Exam 2011

Problem 1:Two different phases of the same solid have respectively the specific heats $c_1 = aT^3$ and $c_2 = bT$.

a) Assuming that they both satisfy the third law of thermodynamics, find the entropies of the phases.

b) Assuming that their internal energies (per particle) at zero temperature are the same and equal to e_0, find how their energies depend on the temperature.

3) Assuming that the densities are the same, find the temperature of the phase transition and describe which is the low-temperature phase.

Problem 2. Consider the 1d spin chain where spins can have values $\sigma_i = \pm 1, \pm 2$. The Hamiltonian is determined by the interaction of the nearest neighbors: $\beta\mathcal{H} = -K \sum_i \delta_{\sigma_i,\sigma_{i+1}}$. Here $\delta_{a,b} = 1$ when $a = b$ and zero otherwise.

a) Do Renormalization Group decimation of every second site ($k = 2$) and find the RG recursion relations $g(K)$ and $K'(K)$.

b) Find the fix points and describe their stability.

c) Find the correlation radius as a function of K.

Problem 3. Consider the over-damped Brownian particle in the potential $V(q) = q^2/2 + q^3/3$ so that the respective equation of motion is

$$\dot{q} = -q - q^2 + \eta \, . \tag{1}$$

Here the noise is white Gaussian with $\langle \eta(0)\eta(t)\rangle = 2\delta(t)$. The space q has the topology of a circle i.e. $q = \infty$ and $q = -\infty$ is the same point. Find stationary probability distribution and describe its asymptotics at large $|q|$.