Statistical Mechanics 2011–12 Exercise on series expansions

This exercise sheet is for anyone who would like to practice expanding functions into power series.

Derive the following expansions for $x \ll 1$:

1.
$$\frac{2\sinh x}{1+2\cosh x} = \frac{2}{3}x - \frac{1}{9}x^3 + O(x^5).$$

2.
$$\log(1 + 2\cosh x) = \log 3 + \frac{x^2}{3} - \frac{x^4}{36} + O(x^6)$$
.

3.
$$2e^{x+2x^2} - e^{2x+2x^2} = 1 + x^2 + O(x^4)$$
.

Derive the following expansion for $x \gg 1$:

1.
$$\sqrt{1-2e^{-x}+11e^{-2x}}=1-e^{-x}+5e^{-2x}+O(e^{-3x})$$
.