# Energy Transfer by Inertial waves in Rotating Turbulence

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**The context**: Want to understand turbulence in 3D rotating systems.
Atmosphere, Oceans, Flows within the Earth's mantle other planetary flows.

Students involved:
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Sergio Lupo

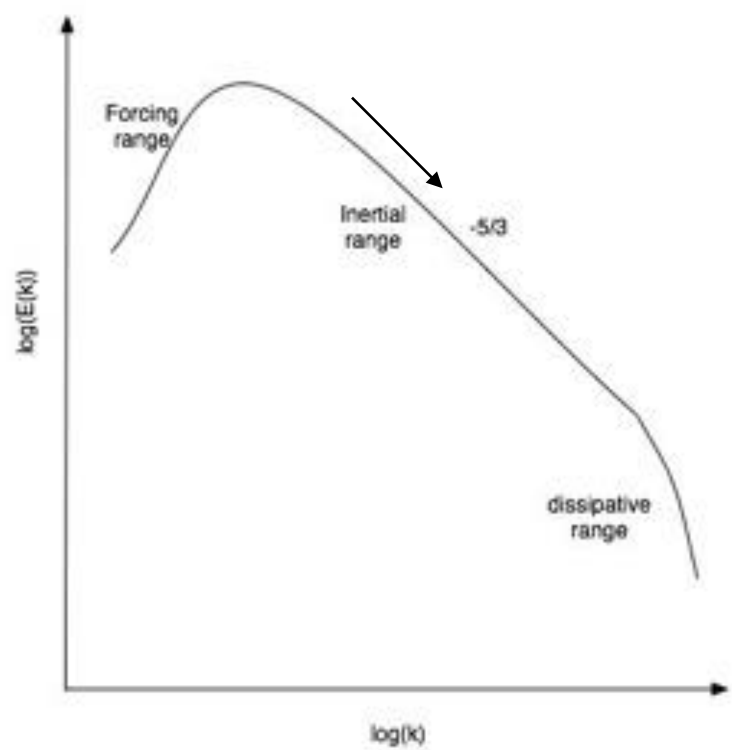
## Outline

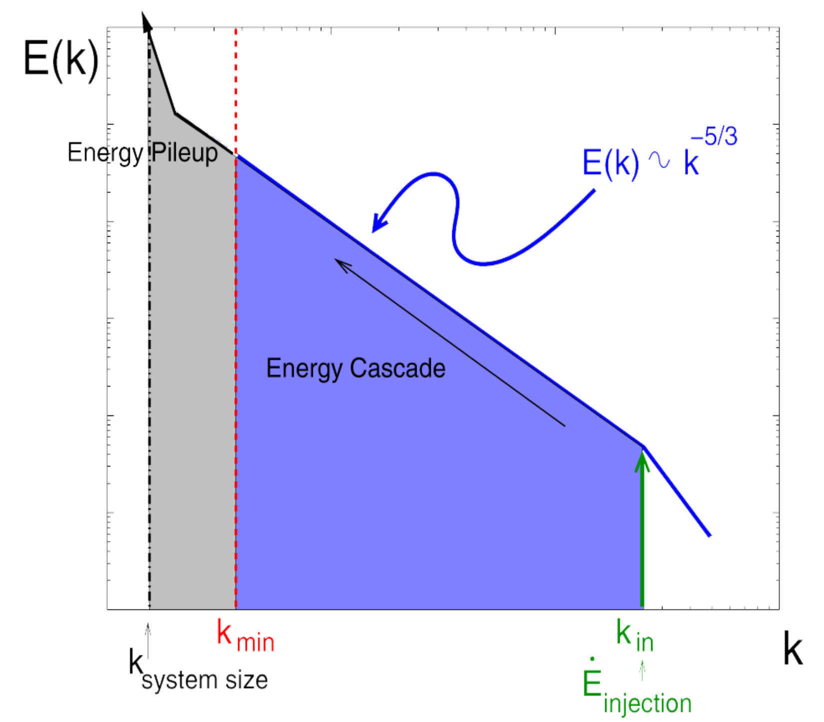
- The connection between rotating and 2D turbulence
- Inertial waves in rotating fluid
- Energy transfer in rotating turbulence
- Statistics
- A call for wave turbulence description

## What is the right framework to describe 3D rotating turbulence?

**3D isotropic turbulence – Richardson, Klmogorov, forward energy cascade…**

**2D isotropic turbulence – Batchelore, Kraichnan, inverse energy cascade…**



## Rotating 3D turbulence is often described in terms of 2D turbulence

# A central question:

## The equivalence and differences between rotating and 2D turbulence

The suggestion for equivalence is sometimes justified using **Taylor-Proudman's theorem**

$$\cancel{\frac{\partial \vec{u}}{\partial t}} + \cancel{\vec{u}\nabla}\vec{u} = -\frac{\nabla p}{\rho} + \cancel{\nu\nabla^2}\vec{u} - 2\vec{\Omega}\times\vec{u} \qquad \Rightarrow \vec{\Omega}\cdot\nabla\vec{u} = 0$$

For $\vec{\Omega} = \Omega_z$ We have: $\frac{\partial}{\partial z}\vec{u} = 0$ Quasi 2D

**Another question:**

**What is the mechanism that maintains two dimensionality of the flow?**
**(How energy and momentum are transferred along the axis of rotation?)**

$$\frac{\partial \vec{u}}{\partial t} + \cancel{\vec{u}\nabla\vec{u}} = -\frac{\nabla p}{\rho} + \cancel{\nu\nabla^2\vec{u}} - 2\vec{\Omega}\times\vec{u}$$

For small and slow perturbations to fluid at rest (in the rotating system)

Coriolis driven inertial waves propagate along the axis of rotation (Greenspan 1968) with group velocity:

$$v_g = 2\bar{k}\times\left(\bar{\Omega}\times\bar{k}\right)/\left|\bar{k}\right|^3$$

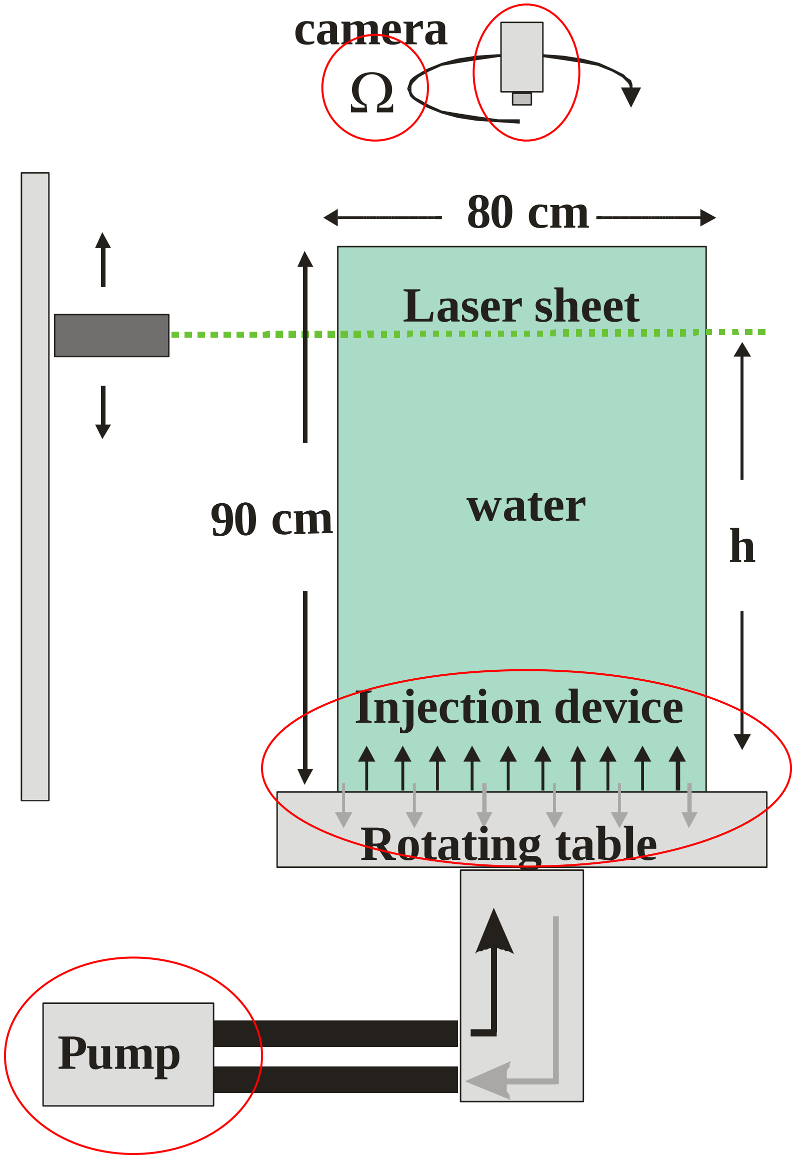
$$\boxed{\left|v_g(k)\right| = \frac{2\Omega}{|k|}}$$

The frequency of inertial waves must satisfy:

$$\boxed{\omega \le 2\Omega}$$

Are these waves important in turbulent rotating flow?

# Experimental system

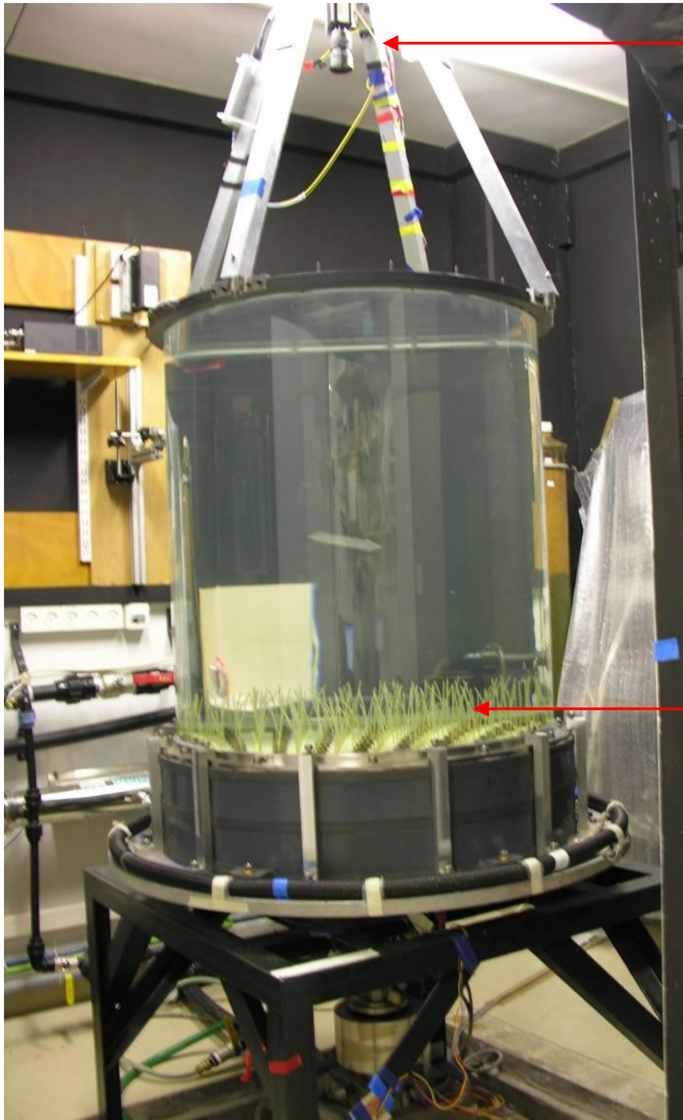



Ω up to 16 Rad/s

Max. flow rate 3 L/s => ~ 300 W

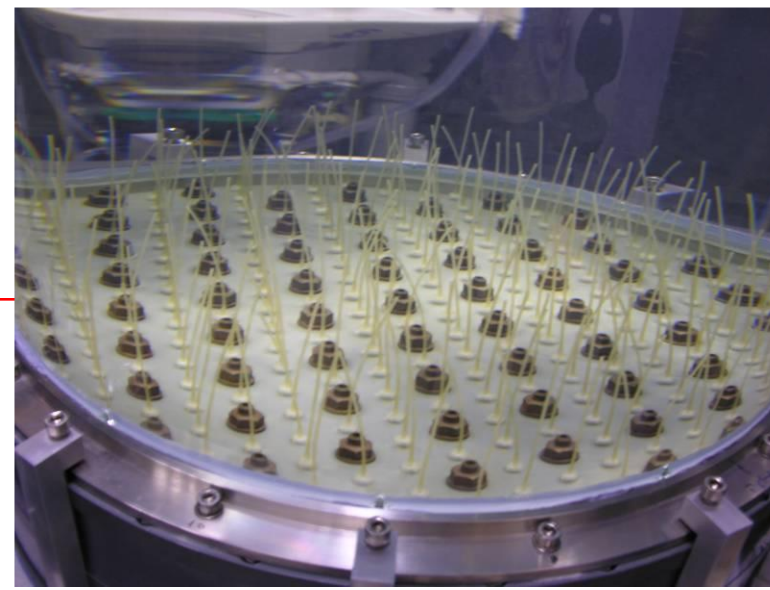
250 outlets and 70 inlets in hexagonal lattice

1 Mpix, 30 f/s, with less than 1ms “dead time”, for PIV measurements

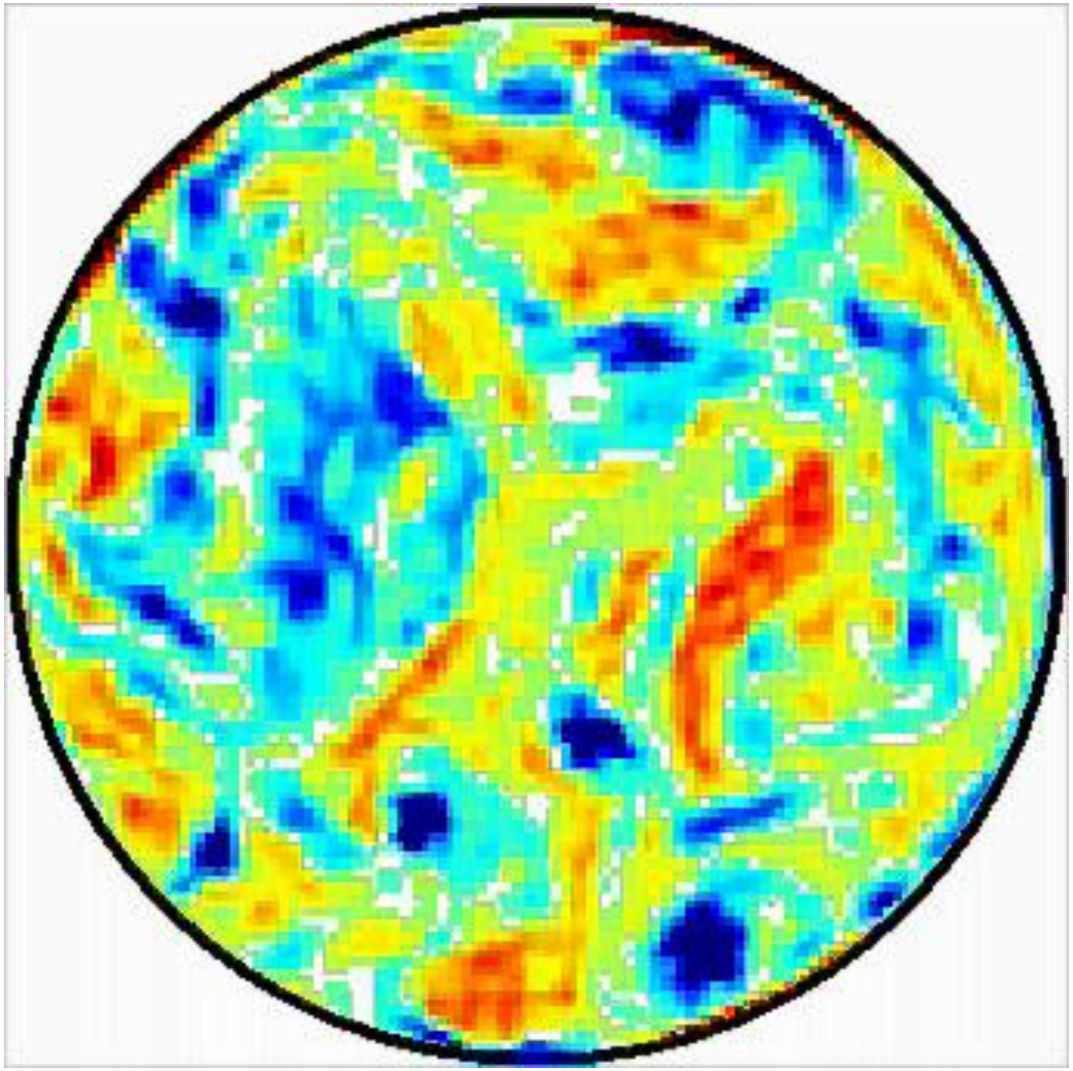


Camera

Injection Nozzles



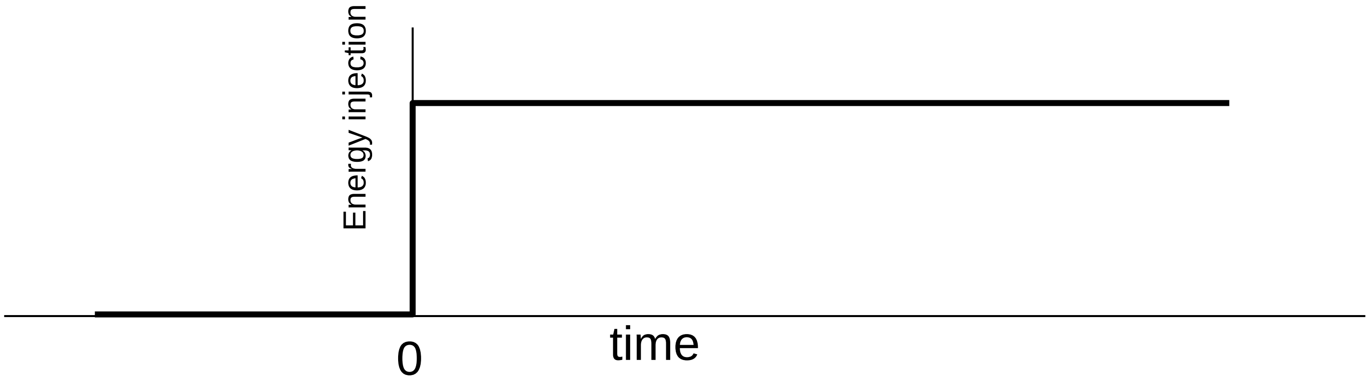
# In a steady state (vorticity field)



# Experiment 1

The system is brought to a solid body rotation (u=0) at a given rotation rate $\Omega$.

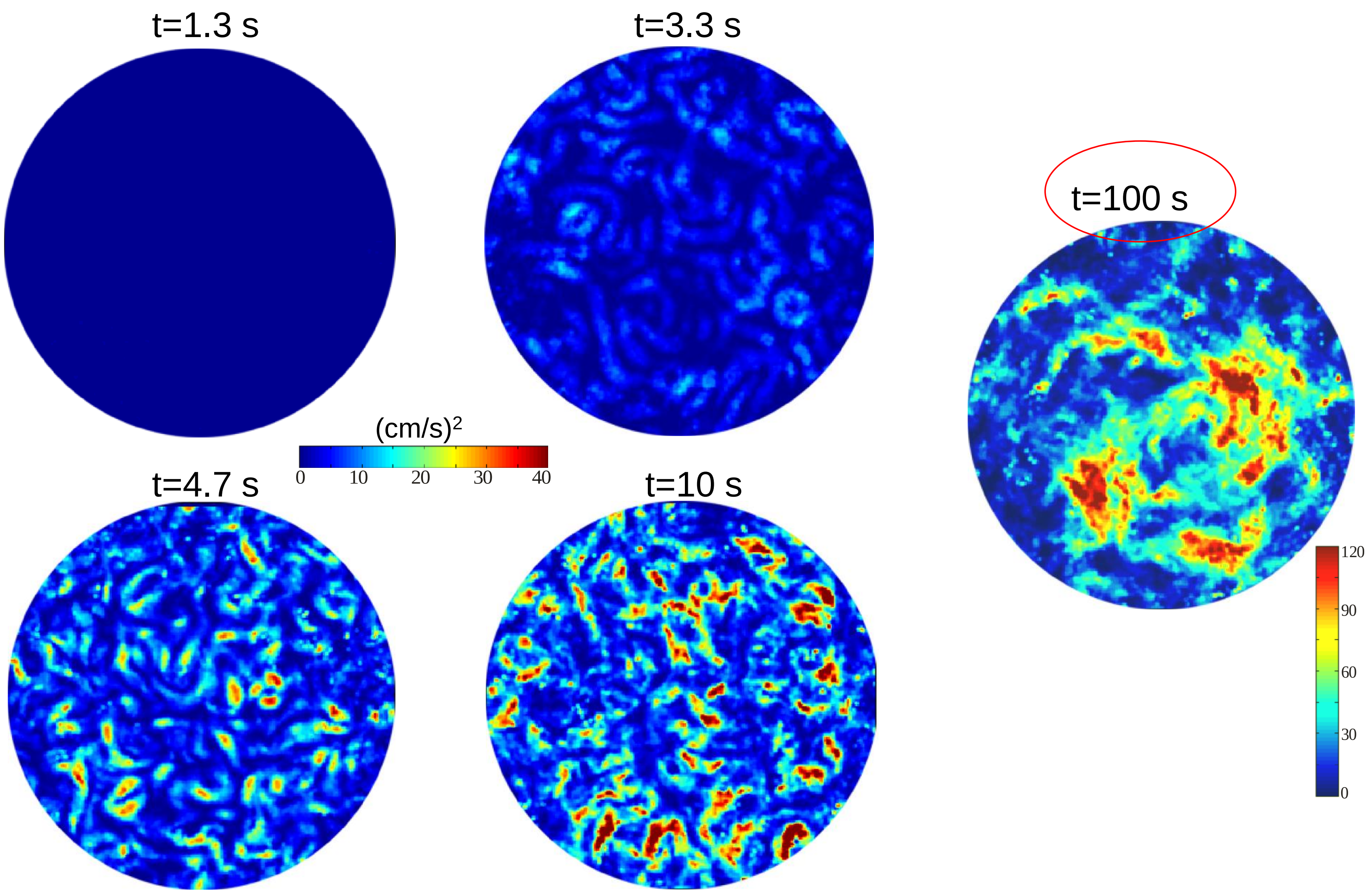
At t=0, we start injecting energy at a given flow rate (generating a step function in the injected power)




We measure the horizontal velocity (u,v) field at height H

Deriving energy power spectrum, E(k) and energy density "map", $(u^2+v^2)$, as functions of time.

# Energy Density evolution

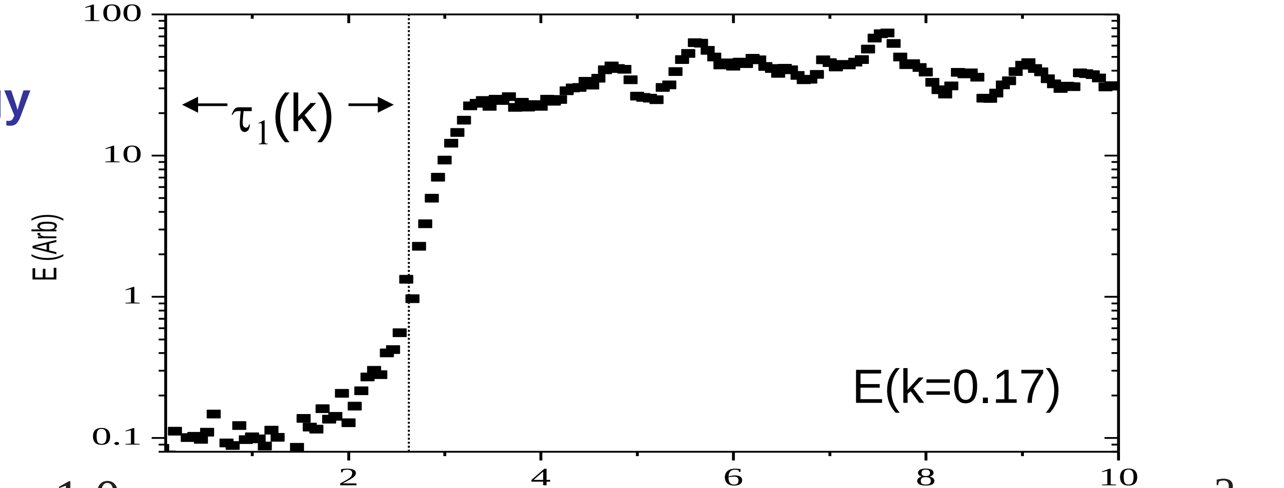


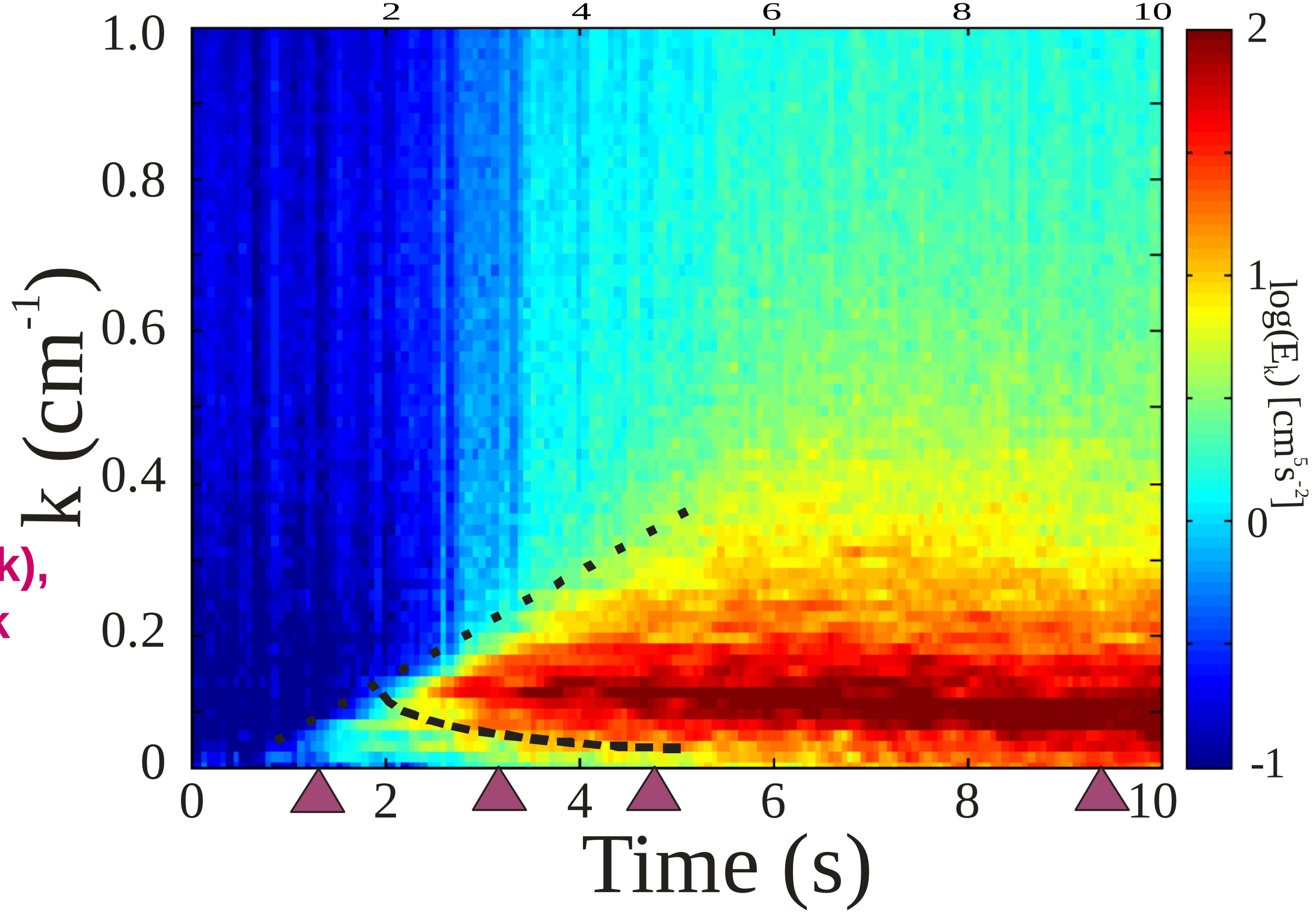
**Sharp "arrival" of energy**

**Two "Populating fronts" in the spectrum**

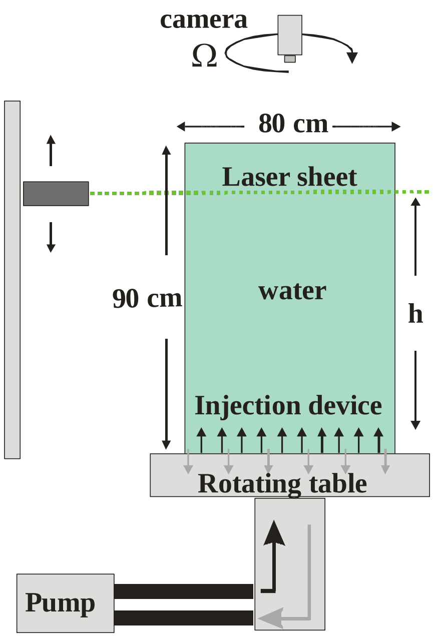
**One is linear in k, defining $\tau_1(k)$**

**The second defines $\tau_2(k)$, which decreases with k**



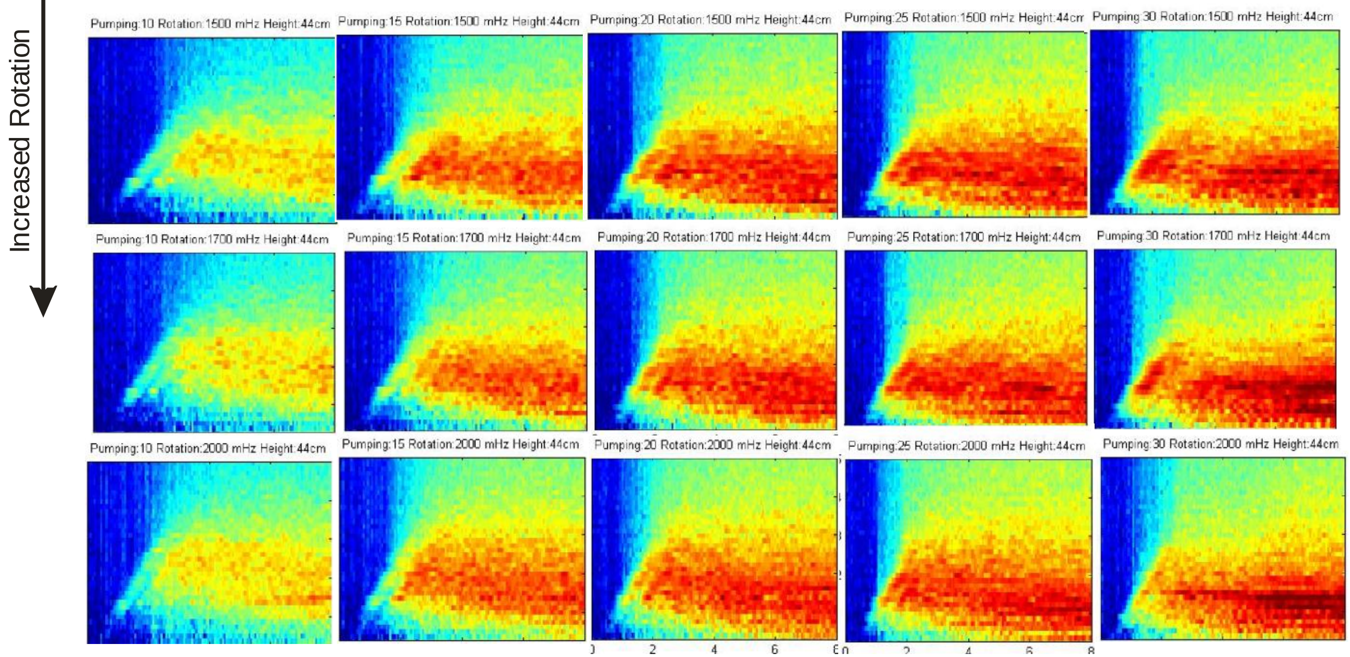



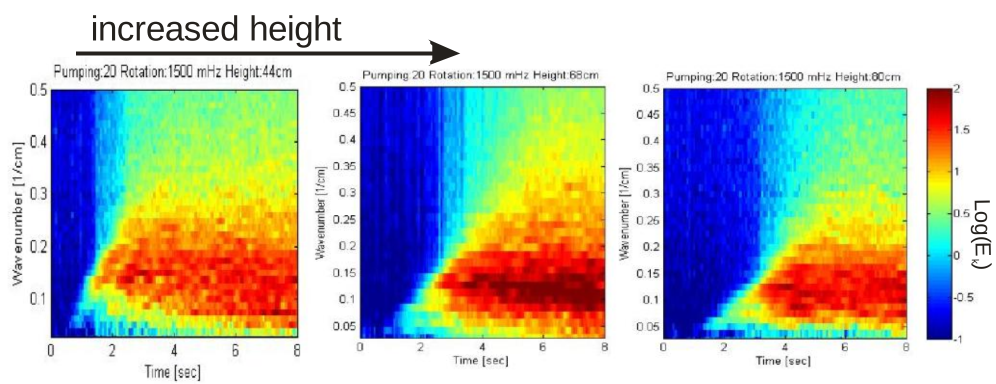
# Variation of fronts properties with rotation, energy injection and height




Examples of measurements at lower height:
The more energetic the flow - the shorter $\tau 2$ is
the higher the rotation the shorter $\tau 1$ is (color bars and axis, as above)

Increased energy injection

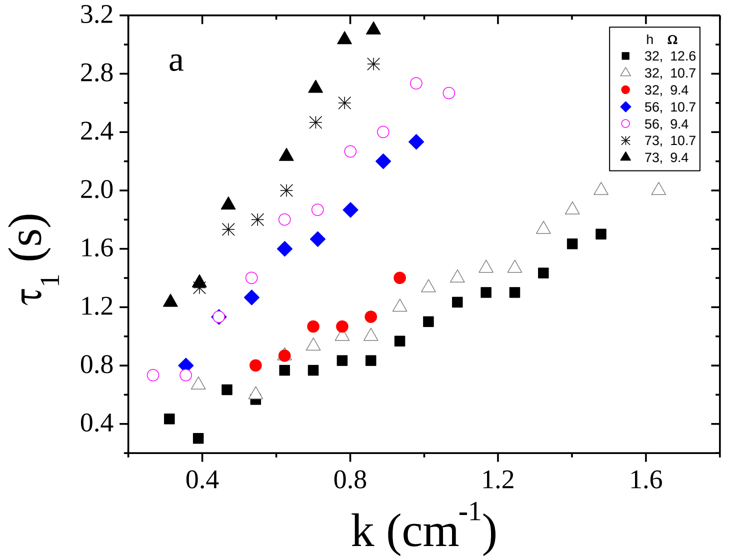


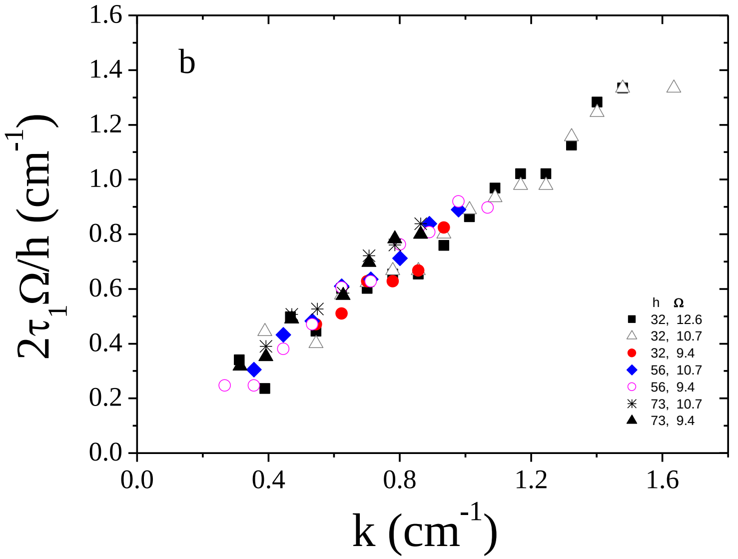



# Scaling of $\tau_1$

The energy carried by an inertial wave of wave number k would arrive at h by a time:

$$t = \frac{h}{v_g} = \frac{hk}{2\Omega}$$




**$\tau_1$ is the traveling time of inertial waves to the measuring plane**

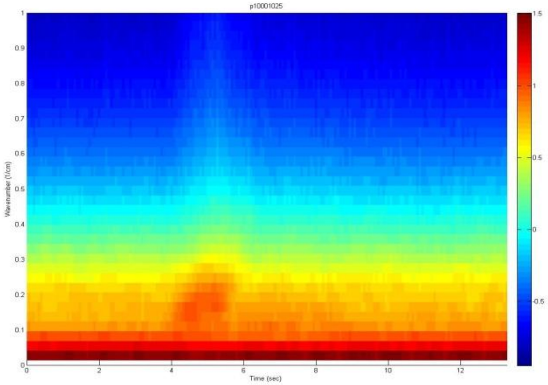
↓

**All the energy transfer along z is done by inertial waves**

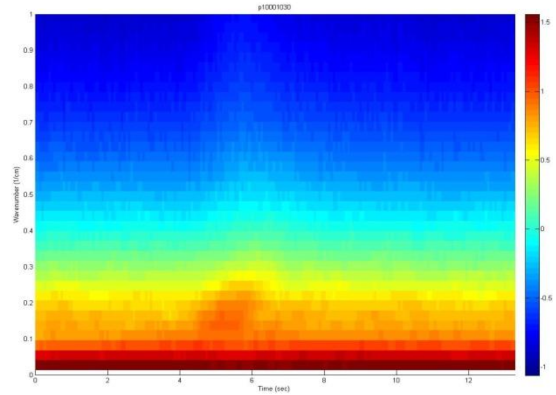

I. Kolvin, K. Cohen, Y. Vardi and E. S., *Phys. Rev. Lett*. 102**,** 014503,(2009).

# Waves propagate within existing turbulence

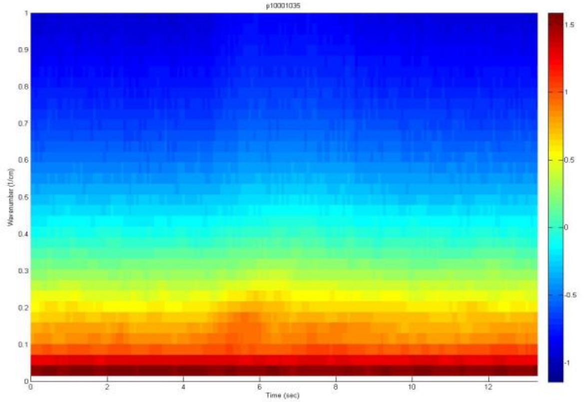
H=25 cm



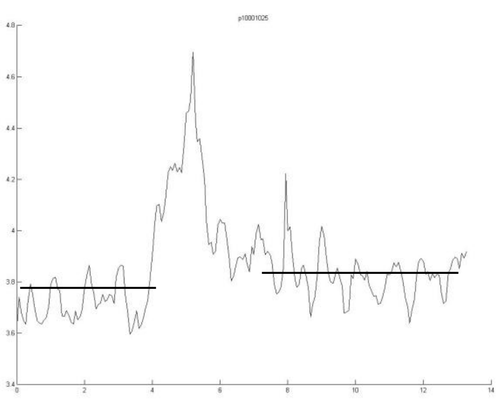
H=30 cm

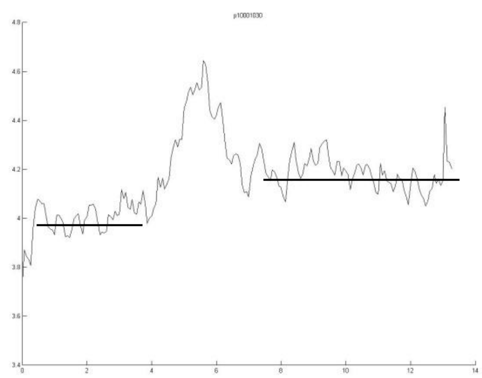


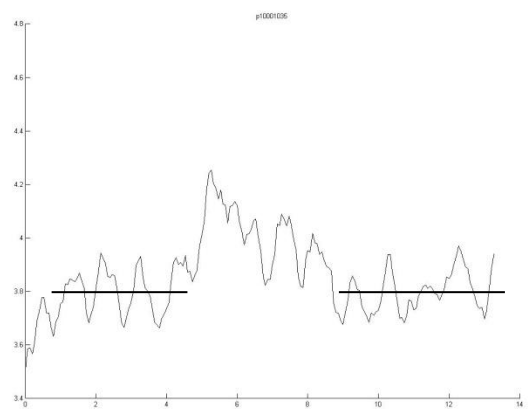
H=35 cm



Energy spectrum vs. time



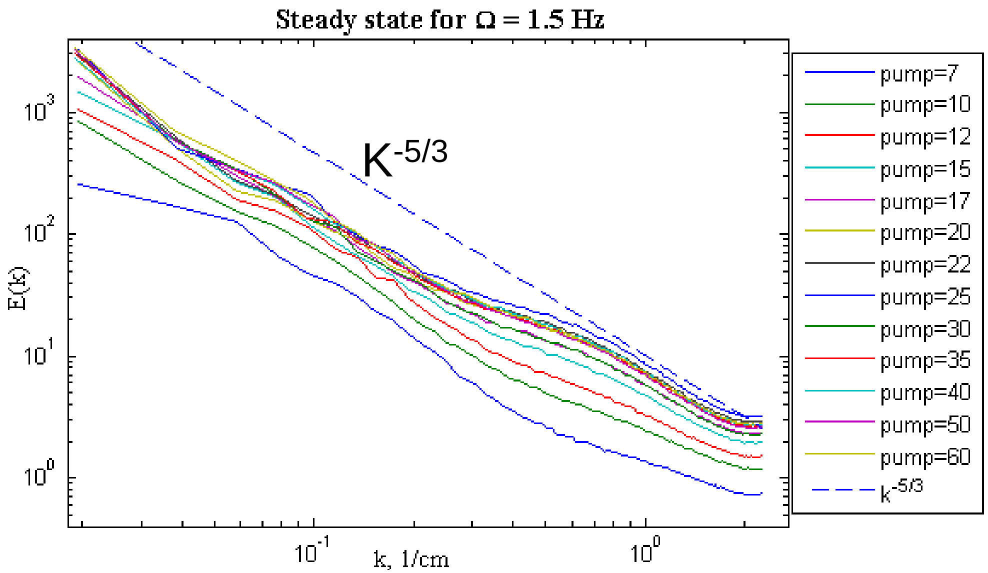


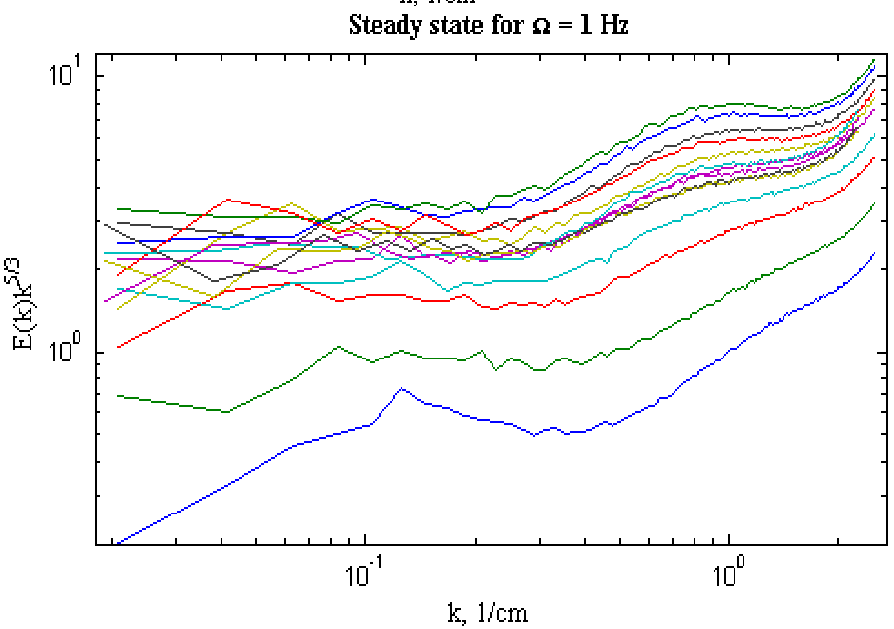


Total Energy vs. time

# Are inertial waves significant in steady state?

Energy spectrum seems consistent with 2D turbulence

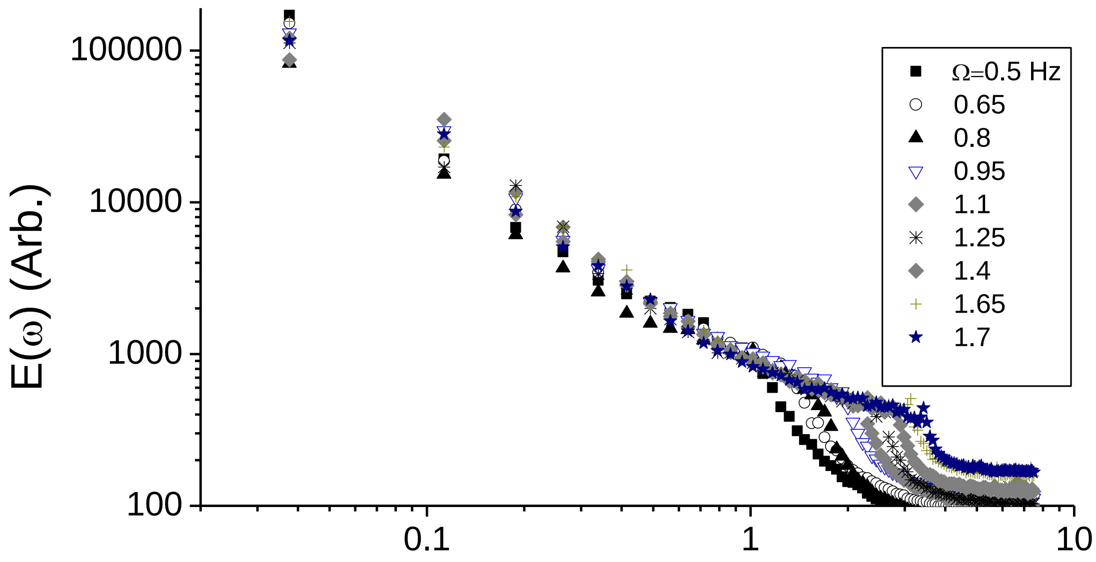


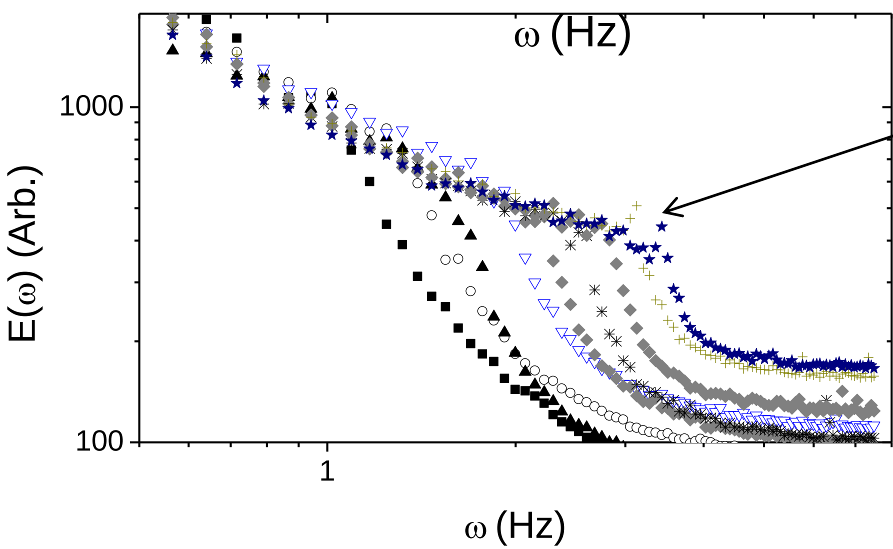




“compensated” spectrum

Measurements by Yuval Vardi

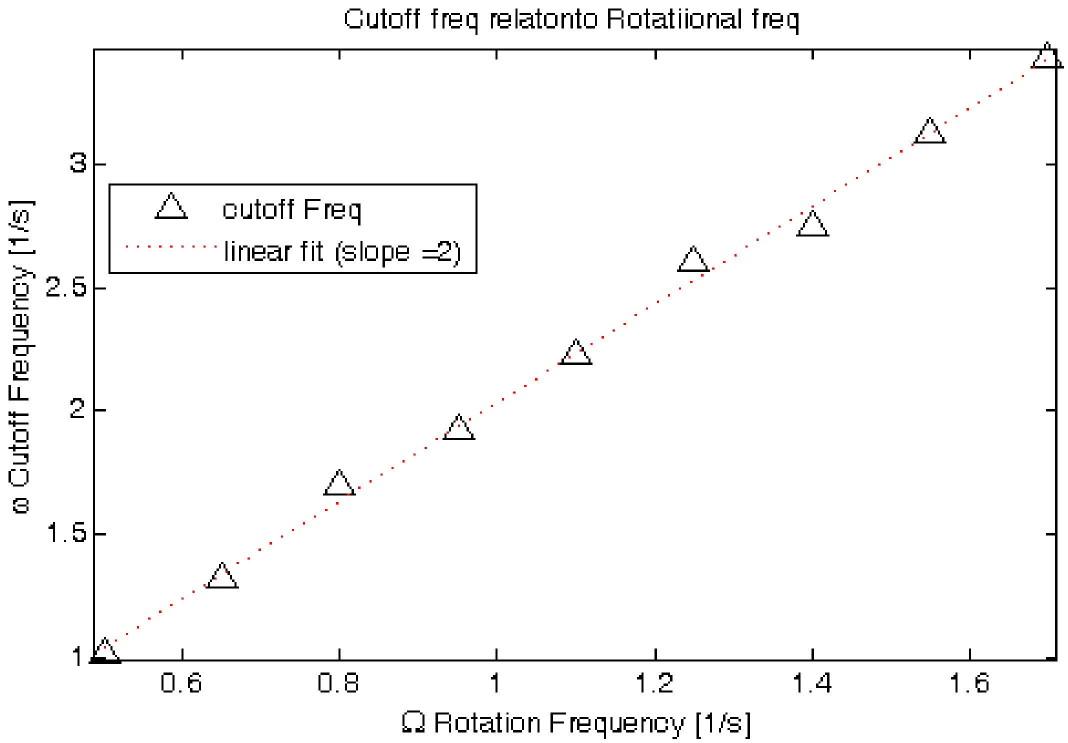
The temporal energy spectrum for different rotation rates




A clear “cutoff frequency for each rotation rate

# The cutoff frequency vs. Ω



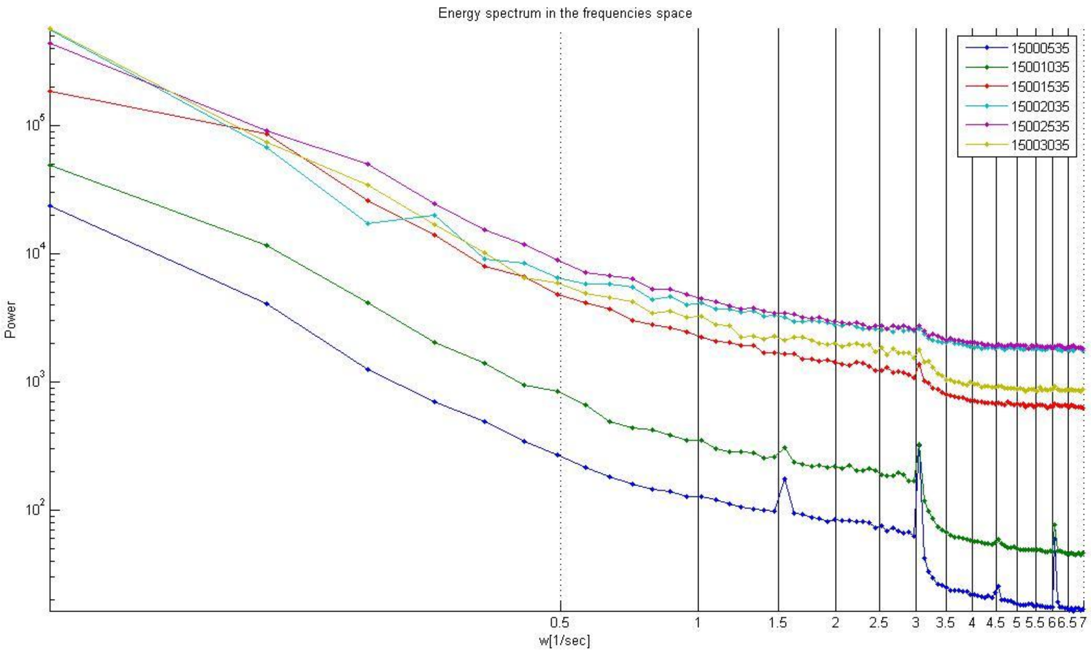

$$\omega_{\text{cutoff}}=2\Omega$$

Consistent with the cutoff in the dispersion relation of inertial waves:

$$\boxed{\omega \leq 2\Omega}$$

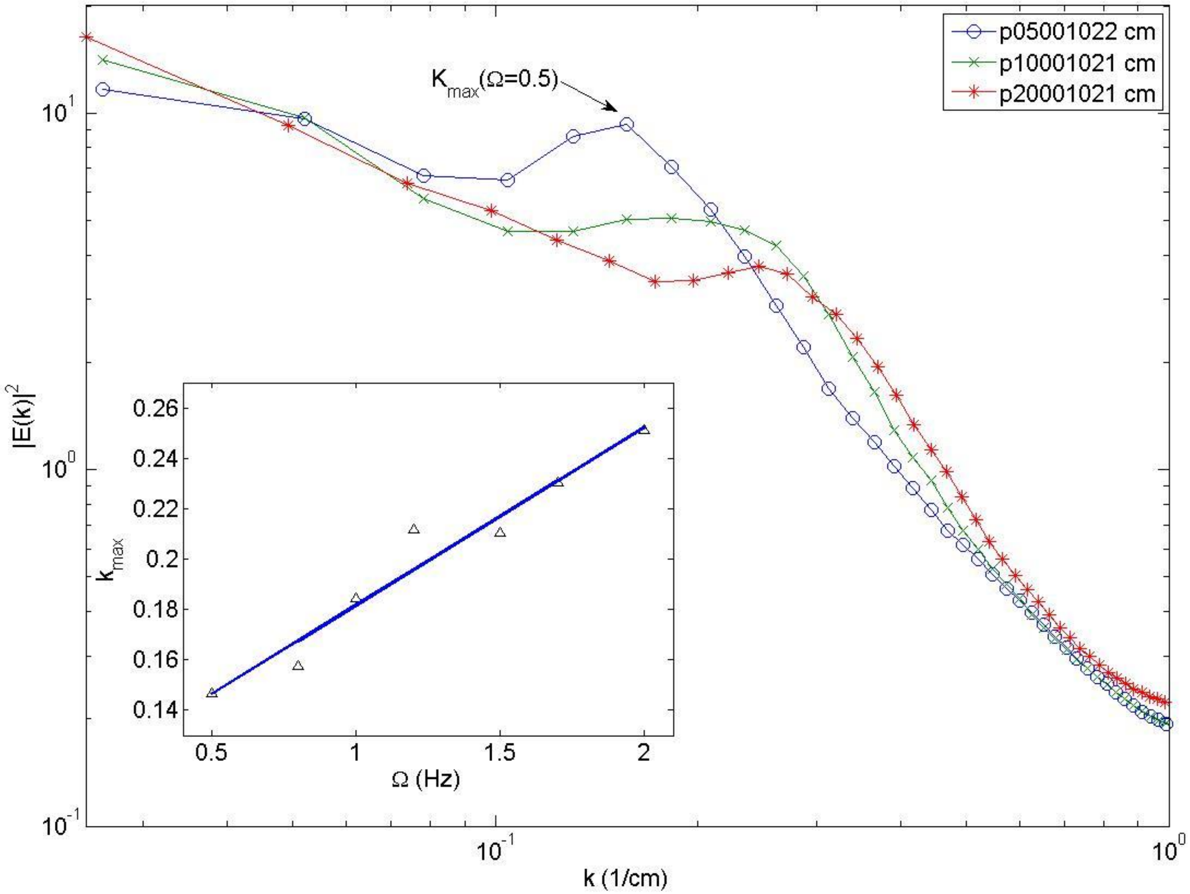
Suggests that inertial waves strongly affect the statistics even in a steady state

The temporal energy spectrum for different pumping rates




What is the dependence on the dissipation rate?

The injected energy spectrum at different rotations (same pumping rate)




A shift of $k_{max}$ with $\Omega$

# Conclusions

**High amplitude inertial wave packets can propagate in a turbulent rotating fluid.**

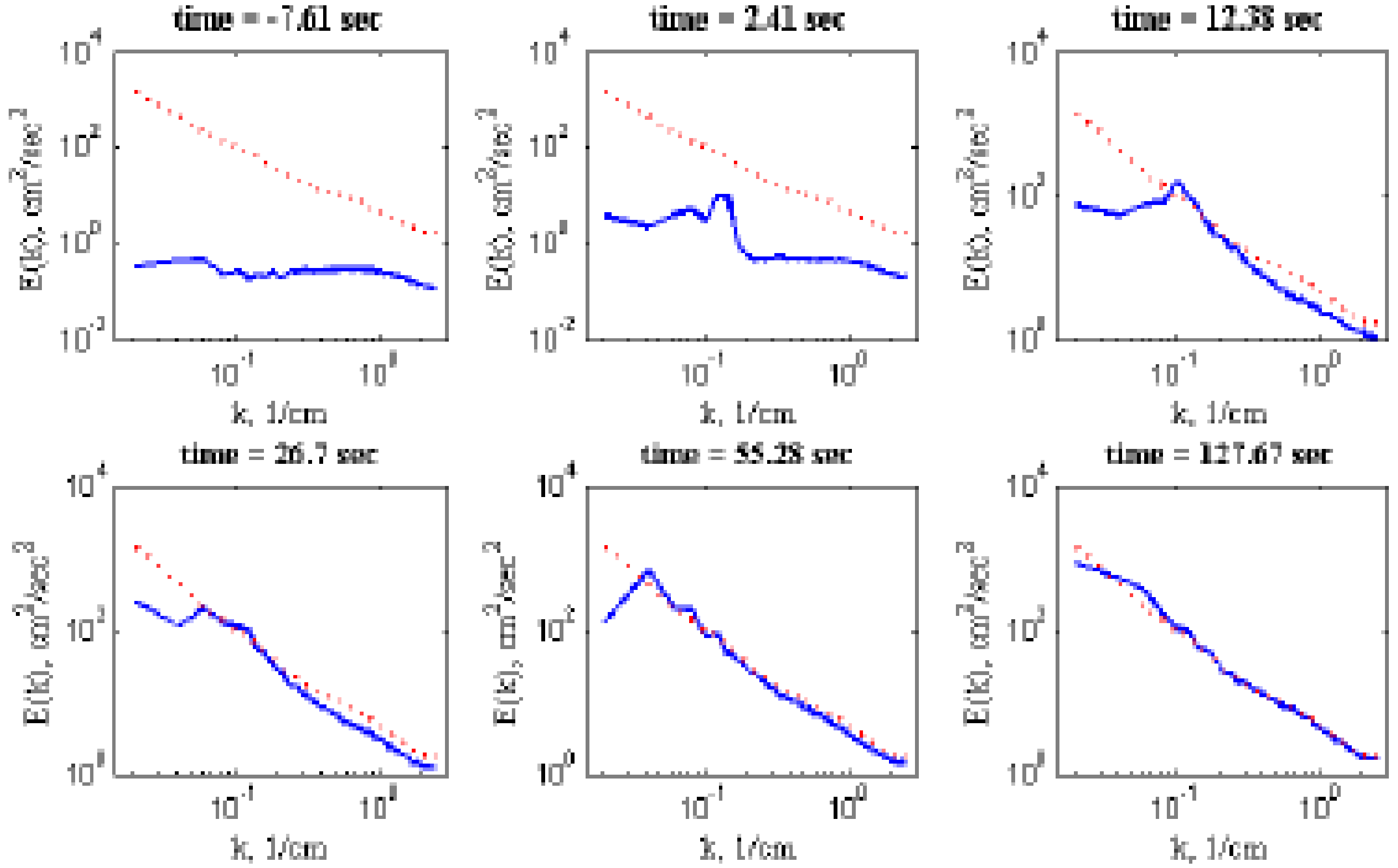
**They propagate with the group velocity calculated for small perturbations**

**In the flow that was studied, energy is transported by inertial waves**

**Statistics of rotating turbulence, contains “finger prints” of inertial waves.**

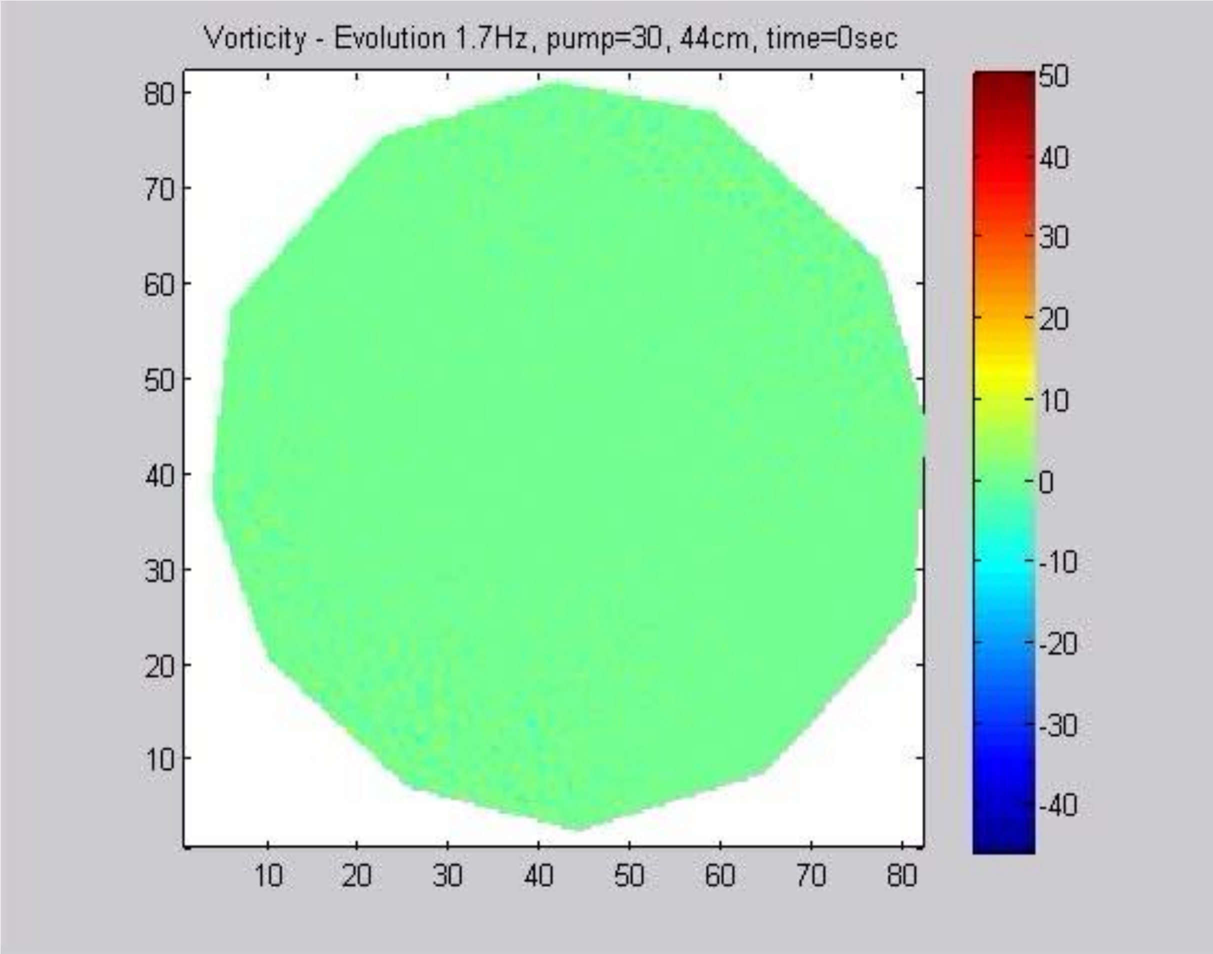
**Can we write an “inertial wave turbulence” description?**

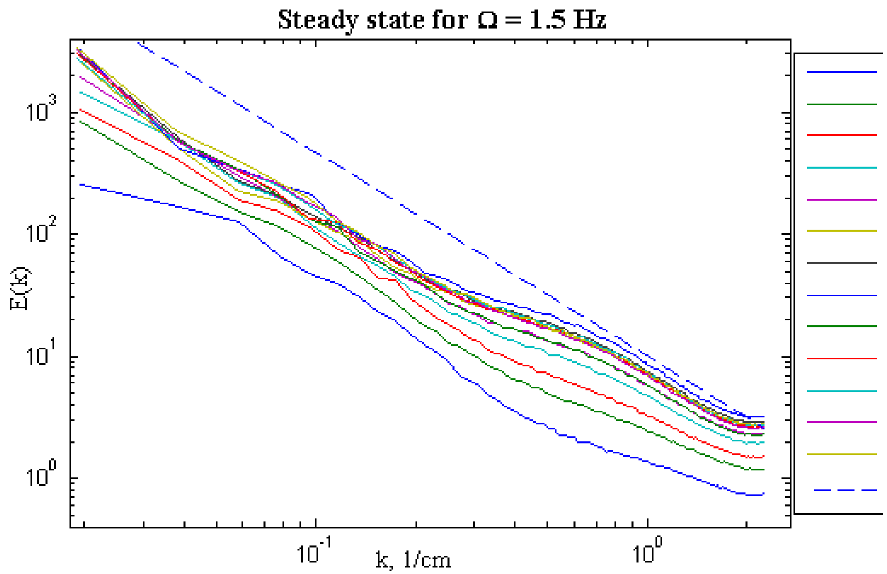
# Spectrum evolution over loner times



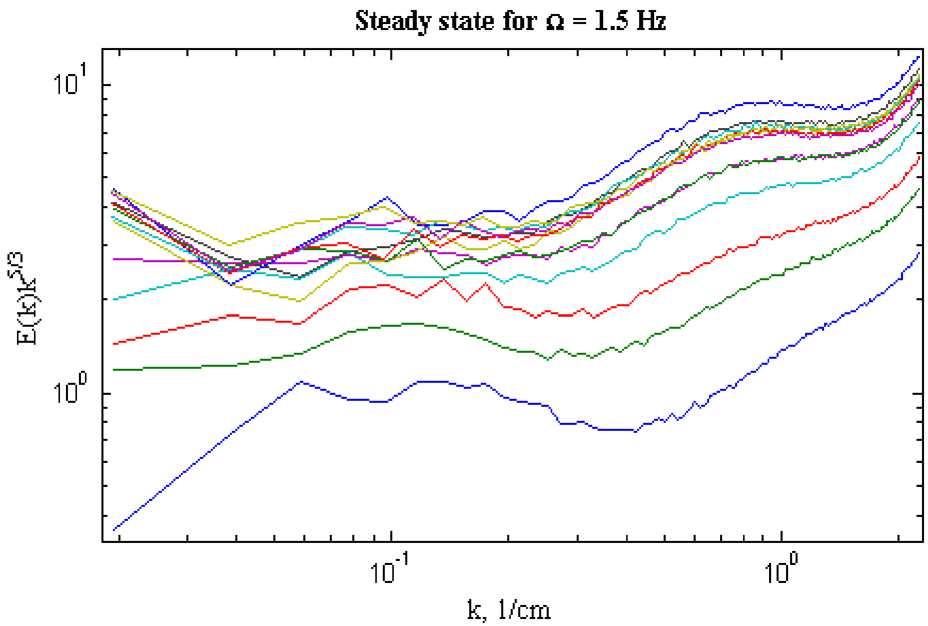
# Vorticity

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

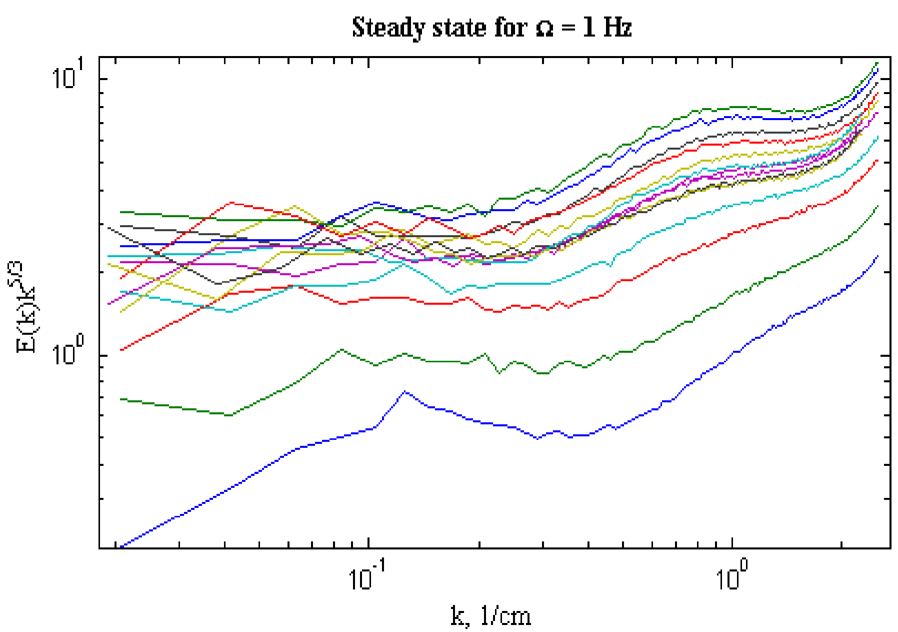




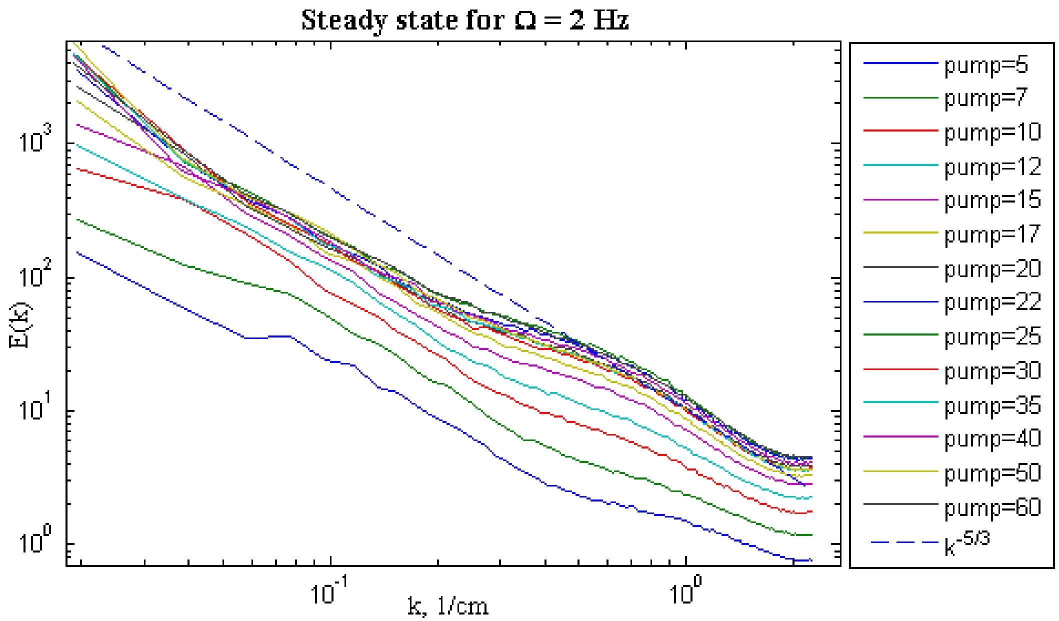
Steady state for Ω = 1.5 Hz
E(k)
k, 1/cm

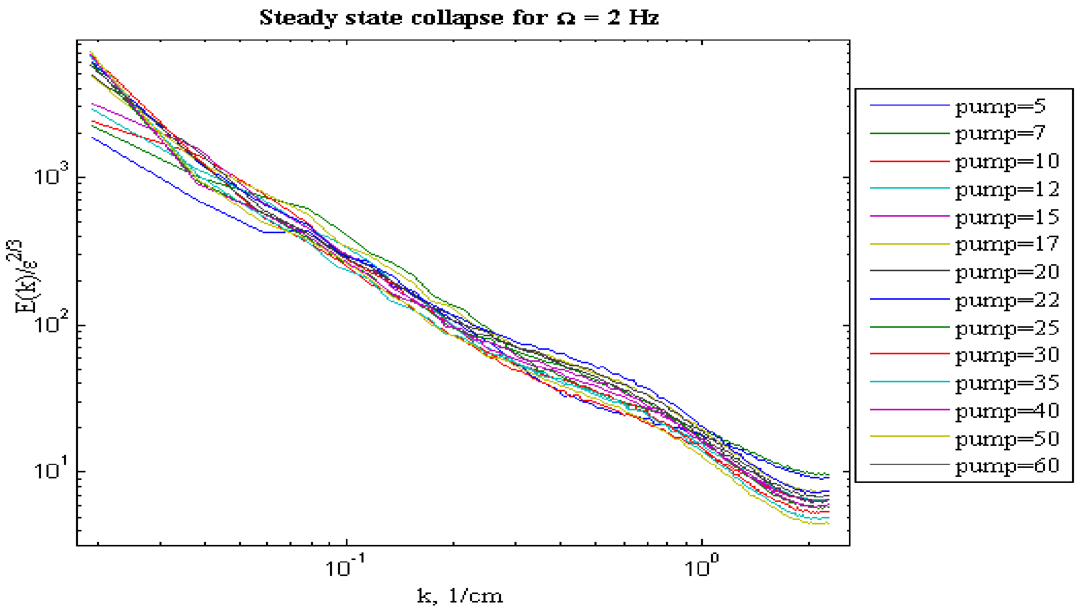



Steady state for Ω = 1.5 Hz
E(k)k^5/3
k, 1/cm

Steady state for Ω = 1 Hz
E(k)k^5/3
k, 1/cm

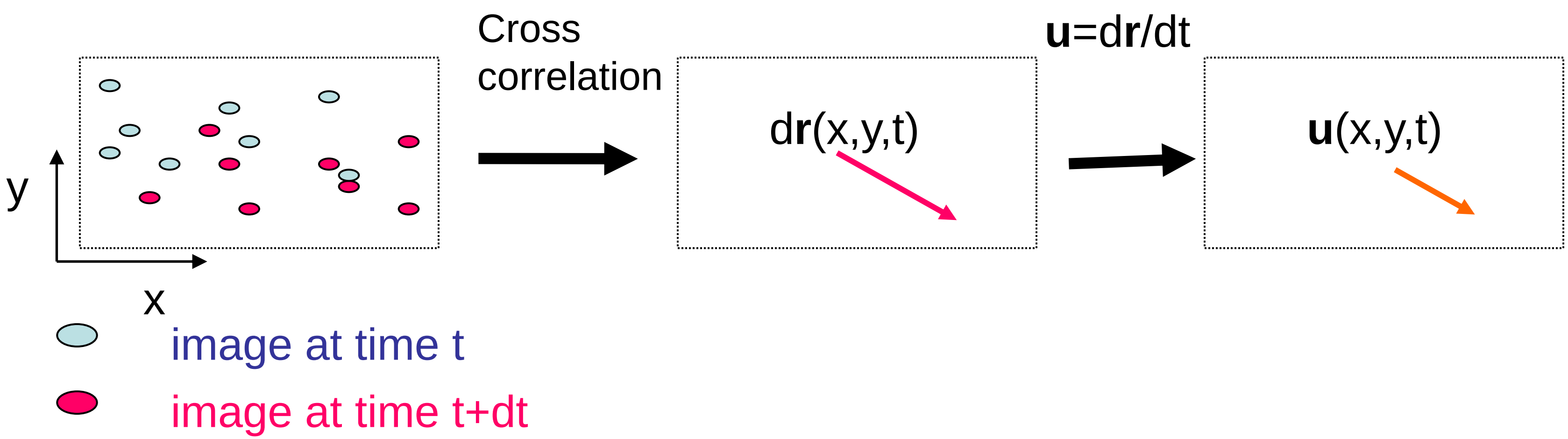



Scaling by $\varepsilon^{2/3}$

# PIV system

- Using ~50 $\mu$m Polystyrene particles ($\rho$=1.06 g/cm$^3$) for light scattering
- Dividing an image to ~200* 200 “cells”, each with ~10 particles
- Calculating the cross correlation between images of different times, t and t+dt, within each cell




- Image 1 Megapixel, 10 bit
- Shortest dt ~ 1 ms (by strobbing the laser)
- $\mathbf{u}_{Max}$ ~ 1 m/s

# Suggested scenario

**$\tau_1 \sim Hk/\Omega$ – linear propagation time**

**$\tau_2 \sim (vk)^{-1}$ – nonlinearities of inertial waves**

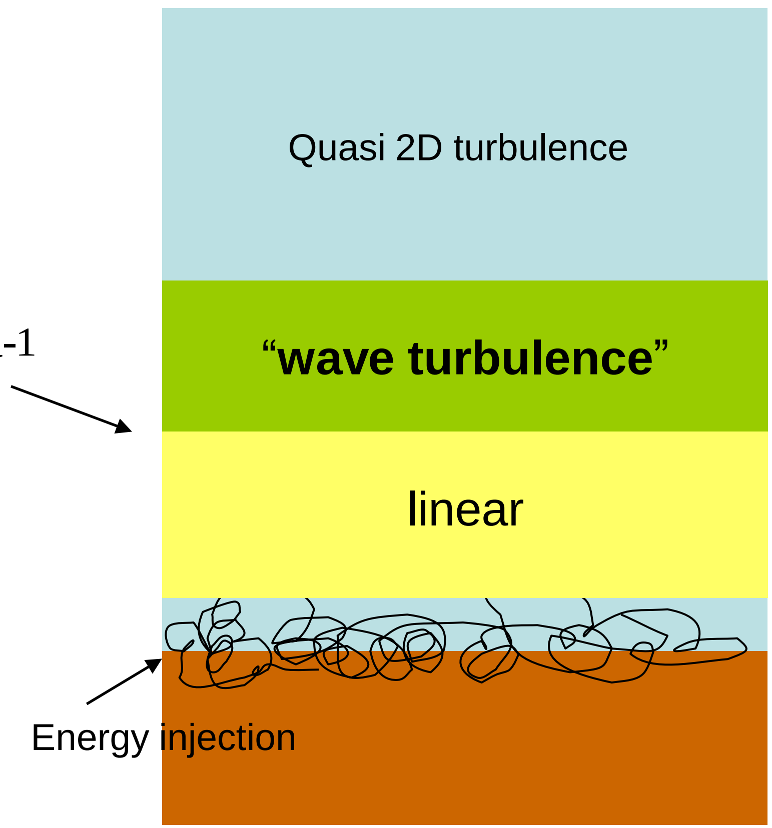
**$\tau_3 \sim ?$ – time to achieve steady state**

**We have dominance of linear behavior for**

$$k < \left(\frac{\Omega}{H\text{v}}\right)^{1/2}$$

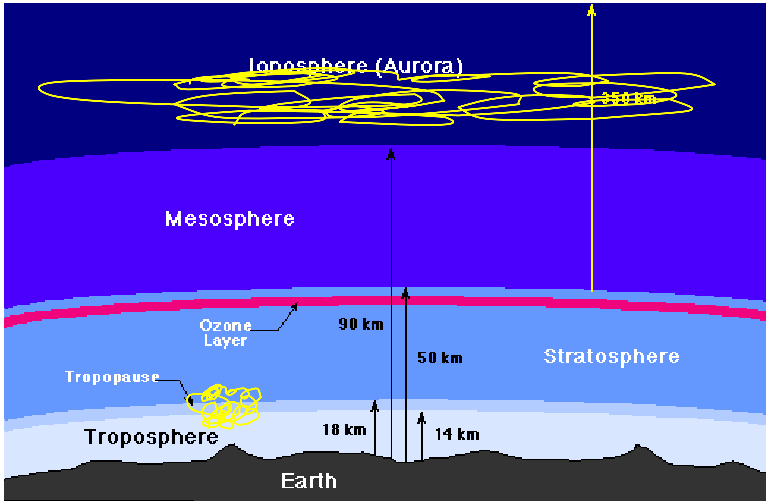
**Or, for heights smaller than** $H^* \sim \Omega k^{-2} \text{v}^{-1}$

**Above this height we expect to find a region dominated by (resonant?) nonlinear interaction of inertial waves – "wave turbulence"?**



# "Shooting" turbulence

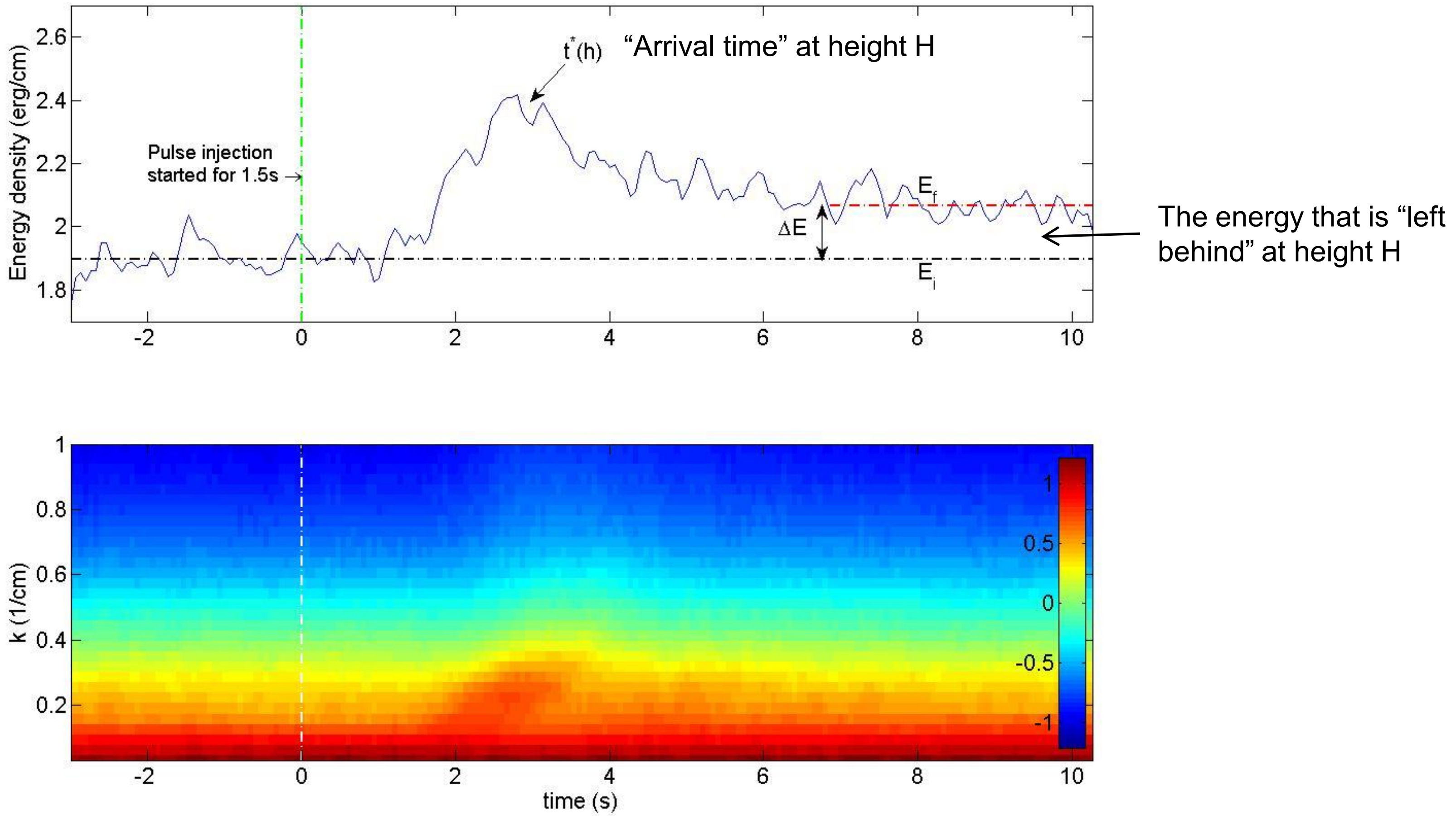
**What happens in a turbulent atmosphere when there is a "sudden" blast of energy from a source?**



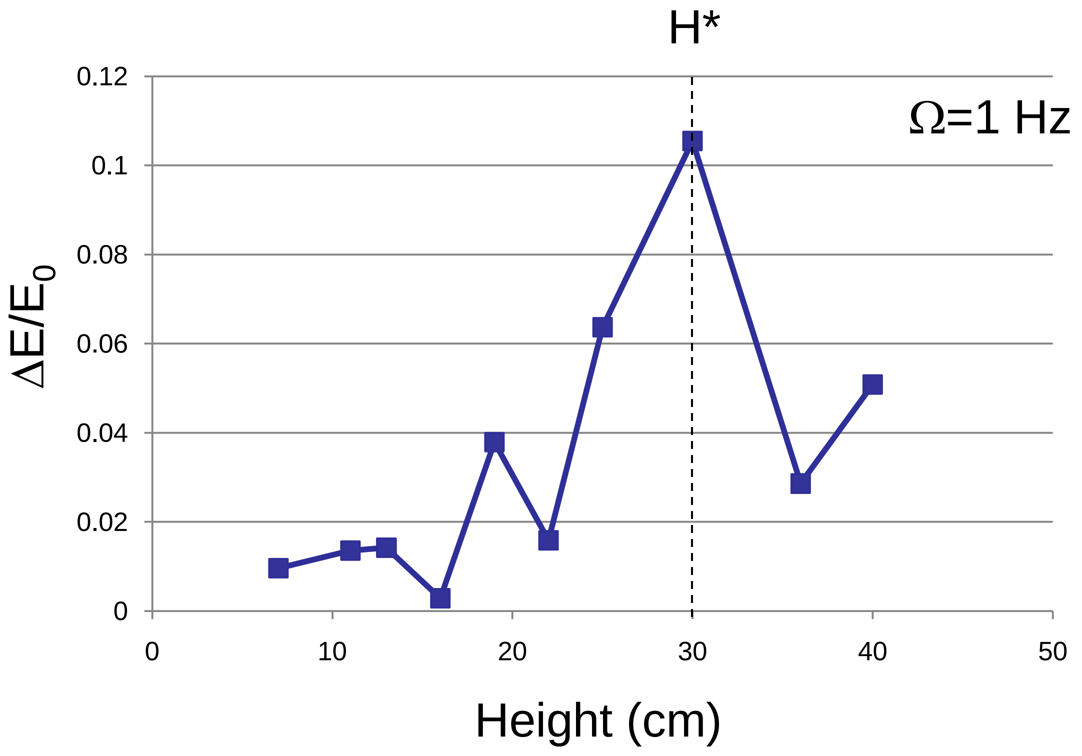

**Intuitively we would guess that most of the energy will accumulate near the source**

**However, now we can predict, that the energy will propagate linearly upwards and will be transferred to the turbulent field only at height larger than H***

# Measuring the propagating pulse



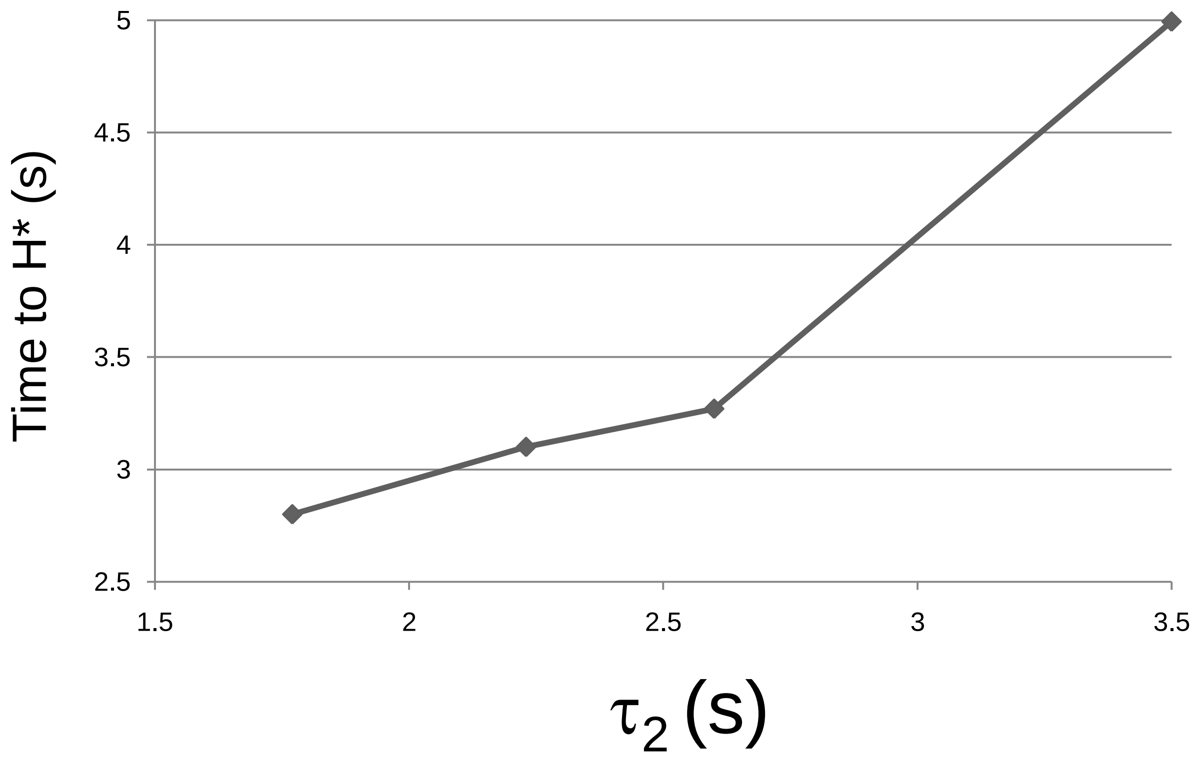
The energy “left behind” by the pulse vs. height



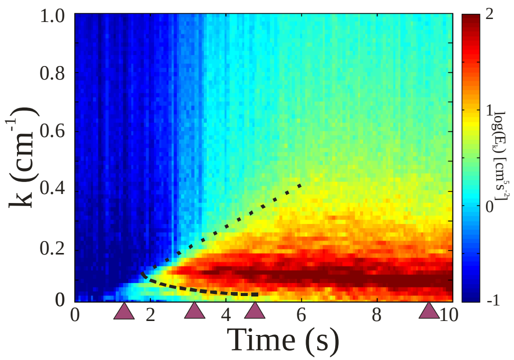

The energy of the pulse is concentrated at a selected height H*

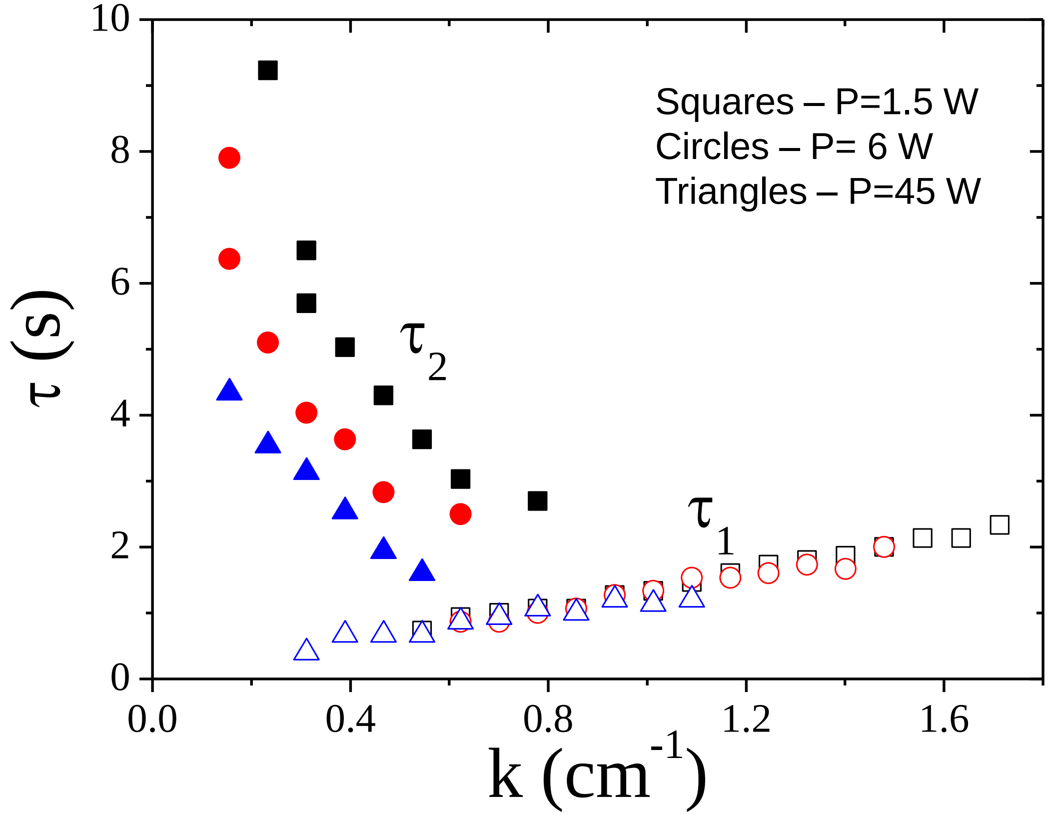
# H* is determined by $\tau_2$

Find the traveling time to H* under different conditions



# Dependence of $\tau_1$ and $\tau_2$ on injected Power - P

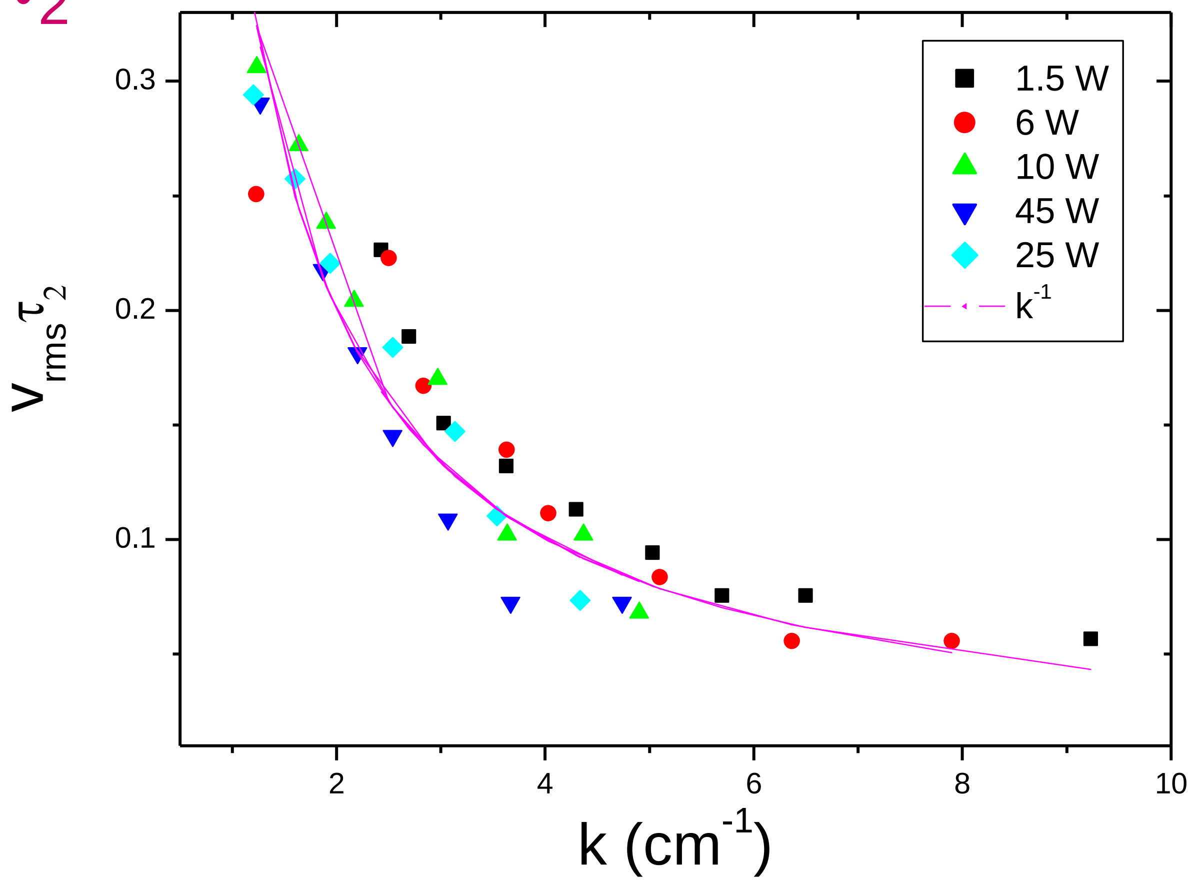



**$\tau_1$ is independent of P** (as expected from a linear mechanism)

**$\tau_2$ decreases with P** (An indication of the nonlinear nature of the underlying process)

# Scaling of $\tau_2$

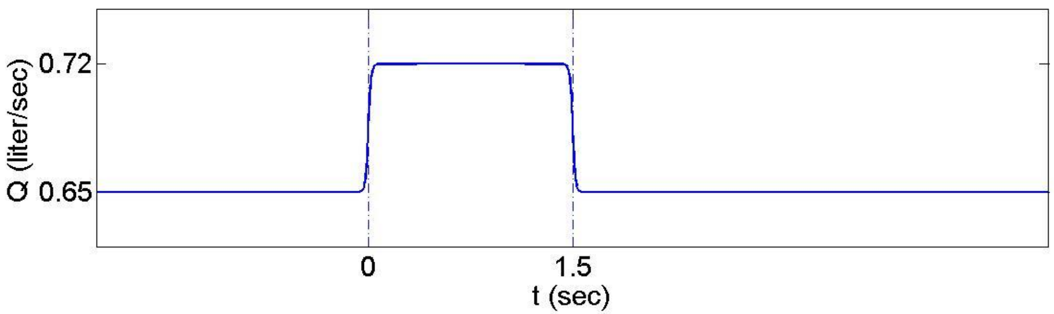


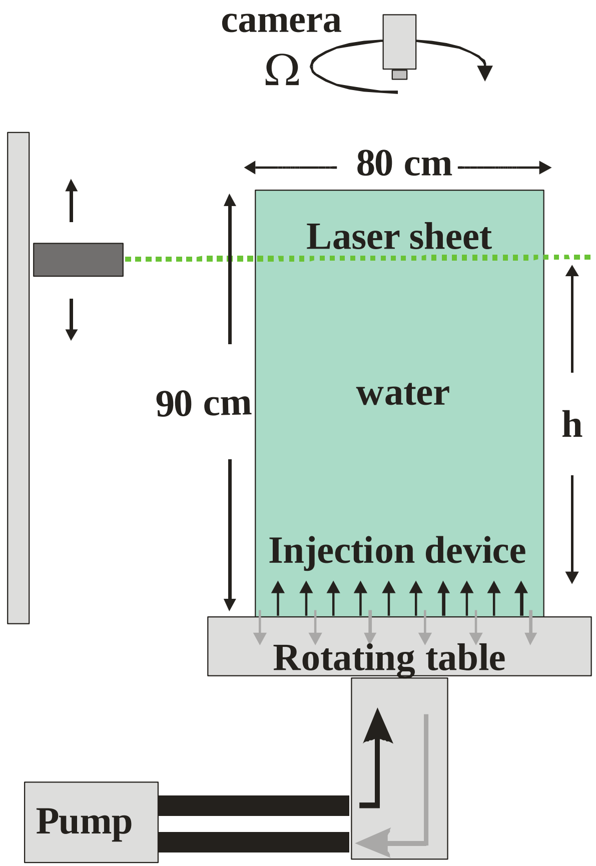

**$\tau_2$ scales roughly as the "eddy turnover time" - $(vk)^{-1}$**

**Should remember that steady state is achieved at times much longer than $\tau_2$– is there a third time scale?**

# The experiment

Injecting a pulse of energy into a steady state turbulence field at H=t=0
And measure the velocity field at H>0.

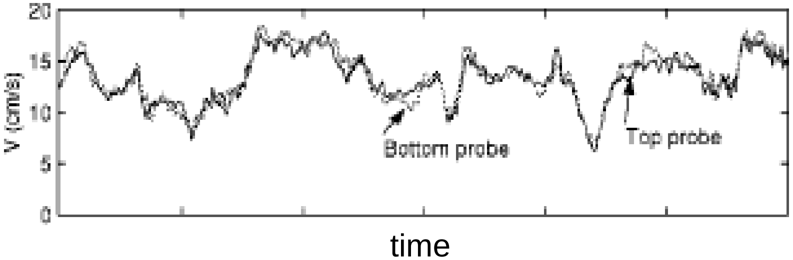


**It is known:**

Built up of large scales

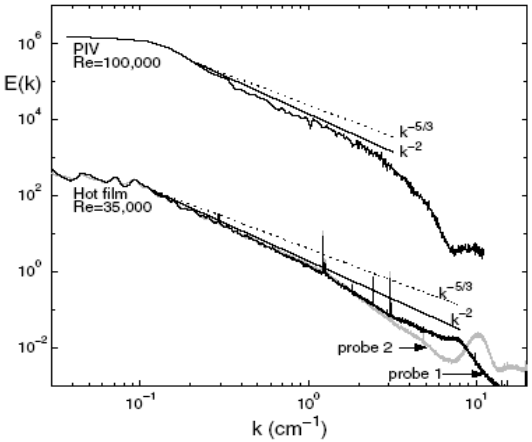
2D in the large scales (Baroud, Plapp and Swinney, 2003)




**But also:**

Different energy power spectra:

Baroud et. al. 2002




Smith and Waleffe 1999