Energy Transfer by Inertial waves in Rotating Turbulence

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Statistical Mechanics day
Weizmann inst. 16.6.2010

The context: Want to understand turbulence in 3D rotating systems. Atmosphere, Oceans, Flows within the Earth’s mantle other planetary flows.

Students involved:
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Outline

• The connection between rotating and 2D turbulence
• Inertial waves in rotating fluid
• Energy transfer in rotating turbulence
• Statistics
• A call for wave turbulence description
What is the right framework to describe 3D rotating turbulence?

3D isotropic turbulence – Richardson, Klimogorov, forward energy cascade...

2D isotropic turbulence – Batchelor, Kraichnan, inverse energy cascade...
Rotating 3D turbulence is often described in terms of 2D turbulence

A central question:

The equivalence and differences between rotating and 2D turbulence

The suggestion for equivalence is sometimes justified using Taylor-Proudman’s theorem

\[ \frac{\partial \tilde{u}}{\partial t} + \tilde{u} \cdot \nabla \tilde{u} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \tilde{u} - 2\tilde{\Omega} \times \tilde{u} \quad \Rightarrow \tilde{\Omega} \cdot \nabla \tilde{u} = 0 \]

For \( \tilde{\Omega} = \Omega_z \), we have: \( \frac{\partial}{\partial z} \tilde{u} = 0 \) Quasi 2D
Another question:

What is the mechanism that maintains two dimensionality of the flow? (How energy and momentum are transferred along the axis of rotation?)

\[ \frac{\partial \vec{u}}{\partial t} + \vec{u} \nabla \vec{u} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \vec{u} - 2\vec{\Omega} \times \vec{u} \]

For small and slow perturbations to fluid at rest (in the rotating system)

Coriolis driven inertial waves propagate along the axis of rotation (Greenspan 1968) with group velocity:

\[ v_g = \frac{2\vec{k} \times (\vec{\Omega} \times \vec{k})}{|\vec{k}|^3} \]

The frequency of inertial waves must satisfy:

\[ \omega \leq 2\Omega \]

Are these waves important in turbulent rotating flow?
Experimental system

Ω up to 16 Rad/s

Max. flow rate 3 L/s => ~ 300 W

250 outlets and 70 inlets in hexagonal lattice

1 Mpix, 30 f/s, with less than 1ms “dead time”, for PIV measurements
In a steady state (vorticity field)
Experiment 1

The system is brought to a solid body rotation \((u=0)\) at a given rotation rate \(\Omega\).

At \(t=0\), we start injecting energy at a given flow rate (generating a step function in the injected power)

We measure the horizontal velocity \((u,v)\) field at height \(H\)

Deriving energy power spectrum, \(E(k)\) and energy density “map”, \((u^2+v^2)\), as functions of time.
Energy Density evolution

$t=1.3\, s$

$t=3.3\, s$

$t=4.7\, s$

$t=10\, s$

$t=100\, s$
Sharp “arrival” of energy

Two “Populating fronts” in the spectrum

One is linear in $k$, defining $\tau_1(k)$

The second defines $\tau_2(k)$, which decreases with $k$
Increased energy injection
Increased Rotation
Examples of measurements at lower height:
The more energetic the flow - the shorter $\tau_2$ is
the higher the rotation the shorter $\tau_1$ is (color bars and axis, as above)

Variation of fronts properties with rotation, energy injection and height

Pumping 1 Rotation 1600 mHz Height 44 cm
Pumping 15 Rotation 1600 mHz Height 44 cm
Pumping 20 Rotation 1600 mHz Height 44 cm
Pumping 25 Rotation 1600 mHz Height 44 cm
Pumping 30 Rotation 1600 mHz Height 44 cm
Pumping 1 Rotation 1700 mHz Height 44 cm
Pumping 15 Rotation 1700 mHz Height 44 cm
Pumping 20 Rotation 1700 mHz Height 44 cm
Pumping 25 Rotation 1700 mHz Height 44 cm
Pumping 30 Rotation 1700 mHz Height 44 cm
Pumping 1 Rotation 2000 mHz Height 44 cm
Pumping 15 Rotation 2000 mHz Height 44 cm
Pumping 20 Rotation 2000 mHz Height 44 cm
Pumping 25 Rotation 2000 mHz Height 44 cm
Pumping 30 Rotation 2000 mHz Height 44 cm

Increased energy injection
Increased Rotation
Examples of measurements at lower height:
The more energetic the flow - the shorter $\tau_2$ is
the higher the rotation the shorter $\tau_1$ is (color bars and axis, as above)

Pumping 20 Rotation 1500 mHz Height 44 cm
Pumping 20 Rotation 1500 mHz Height 68 cm
Pumping 20 Rotation 1500 mHz Height 90 cm
Pumping 20 Rotation 1600 mHz Height 54 cm
Pumping 20 Rotation 1600 mHz Height 64 cm
Pumping 20 Rotation 1600 mHz Height 80 cm
Pumping 20 Rotation 1700 mHz Height 44 cm
Pumping 20 Rotation 2000 mHz Height 44 cm
Pumping 20 Rotation 2000 mHz Height 64 cm
Pumping 20 Rotation 2000 mHz Height 80 cm

increased height

infrared camera

Pump

Rotating table

Injection device

Laser sheet

water

$\Omega$

$\Omega$

80 cm

90 cm

h

Log($E$)

Log($E$)

$\tau_1$

$\tau_2$
Scaling of $\tau_1$

The energy carried by an inertial wave of wave number $k$ would arrive at $h$ by a time:

$$t = \frac{h}{v_g} = \frac{hk}{2\Omega}$$

$\tau_1$ is the traveling time of inertial waves to the measuring plane

All the energy transfer along $z$ is done by inertial waves

Waves propagate within existing turbulence

Energy spectrum vs. time

H=25 cm

H=30 cm

H=35 cm

Total Energy vs. time
Are inertial waves significant in steady state?

Energy spectrum seems consistent with 2D turbulence

Measurements by Yuval Vardi
The temporal energy spectrum for different rotation rates

A clear “cutoff frequency for each rotation rate
The cutoff frequency vs. $\Omega$

$\omega_{\text{cutoff}} = 2\Omega$

Consistent with the cutoff in the dispersion relation of inertial waves:

$\omega \leq 2\Omega$

Suggests that inertial waves strongly affect the statistics even in a steady state.
The temporal energy spectrum for different pumping rates

What is the dependence on the dissipation rate?
The injected energy spectrum at different rotations (same pumping rate)

A shift of $k_{\text{max}}$ with $\Omega$
Conclusions

**High amplitude** inertial wave packets can propagate in a turbulent rotating fluid.

They propagate with the group velocity calculated for small perturbations.

In the flow that was studied, **energy is transported by inertial waves**.

**Statistics** of rotating turbulence, contains “finger prints” of inertial waves.

Can we write an “**inertial wave turbulence**” description?
Spectrum evolution over longer times

- time = -7.61 sec
- time = 2.41 sec
- time = 12.38 sec
- time = 26.7 sec
- time = 55.28 sec
- time = 127.6 sec
Vorticity

\[ \omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \]
Scaling by $e^{2/3}$
PIV system

• Using ~50 μm Polystyrene particles (ρ=1.06 g/cm³) for light scattering

• Dividing an image to ~200*200 “cells”, each with ~10 particles

• Calculating the cross correlation between images of different times, \( t \) and \( t+dt \), within each cell

\[
\begin{align*}
\text{Cross correlation} & \quad \longrightarrow \\
\text{dr}(x,y,t) & \quad \longrightarrow \\
\text{u}(x,y,t) & \quad \longrightarrow \\
\text{Image at time } t & \\
\text{Image at time } t+dt
\end{align*}
\]

• Image 1 Megapixel, 10 bit
• Shortest \( dt \sim 1 \) ms (by strobbing the laser)
• \( u_{\text{Max}} \sim 1 \) m/s
Suggested scenario

\[ \tau_1 \sim \frac{H k}{\Omega} \] – linear propagation time

\[ \tau_2 \sim (v k)^{-1} \] – nonlinearities of inertial waves

\[ \tau_3 \sim ? \] – time to achieve steady state

We have dominance of linear behavior for

\[ k < \left( \frac{\Omega}{H v} \right)^{1/2} \]

Or, for heights smaller than

\[ H^* \sim \Omega k^{-2} v^{-1} \]

Above this height we expect to find a region dominated by (resonant?) nonlinear interaction of inertial waves – “wave turbulence”?
“Shooting” turbulence

What happens in a turbulent atmosphere when there is a “sudden” blast of energy from a source?

Intuitively we would guess that most of the energy will accumulate near the source.

However, now we can predict, that the energy will propagate linearly upwards and will be transferred to the turbulent field only at height larger than $H^*$.
Measuring the propagating pulse

The energy that is "left behind" at height H

"Arrival time" at height H

Pulse injection started for 1.5s

The energy that is "left behind" at height H
The energy “left behind” by the pulse vs. height

The energy of the pulse is concentrated at a selected height $H^*$
$H^*$ is determined by $\tau_2$

Find the traveling time to $H^*$ under different conditions
Dependence of $\tau_1$ and $\tau_2$ on injected Power - $P$

$\tau_1$ is independent of $P$ (as expected from a linear mechanism)

$\tau_2$ decreases with $P$ (An indication of the nonlinear nature of the underlying process)
Scaling of $\tau_2$

$\tau_2$ scales roughly as the “eddy turnover time” $- (vk)^{-1}$

Should remember that steady state is achieved at times much longer than $\tau_2$—is there a third time scale?
The experiment

Injecting a pulse of energy into a steady state turbulence field at H=t=0
And measure the velocity field at H>0.
It is known:

Built up of large scales

2D in the large scales (Baroud, Plapp and Swinney, 2003)

But also:

Different energy power spectra:

Baroud et. al. 2002

Smith and Waleffe 1999