Guided wave optics

The slab waveguide from a geometrical optics point of view – modes

The slab waveguide from a wave optics point of view

Single-mode waveguides ; Multimode waveguides ; Dispersion curves

The step index optical fiber – 1D waveguide in cylindrical coordinates

Mode dispersion ; material dispersion ; polarization in optical fibers

Guided wave devices – couplers, junctions, mach-zender, ring resonator

The slab waveguide from a geometrical optics point of view.

Lets consider the simplest case of a waveguide – A layer of material with refractive index n1 sandwiched between two layers of material of refractive index n2, where n1>n2.

The total internal reflection angle is given by:

$$n_1 \sin(\pi/2-\varphi) \geq n_2$$

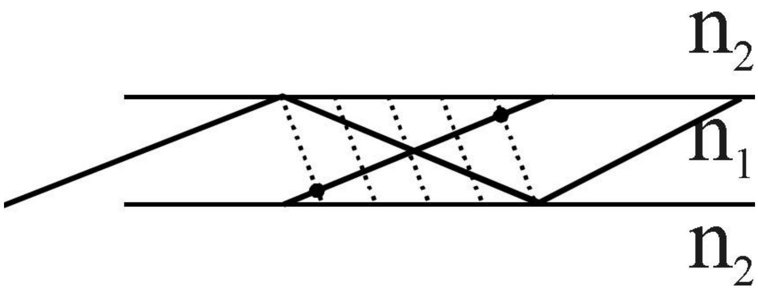
Since applying Snell's law upon entrance to the waveguide relates θ, the angle on the outside to φ by:

$$\sin\theta = n_1 \sin(\varphi)$$

we get:

$$\sin\theta = \sqrt{n_1^2 - n_2^2} \approx n_1\sqrt{2\Delta}$$

Defining Δ as the index contrast Δn/n. Typically, angles are small so that sinθ=θ.



In considering the mode structure of waveguides we must solve for a self-consistent (phase matched) solution for a plane wave propagating at an angle to the waveguide. In doing this we have to take into account the phase shift following total internal reflection (remember polarization, beginning of semester).

This condition gives:

$$\left(kn_1 l_2 + 2\Phi\right) - kn_1 l_1 = 2m\pi$$

Since:

$$l_1 = 2a\left(\frac{1}{\sin\phi} - 2\sin\phi\right); l_2 = \frac{2a}{\sin\phi}; \Phi = -2\tan^{-1}\sqrt{\frac{2\Delta}{\sin^2\phi} - 1}$$

from which we get the condition:

$$\tan\left(kn_1 a\sin\phi - \frac{m\pi}{2}\right) = \sqrt{\frac{2\Delta}{\sin^2\phi} - 1}$$

for allowed (discrete) angles.

The mode for which m=0 is called the fundamental mode and other solutions are higher order modes. These do not necessarily exist. To determine the cutoff for their existence we can rewrite the above equation in normalized coordinates.

Defining

$$\xi = \frac{\sin\phi}{\sqrt{2\Delta}}$$

and

$$v = kn_1 a\sqrt{2\Delta}$$

we can rewrite the phase matching equation as:

$$v = \frac{\cos^{-1}\xi + m\pi/2}{\xi}$$

The normalized frequency v depends only on the wavelength and the waveguide parameters.

Since for ξ=1 the arcos term is zero, there is always a bound m=0 solution for any finite value of v. The existence of the m=1 solution requires $v>\pi/2$. Thus the cutoff wavelength for single mode operation of a waveguide is:

$$\lambda_c = 4an_1\sqrt{2\Delta}$$

The slab waveguide from a wave optics point of view.

Formally, we need to solve Maxwell's equations again, for the case of a slab waveguide. Unlike our discussion of Fresnel propagation we now have to deal with the different indices of refraction in the various parts. Let us assume now a waveguide with a core refractive index of n1 and thickness 2a, embedded in a cladding material with a refractive index n0.

Maxwell's equations are:

$$\nabla \times \vec{E} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\nabla \times \vec{H} = n^2 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

and we assume a solution of the form:

$$\vec{E} = \vec{E}(x)e^{i(\omega t - \beta z)}$$
$$\vec{H} = \vec{H}(x)e^{i(\omega t - \beta z)}$$

This gives two independent solutions of the general form:

$$\frac{d^2 E_y}{dx^2} + \left(k^2 n^2 - \beta^2\right)E_y = 0$$

with Hx,Hz components derived from Ey and the other three components are zero (these are called TE modes)

and:

$$\frac{d}{dx}\left(n^2 \frac{dH_y}{dx}\right) + \left(k^2 - \frac{\beta^2}{n^2}\right)H_y = 0$$

with Ex,Ez components derived from Hy and the other three components are zero (these are called TM modes).

Let us now solve in more details for the TE case.

We assume a solution of the form:

$$E_y = \begin{cases} A\cos(\kappa a - \phi)e^{-\sigma(x-a)} & x > a \\ A\cos(\kappa x - \phi) & |x| < a \\ A\cos(\kappa a + \phi)e^{\sigma(x-a)} & x < -a \end{cases}$$

With:

$$\kappa = \sqrt{k^2 n_1^2 - \beta^2}$$
$$\sigma = \sqrt{\beta^2 - k^2 n_0^2}$$

From the continuity conditions of Ey and its derivative at +-a, we get:

$$\kappa \sin(\kappa a + \phi) = \sigma \cos(\kappa a + \phi)$$
$$\kappa \sin(\kappa a - \phi) = \sigma \cos(\kappa a - \phi)$$

or:

$$\tan(u + \phi) = \frac{w}{u}$$

$$\tan(u - \phi) = \frac{w}{u}$$

where u=κa and w=σa. Overall we get:

$$u = \frac{m\pi}{2} + \arctan\left(\frac{w}{u}\right)$$
$$\phi = \frac{m\pi}{2}$$

These equations can be easily generalized for the case of an asymmetric waveguide (upper and lower cladding having different refractive index).

It is important to note that the other restriction on the values of u and w is that:

$$u^2 + w^2 = k^2 a^2 \left(n_1^2 - n_0^2\right)$$

Thus, we can geometrically determine the dispersion relation and the mode structure of the waveguide, plotting w vs. u, and finding the crossing points with a circle whose diameter is v. For each point, by knowing u and w, we can also determine the propagation constant β.

Clearly, single-mode operation is only achieved for v<vc=π/2.

Considering the power distribution in the waveguide we get the following: Since f jumps by p/2 for each additional mode, the distribution of intensity in the waveguide has an antinode (node) for odd (even) mode numbers. The fraction of the power contained in the core vs the cladding can also be calculated.

2D confined waveguides

The solution for a 2D rectangular waveguide closely follows that of the 1D one, separating the solution to two separate dependencies on x and y.

Rib waveguides are a very common type of waveguide which cannot be easily solved in this approximation. The utility of Rib waveguides is due to the ease of fabrication by lithographic techniques. In this case, the usual way of handling the solution is by the effective index method. Since we need to solve the equation:

$$\frac{d}{dx}\left(n^2 \frac{dH_y}{dx}\right) + \frac{d}{dy}\left(n^2 \frac{dH_y}{dy}\right) + \left(k^2 - \frac{\beta^2}{n^2}\right)H_y = 0$$

which is difficult, we make the Ansatz of separation of variables:

$$H_y(x, y) = X(x)Y(y).$$

This leads to the following equation:

$$\frac{1}{X}\frac{d^2X}{dx^2}+\frac{1}{Y}\frac{d^2Y}{dy^2}+\left[k^2n^2(x,y)-\beta^2\right]=0$$

We now introduce an effective index neff(x), such that we get:

$$\frac{1}{X}\frac{d^2X}{dx^2}\left[k^2n_{eff}^2(x)-\beta^2\right]=0$$
$$\frac{1}{Y}\frac{d^2Y}{dy^2}\left[k^2n^2(x,y)-k^2n_{eff}^2(x)\right]=0$$

From which we can derive both the effective index and the full solution as piecewise constant functions.

Briefly go over types of fibers:

Step index
Polarization maintaining
Gradient index
Omniguides
Hollow core
PCFs (air guiding, index guiding).

Full analysis of the step-index optical fibers

Basically, in this case we need to solve Maxwell's equations in cylindrical coordinates.

$$\vec{E}=\vec{E}(r,\theta)e^{i(\omega t-\beta z)}$$
$$\vec{H}=\vec{H}(r,\theta)e^{i(\omega t-\beta z)}$$

We therefore get two sets of equations:

$$\frac{\partial^2E_z}{\partial r^2}+\frac{1}{r}\frac{\partial E_z}{\partial r}+\frac{1}{r}\frac{\partial^2E_z}{\partial\theta^2}+\left(k^2n^2-\beta^2\right)E_z=0$$
$$\frac{\partial^2H_z}{\partial r^2}+\frac{1}{r}\frac{\partial H_z}{\partial r}+\frac{1}{r}\frac{\partial^2H_z}{\partial\theta^2}+\left(k^2n^2-\beta^2\right)H_z=0$$

From which we can derive all other quantities:

$$E_r = -\frac{i}{k^2n^2 - \beta^2}\left(\beta\frac{\partial E_z}{\partial r} + \frac{\omega\mu_0}{r}\frac{\partial H_z}{\partial \theta}\right)$$
$$E_\theta = -\frac{i}{k^2n^2 - \beta^2}\left(\frac{\beta}{r}\frac{\partial E_z}{\partial \theta} - \omega\mu_0\frac{\partial H_z}{\partial r}\right)$$
$$H_r = -\frac{i}{k^2n^2 - \beta^2}\left(\beta\frac{\partial H_z}{\partial r} - \frac{\omega\varepsilon_0 n^2}{r}\frac{\partial E_z}{\partial \theta}\right)$$
$$H_\theta = -\frac{i}{k^2n^2 - \beta^2}\left(\frac{\beta}{r}\frac{\partial H_z}{\partial \theta} + \omega\varepsilon_0 n^2\frac{\partial E_z}{\partial r}\right)$$

There are "simple" TE and TM solutions for these (Ez=0 or Hz=0) as well as hybrid solutions, where both Ez and Hz are nonzero.

Lets start by looking at the TE solution:

Hz has the form:

$$H_z = \begin{cases} g(r)\cos(m\theta + \psi) & r < a \\ h(r)\cos(m\theta + \psi) & r > a \end{cases}$$

and the radial equation gives:

$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r}\frac{\partial H_z}{\partial r} + \left(k^2n^2 - \beta^2 - \frac{m^2}{r^2}\right)H_z = 0$$

Continuity conditions give:

$$g(a) = h(a)$$
$$\frac{i\beta}{\left[k^2 n(a)^2 - \beta^2\right]}\frac{m}{a}g(a)\sin(m\theta + \psi) = \frac{i\beta}{\left[k^2 n_0{}^2 - \beta^2\right]}\frac{m}{a}h(a)\sin(m\theta + \psi)$$

Since the first term is not similar, for this to hold for all values of theta we need to have m=0. Hence, Er=Hθ=0.

We finally get, then:

$$\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r}\frac{\partial H_z}{\partial r} + \left(k^2n^2 - \beta^2\right)H_z = 0$$
$$E_\theta = \frac{i\omega\mu_0}{\left[k^2 n(r)^2 - \beta^2\right]}\frac{dH_z}{dr}$$
$$H_r = -\frac{i\beta}{\left[k^2 n(r)^2 - \beta^2\right]}\frac{dH_z}{dr}$$

Defining:

$$\kappa = \sqrt{k^2 n_1^2 - \beta^2}$$
$$\sigma = \sqrt{\beta^2 - k^2 n_0^2}$$

The equation for Hz becomes:

$$\frac{d^2 g}{dr^2} + \frac{1}{r}\frac{dg}{dr} + \kappa^2 g = 0$$
$$\frac{d^2 h}{dr^2} + \frac{1}{r}\frac{dh}{dr} - \sigma^2 h = 0$$

The solutions of the first are zeroth order Bessel and Neumann functions, of which only the Bessel function is finite at r=0.
The solutions for the second are Bessel functions of the first kind, of which only one does not diverge at infinity. The solution is thus:

$$H_z = \begin{cases} AJ_0(\kappa r) & r < a \\ BK_0(\sigma r) & r > a \end{cases}$$

The continuity conditions can now be expressed using the normalized variables u=κa and w=σa as:

$$\frac{J_1(u)}{uJ_0(u)} = -\frac{K_1(w)}{wK_0(w)}$$

A similar analysis can be performed for the TM modes.

Notably, linearly polarized propagation is NOT strictly a mode in an optical fiber. However, under the approximation Δn<<n, there are groups of TE, TM and hybrid modes which have the same dispersion relation. Thus, for each of these groups an alternate basis can be found. In this approximation, therefore, LP modes are also approximate solutions of the above equations.

Dispersion in optical fibers:

The signal delay in an optical fiber follows:

$$t = \frac{L}{v_g} = \left(\left.\frac{d\beta}{d\omega}\right|_{\omega_0} + (\omega - \omega_0) \left.\frac{d^2\beta}{d\omega^2}\right|_{\omega_0} \right) L$$

There are a variety of sources of dispersion:

1. Material dispersion
2. Waveguide dispersion

also, for non-single-mode case:

3. Polarization mode dispersion
4. Multimode dispersion

Typical parameters for step-index fibers – the zero-dispersion wavelength is about 1.3 microns. Typical values for the visible range are 100s ps/nm*km. Talk a bit about dispersion shifted fibers, dispersion compensating fibers and dispersion flattened fibers (core/clad/2nd region of intermediate refractive index) for canceling the effects of waveguide and material dispersion.

A few words about devices:

Y junction (splitter)
Beamsplitter
Mach-Zehnder interferometer
MMI device (coupling from one mode to another in waveguides)
Ring resonator
Fiber Bragg grating