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The Dynamics and Interaction of Laminar Thermal Plumes.

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Abstract. – We present an experimental study of the dynamics and interactions of laminar plumes emitted from a localized heat source. The observations are explained by a simple model of the flow structure around a plume. Using sources and sinks in a uniform flow, we reproduce the experimental shapes and extract the scaling behavior of the size of the plume. The model describes the initial stage of the interaction between plumes.

The shape of the interface between one fluid penetrating into another is a problem of pattern formation that has received much attention lately [1]. By now classical examples of similar systems are those of dendritic growth, directional solidification, and of Saffman-Taylor fingering. Thermal plumes, or convection from a localized heat source, where hot fluid penetrates into the colder region above, is another system which displays the formation of a stable, large amplitude, essentially nonlinear object. It differs from the other examples in that the fluids on the two sides of the interface are essentially the same. This form of flow occurs abundantly in nature, for example in problems of mantle motion, cloud formation and in turbulent convection.

The physical problem of a heat source in a fluid initially at rest is appealing in its conceptual and experimental simplicity. To excite the plumes we ran a d.c. current through a resistor immersed in water. When the resistor heats up, it creates a column of rising hot fluid (the stem). The tip of the stem is a propagating front, in the form of a cap. The cap grows due to an excess of the vertical velocity in the stem $v_s$ over the vertical velocity $v_c$ of the cap. We have used a variety of heater sizes, ranging from diameters of 150 μm to 10 mm, with no essential difference in the form of the emitted plumes. The measurements described below were taken using a 100 Ω, 1/8 W metal film resistor 2 mm in diameter and 6 mm long placed in water in a (10 × 10 × 20) cm³ covered glass container. We used standard computer enhanced shadowgraph techniques to visualize the flow.

Figure 1 shows the evolution of such a plume. Once the power is switched on the heat diffuses outwards from the heater and a boundary layer begins to form. The boundary layer then breaks, and a spherical protuberance appears (Fig. 1a), as fluid begins to flow up. The plume rises at a constant velocity $v_c$ [2], while the cap size grows in time.
Fig. 1. – A rising plume superposed by a fit to the Rankine fairing. $P = 1.25$ W. a) $t/\tau = 0.03$, $a/R = 1.4$; b) $t/\tau = 0.22$, $a/R = 2.4$; c) $t/\tau = 0.40$, $a/R = 3.1$; d) $t/\tau = 0.55$, $a/R = 3.8$; d') $t/\tau = 0.55$, $a/R = 4.1$, $d/R = 5.5$.

We put our results in nondimensional form by using $R$, $\tau = R^2/\kappa$ and $P/(\rho C_p \kappa R)$, as units of length, time and temperature, respectively. Here $R$ is the heater size, $P$ the power, $\rho$ the fluid density, $\kappa$ its thermal diffusivity, and $C_p$ its specific heat. A detailed justification of the ensuing scalings will be given elsewhere [3]. For the cap velocity we find

$$\frac{v_c}{\kappa/R} = c_1 \left(\frac{\alpha g PR^2}{\nu \rho C_p \kappa^2}\right)^{\varepsilon}$$

with $\varepsilon = 0.50 \pm 0.03$ and $c_1 = 0.2 \pm 0.02$. Here $\nu$ is the kinematic viscosity, $\alpha$ the thermal expansion coefficient and $g$ the gravitational acceleration.

Plumes (produced by a continuous heat input at a point) and thermals (one-shot input of heat) have been studied extensively in the past, most notably by Taylor in the context of an atomic explosion [4]. Most previous work on plumes has centered on the stem in steady state [4-9b]. Taylor used a scaling argument, relying on geometrical similarity of the solution at different times [5-7]. No attempt, however, has been made to explain the shape of the cap. Thermals have been treated [6] as buoyant vortex rings [8], but applying this approach to plumes [7] does not yield a constant $v_c$. A height-independent $v_b$ was found in an analytical solution of similarity equations for the stem under special conditions [10].

In what follows we introduce a new approach to the cap of the plume, describing it as a source and sink in a uniform flow [11], its border defined by the resulting Rankine shape. We use this shape to measure new scaling laws for the plume size, which indicate that geometrical similarity of the whole plume does not exist. We then turn to interactions between plumes, where we observe that two plumes coalesce and form a new plume. Continuing in this line, we find that four plumes go to two plumes that go to one plume, signaling an inverse cascade. These interactions are accounted for in terms of dynamics of sources and sinks.

To obtain a model for the cap, we consider a rising plume in its rest frame. There the ambient fluid is pushing down at a constant velocity $-v_c$, separated from a «source flow» emanating from the stem into the cap. Such a flow was considered by Rankine [12] for...
building better ("streamlined") ships. Rankine used a simple potential flow: the superposition of a point source and a uniform flow. The corresponding velocity potential (with the source at the origin and the uniform flow in the z direction) is \( \Phi = vz - J/4\pi r \), where \( v \) is the uniform velocity, \( J \) the strength of the source and \( r \) the distance from it. This flow has a stagnation point on the z-axis at \( z = a/2 \), where \( a = (J/\pi v)^{1/2} \). The streamline through this point defines a cigarlike shape of semi-infinite extent (the Rankine fairing). This is a dividing line of the flow, in the sense that all the fluid emitted from the source stays within it. The Rankine fairing is completely determined by the parameter \( a \), with the width of the cigar quickly asymptoting to \( 2a \).

In fig. 1 the corresponding fits to a Rankine fairing are superposed on pictures of the plume at different times. The single fit parameter at each time is the length \( a \). The time evolution of \( a \) also gives the evolution of the size of the plume, and is given below (fig. 2). We have checked that fitting to different models yields significantly worse fits.

There is a basic difference between the plume and the Rankine fairing, in that a plume does not expel any of its fluid to \(-\infty\). Rather, fluid that is in the cap is still buoyant and continues up in the laboratory frame. A potential flow with this feature is the Rankine ovoid, composed of a source and sink a distance \( d \) apart in a uniform flow. Figure 1d') shows the fit of a Rankine ovoid superposed on a developed stage of the plume, using two parameters (the size \( a \) and the distance \( d \)).

Using one-parameter fits like those of fig. 1a)-d) we can find the behavior of the plume size with time. Figure 2 shows the dimensionless size of the cap \( a/R \) as a function of dimensionless time \( t/(R^2/k) \) for almost two decades of \( P \). The origin of time for each plume shifts due to a power-dependent transient, that occurs between the switch on time for \( P \), and the actual appearance of the plume. The plume appears once the convective transport exceeds the conduction heat transport in the outer part of the forming boundary layer. This is formally the same as assigning a critical Rayleigh number \( Ra_c \) for the boundary layer to break [14], and estimating the transient time as \( t_0(P) = \delta^2/k \), where \( \delta(P) \) is the size of the boundary layer when it breaks. One then finds \( t_0/(P = (Ra_c \cdot 4\pi R^2 \rho C_p v)/(P g)) \). We adjust \( Ra_c \) so that the points all collapse on one curve. Fitting the data of fig. 2 to the form

\[
\frac{a}{R} = c_2 \left( \frac{t - t_0(P)}{R^2/k} \right)^{\beta},
\]

we find \( \beta = 0.47 \pm 0.05 \), \( c_2 = 4.2 \pm 1.0 \) and \( Ra_c \approx 200 \) for \( R = 0.16 \) cm.

Fig. 2. – Nondimensional size of the Rankine fairing as a function of time. Crosses 0.08 W, rhomboids 0.16 W, circles 0.31 W, squares 0.63 W, triangles 1.25 W, plussed squares 2.45 W, plusses 4.92 W.
In the context of this model we can interpret the value $\beta = 1/2$ by locking at $a = (J/\pi \nu)^{1/2}$. If we assume that $J$ is a measure of how much fluid is circulated within the plume cap, then we need to introduce a characteristic time scale $\tau_c$ for the circulation. The cap is fed through the stem of area $s$, so $J = (1/\tau_c) \int (v_s - v_c) s \, dt'$. Empirically, $v_s$, $v_c$ and $s$ do not depend on $t$. We therefore have $J \sim t$, hence $a \sim t^{1/2}$. If $\tau_c$ grows with $a$ (and $t$), then $\beta$ will be less than $1/2$. We note that $\beta = 1/2$ is predicted by similarity arguments for thermals [6]. To explain the data collapse all we need is that $v_s^2 \sim P$, which implies a simple energy balance. We have also checked that (within our resolution) fitting the plume to a Rankine ovoid does not change this scaling behavior for $a$.

A test of this model is its ability to describe the dynamics of two plumes. The stems of both turbulent and laminar plumes are known to attract [9]. Figure 3 shows the interaction in a system of two laminar plumes. Two heaters were placed at a distance $r_0 = 9$ mm from each other. Equal power was applied simultaneously to both of the resistors, so that two identical plumes were produced. The interaction is simplest for this symmetric situation.

Looking at fig. 3, we see that while the size of the plumes, $a$, is small relative to their distance $r_0$, the interaction is weak and there is essentially no effect (fig. 3a). As the plumes grow the caps begin to tilt. They move away from each other at the top, while coming closer at the bottom (fig. 3b, c). At this point the two plumes begin to penetrate each other (fig. 3d), and this causes a transition in the type of flow. The two plumes coalesce and the base (fig. 3d, e), and begin to form a new structure. The old caps are shed to the side (fig. 3f, g), and a new single front develops. A Rankine shape forms again (fig. 3f-i), but now it is larger and rises at a higher velocity, since it has double the power feeding it. Finally, a single new plume fed by a double stem emerges (fig. 3h-i). By looking at $90^\circ$, we have been able to verify that the process is restricted to the plane that includes both plumes.

In applying the model to this interaction we consider the relative motion that is induced

![Fig. 3. - Two interacting plumes. a) $t/\tau = 0.22$; b) $t/\tau = 0.51$; c) $t/\tau = 0.90$; d) $t/\tau = 1.13$; e) $t/\tau = 1.38$; f) $t/\tau = 1.61$; g) $t/\tau = 1.75$; h) $t/\tau = 1.96$; i) $t/\tau = 2.18$.](image-url)
in each plume by the velocity fields of the other. We represent the plumes by placing two source-sink pairs in a homogeneous flow, at a distance \( r_0 \) from each other. We assign the dynamics that sources and sinks move like particles in the flow, which we find reasonable because these are the dynamics that apply in a neighborhood of the sources and sinks, and by continuity should apply to them also. With these rules sources repel and sinks attract.

Figure 4A) shows the evolution in time \( t \) of this process. Sources (plus signs) and sinks (minuses) move by integrating numerically the velocity field imposed by the other dipole. The stagnation points (circles) are found, and the streamlines emanating from (or culminating at) these points are found by integrating along the instantaneous velocity field of the two dipoles and the uniform flow. The initial conditions used were \( a = 0.28, d = 1, r_0 = 1.5 \). As the source-sink pairs tilt (\( t = 0 \) in fig. 4A)), the Rankine ovoids lose their symmetry. As a consequence, part of the fluid flows in from \( +\infty \) and out to \(-\infty \). Once the plumes are thus distorted they begin to coalesce. At first the two stagnation points at the base of the plume join (\( t = 8 \)), followed by the two sinks (\( t = 9 \)). At this point the model can no longer follow the experiment, that is create a new cap.

A fit, using parameters close to those of fig. 4A), is shown in fig. 4B). This superposition is only supposed to demonstrate a qualitative correspondence between the model and the experiment, and is not the result of a realistic time evolution of the plume (which should include its growth). For example, note that by d) of fig. 4B) the plumes have tilted beyond the angles given by the model. Furthermore, the lower dividing streamline was not observed experimentally.

What happens with more than two plumes? One way to demonstrate that the plume that emerges from two plumes is again a source-sink pair is to look at the interaction between two such objects. To do that we placed four resistors collinearly, pairing them so that the plumes emitted will first interact in pairs. We find that if the flow is completely symmetric and clean of fluctuations, then indeed the four plumes form two plumes that in turn interact to make one plume (with four stems and an interesting inner structure). Preliminary

Fig. 4. – A) Evolution in time of 2 interacting source-sink pairs; B) as in fig. 3a)-d), with superposed source-sink configurations. The tilt is adjusted to fit the experimental data. a) \( a = 0.14, d = 0.45, r_0 = 1 \); b) \( a = 0.22, d = 0.7, r_0 = 1 \); c) \( a = 0.25, d = 0.7, r_0 = 1 \); d) \( a = 0.27, d = 1.2, r_0 = 1 \).
experiments with a linear resistance have shown that a line plume consists of several source-
sink pairs, and destabilizes to axisymmetric plumes.

Finally, if the ideas of potential flow are relevant, then the method of images might be
applicable. To check this we placed a resistor at a distance from a vertical wall that is
precisely one-half the distance between the two interacting plumes of fig. 3, and excited a
single plume of the same power. The motion of half of the two-plume system, including the
tilting process, is reproduced exactly by the single plume near a wall, until approximately
the time at which the sinks start to coalesce. Beyond that time the plume stays tilted at a
constant angle, and the stem adjusts and clings closely to the wall.

The flow inside the plume is not well described by the model. Indeed, we know that there
is vorticity in the cap itself (some rotation can be seen already in fig. 1d)). The success of the
model lies, rather, in a correct description of the velocity field outside the plume.

In summary, we have shown that we can capture the essential features of laminar plumes
by a model of potential flow that utilizes only sources, sinks, and a homogeneous flow. The
complex structures created in the interaction of two plumes can be reconstructed using
these simple ideas. The constant velocity of the rising plume and its growth rate remain a
puzzling question.

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