Statistical Mechanics 2012/2013 Problem Set 5

Submission date: 14.1.13

5.1 Fluctuation of the number of particles and of the wave intensity (32 pts)

(a) Find the mean squared fluctuation of the number of particles in the state number k of the ideal Fermi and Bose gases.

(b) Consider a meter measuring the amplitude of light coming from a monochromatic noncoherent source. This can be described by a set of classical light waves of a specific frequency, ω, with random phases and amplitudes. Calculate the fluctuation in the mean intensity of the radiation, $\langle(\Delta I)^2\rangle$, by averaging over long time (much longer than ω^{-1}) and over the random phases. Express your result in term of the mean intensity of the radiation, $\langle I\rangle$. How is this result related to the quantum result you have computed in (a) for Bose gas?

5.2 Renormalization group (42 pts)

Consider the partition function of 1d Ising model in an external field

$$Z_N(K,L,C) = e^{NC}\sum_{\{\sigma\}}\exp\Big[K\sum_i \sigma_i\sigma_{i+1} + L\sum_i \sigma_i\Big]\ . \tag{1}$$

(a) Express it via the decimated partition function by summing over all odd spins:

$$Z_N(K,L,C) = Z_{N/2}(K',L',C')\ .$$

Find the recursion relations $K'(K,L)$ and $L'(K,L)$ (remember that there is also a correction to the free energy).

(b) In the variables $x = e^{-4K}$ and $y = e^{-2L}$ find the fixed points of the renormalization procedure and analyze their stability.

5.3 Random walk (26 pts)

Using Fourier representation, show that a random walk on a triangular lattice gives the same continuous limit for the probability on site, $P(x,t)$, as was given by a cubic lattice. **Hint**: It may be useful to use an explicit representation of the lattice vectors.