n^2}}{1 + \frac{n_1 n_2}{n^2}}}$$

So for $n = \sqrt{n_1 n_2}$ we get $r = 0$

Two problems arise in this case:

① $r=0$ only for normal incidence and specific λ

② practically $n_1 = 1$ $n_2 \cong 1.5$ $\Rightarrow n \sim 1.2$

But it's hard to come by such materials

$n_{MgF_2} \cong 1.38$

This gets you from 4% (air $\rightarrow$ glass) to

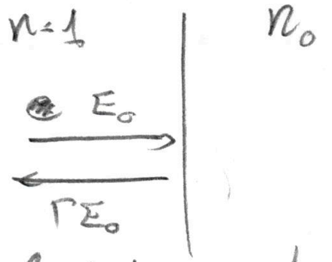
$\sim 1.5\%$

To do better you need a bigger stack.

Drude model and reflection from a metallic surface

We have seen for normal incidence light

(keep) $$r = \frac{1-n_0}{1+n_0}$$



But we have looked only at dielectrics where

$$\begin{cases} n \text{ is real} \\ n > 1 \end{cases}$$

A common thing is using a metallic reflector

The refractive index of a metal

There are some free electrons in a metal.

There EOM is given by

$$\frac{d^2x}{dt^2} - \gamma\frac{dx}{dt} = \frac{eE_0}{m}e^{+i\omega_0 t}$$

acceleration — friction — force by electric field

Use:

$$x(t) = x(\omega_0)\,e^{+i\omega_0 t}$$

And get

$$x(t) = \frac{eE_0/m}{-\omega^2 + i\omega\gamma}e^{+i\omega_0 t}$$

What is the conducitivity?

$$\sigma E = J = N e v = N e \frac{dx}{dt} = \frac{\frac{Ne^2 E_0}{m}}{-\omega_0^2 + i\omega_0 \gamma} e^{+i\omega_0 t} \cdot (+i\omega_0)$$

free electron density (N), velocity (v)

In maxwell's eq.

$$\nabla \times H = J + \varepsilon_0 \frac{\partial E}{\partial t}$$

Assuming no extra dipoles (ε_0)

Going through the usual steps: $\left(\nabla \times \bar{E} = -\frac{\partial \bar{H}}{\partial t} \right)$

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{E}) = -\nabla^2 \bar{E}$$

$$-\nabla^2 E = \frac{\partial J}{\partial t} \quad \varepsilon_0 \frac{\partial^2 E}{\partial t^2}$$

Taking $E(r,t) = E(r) e^{-i\omega t}$

In dielectrics $J \to 0$ but in a metal this becomes important

$$-\nabla^2 E(r) = \left(-i\omega\sigma - \omega^2 \varepsilon_0 \right) E(r)$$

We got the wave eq with an important correction for a conductor. For a plane wave $E = E_0 e^{ikz}$

$$\frac{k^2}{\omega^2} = \underbrace{\left(\varepsilon_0 + \frac{i\sigma(\omega)}{\omega} \right)}_{\text{complex dielectric function}}$$ (keep)

Going back to the Drude model:

$$\sigma = \frac{\sigma_0}{1 - i\omega\tau} \qquad \tau \equiv 1/\gamma$$

$$\sigma_0 \equiv \frac{Ne^2\tau}{m}$$

Divide to image and Re part the dielectric func:

$$\varepsilon_r = \varepsilon_0 + \frac{i\sigma_0}{\omega - i\omega^2\tau} =$$

$$\equiv \left(\varepsilon_0 - \frac{\sigma_0\tau}{1+\omega^2\tau^2}\right) + \frac{i\sigma_0}{\omega(1+\omega^2\tau^2)}$$

For optical freq. $\omega \sim 10^{15}$ Hz

$$\varepsilon_r \cong \varepsilon_0 + \frac{i}{\omega}\frac{\sigma_0}{\cancel{1} - i\omega\tau} = \varepsilon_0 - \frac{\sigma_0}{\omega^2\tau}$$

$$\equiv \varepsilon_0\left(1 - \frac{\omega_p^2}{\omega^2}\right) \qquad \omega_p^2 \equiv \frac{Ne^2}{\varepsilon_0 m}$$

plasma frequency

ω_p in silver is at UV.

So in visible $\omega < \omega_p$

$$\underline{\underline{\varepsilon_r < 0}}$$

Remember

$$\frac{n^2}{c^2} \equiv \varepsilon \cdot \mu \equiv \varepsilon_0\mu_0 \cdot \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

$\Downarrow$

$\boxed{n = \text{purely imaginary}}$

And what about reflections?

$$R = |r|^2 = \left|\frac{1-n}{1+n}\right|^2 = \left|\frac{1-i\alpha}{1+i\alpha}\right|^2 = \frac{1+\alpha^2}{1+\alpha^2} = \underline{1}$$

And therefore a metal acts as a perfect reflector.

In practice:

Aluminum : Good for UV until 250nm
Not good for vis

Silver : Most common
~98% in vis and NIR

Gold: Good for NIR and FIR
Absorptive in most visible