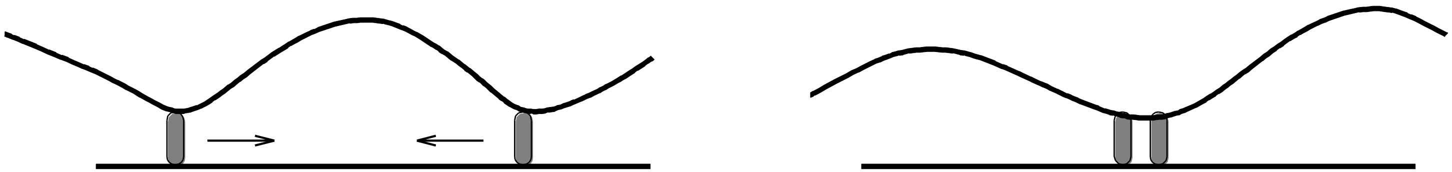
Entropy-driven aggregation of adhesion sites in supported membranes



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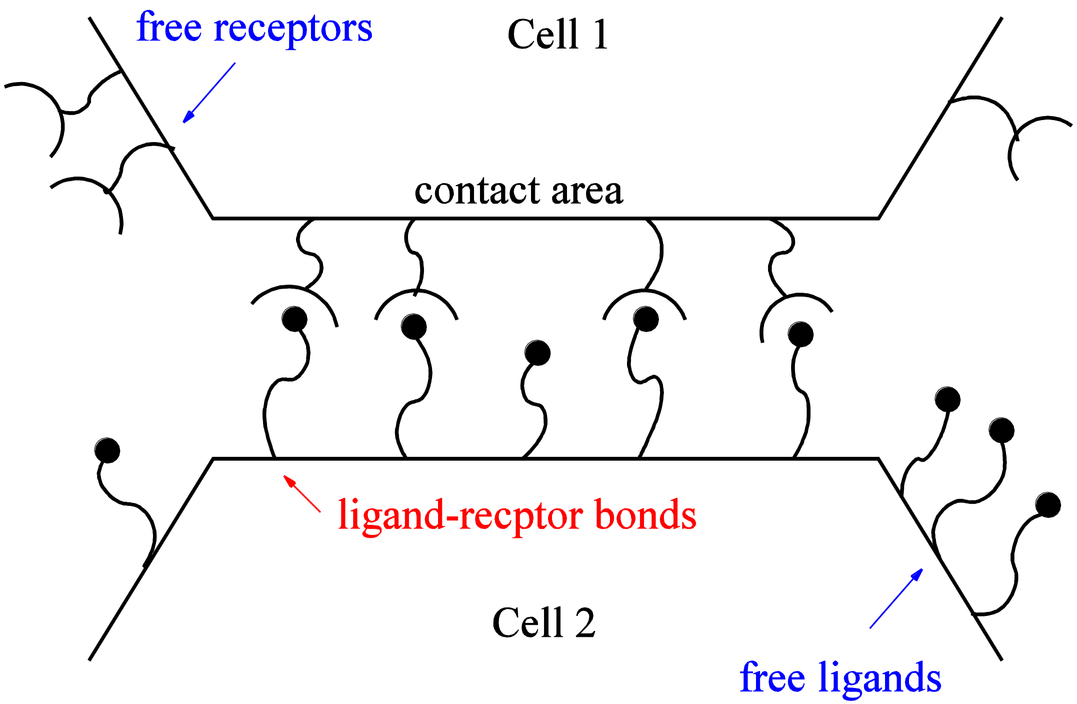
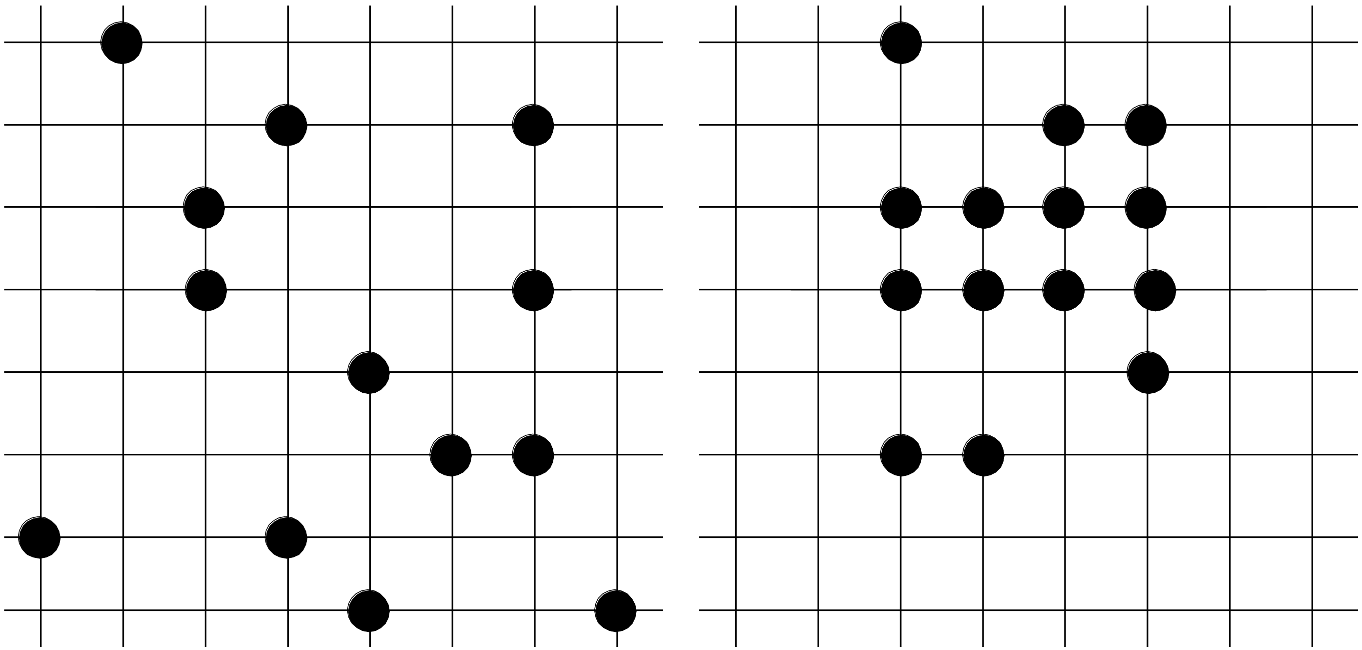
Specific (bio)adhesion



Questions:

1. Which forces drive the formation of adhesion-domains?
2. Can this process be described by the standard lattice-gas model of condensation?

Direct vs. fluctuation-mediated interactions

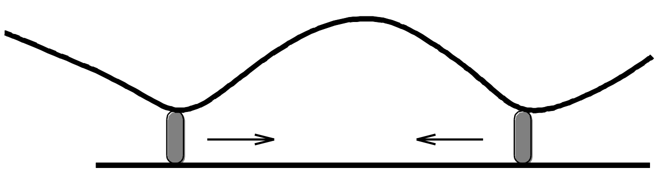


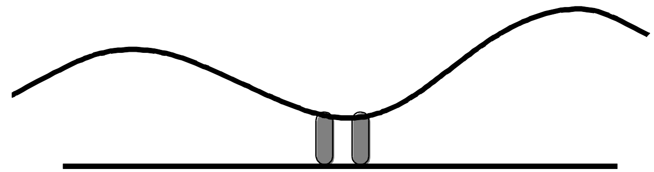
$$\mathcal{H} = \sum_{\langle ij \rangle} -\epsilon s_i s_j + U\left(\vec{r}_1, \ldots \vec{r}_N\right)$$

direct interactions   fluctuation-mediated interactions

Fluctuation-induced attraction

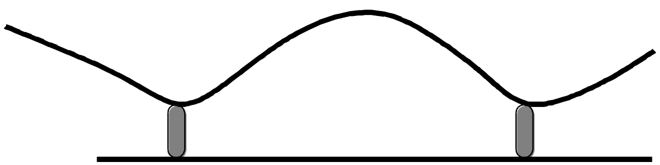
- Increase the conformational entropy of the fluctuating membrane

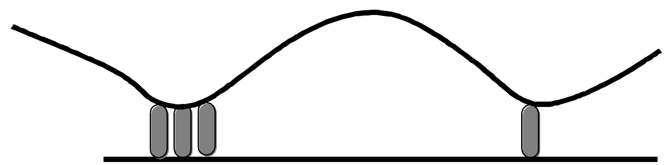




Membrane fluctuations restricted at 2 points   Membrane fluctuations restricted at 1 point

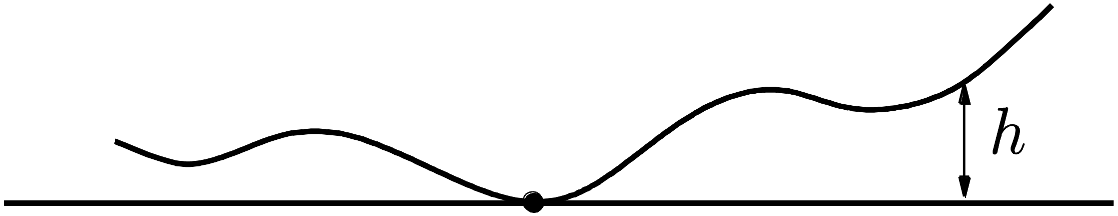
- Many-body (non pairwise) interactions





Right adhesion point is equally attracted to the left adhesion point (left figure) and the adhesion cluster (right figure). The spectrum of thermal fluctuations in both cases is similar.

Supported membranes with one fixed adhesion point

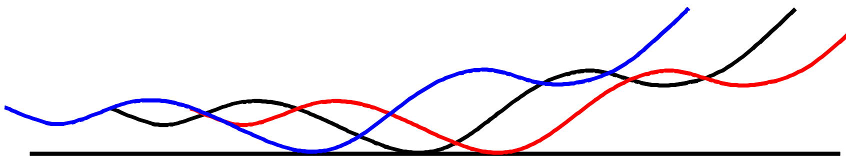


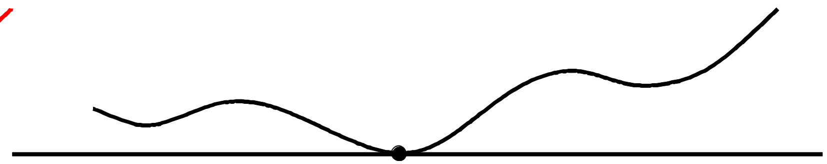
$$\mathcal{H} = \int \left[\frac{\kappa}{2}\left(\nabla^2 h\right)^2\right] \Phi\left(h\right) \delta\left[h\left(r_0\right) - 0\right] d^2\vec{r}$$

Helfrich effective Hamiltonian , hard wall constraint , attachment point

$$\Phi\left(h\right) = \begin{cases} 1 & \text{for } h \geq 0 \\ \infty & \text{for } h < 0 \end{cases}$$

OF, Phys. Rev. E **78**, 051919 (2008).





free membrane – can be translated horizontally

pinned membrane – global minimum of the height function is at the pinning site

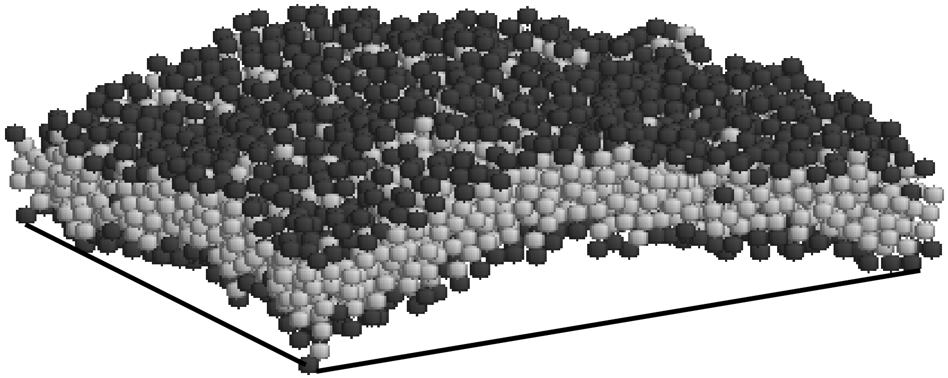
The attachment eliminates the membrane's translational degree of freedom

$$F_{\mathrm{attachment}} = k_B T \ln\left(\frac{L^2}{l^2}\right) = 2k_B T \ln\left(\frac{L}{l}\right)$$

Out of **every** $\Omega = \left(L^2/l^2\right)$ **similar** free membrane configurations, only one is also a configuration of the pinned membrane

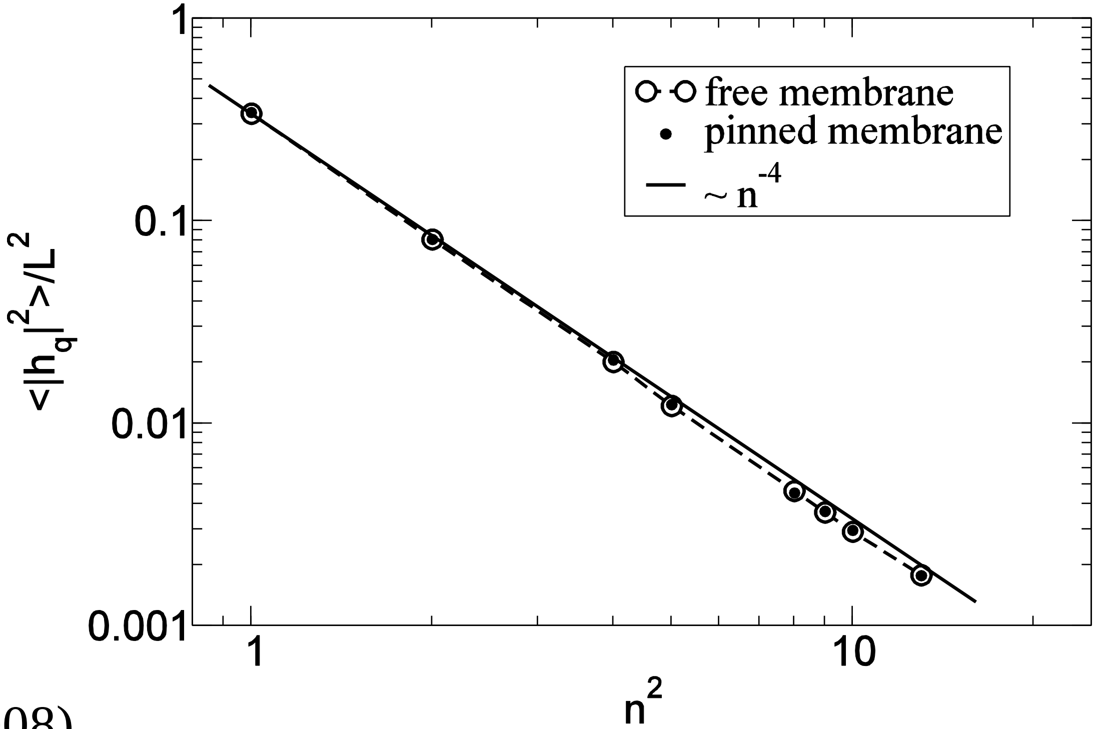
But this mapping between the two problems implies that …

… their statistical properties are the same !



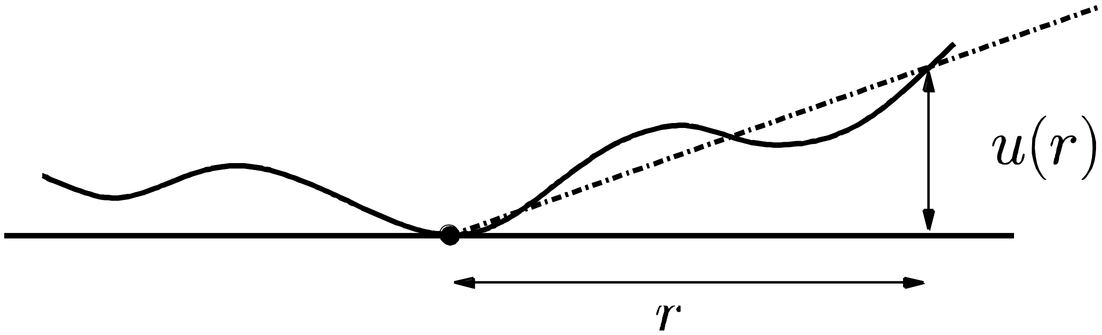
amplitude of thermal fluctuations

$$\left\langle |h_q|^2 \right\rangle = \frac{k_B T L^2}{\kappa} \frac{1}{(2\pi n)^4} \; ; \; q = \frac{2\pi n}{L}$$



OF, Phys. Rev. E **78**, 051919 (2008).

Entropic repulsion force (collision pressure)



$$u(r) = \langle h(r) \rangle \sim r\sqrt{\frac{k_B T}{\kappa}}$$

The attachment free energy density [W. Helfrich, Z. Naturforsch. **33A**, 305 (1978)]

$$V(r) \sim \frac{(k_B T)^2}{\kappa u(r)^2} \sim \frac{k_B T}{r^2}$$

The attachment free energy [Bruinsma, Goulian, Pincus, Biophys. J. **67**, 746 (1994)]

$$F_{\mathrm{attachment}} = \int_l^L 2\pi r V(r) dr = C k_B T \ln\left(\frac{L}{l}\right)$$

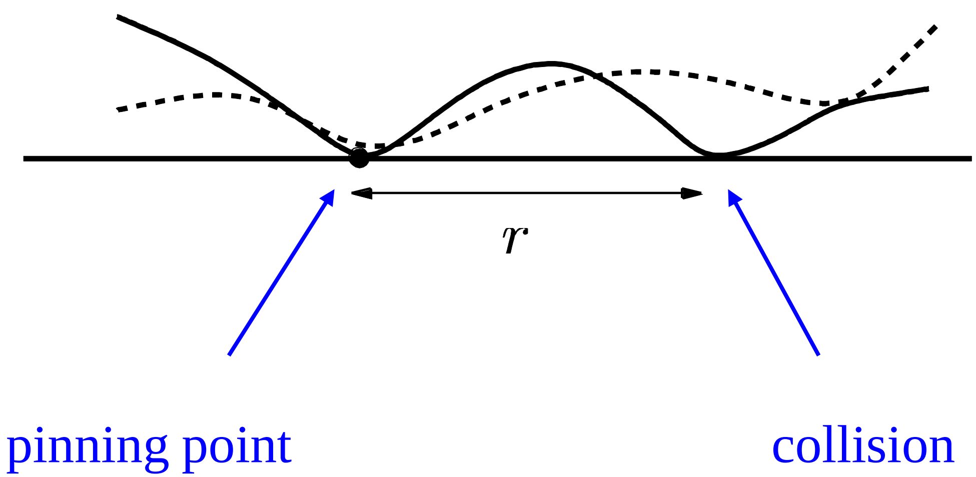
We already found $C = 2$

$$V(r) = \frac{k_B T}{\pi r^2}$$

Fluctuation-induced attraction between two adhesion points

$$V(r) = \frac{k_B T}{\pi r^2} \quad \Longleftrightarrow \quad p\left[h\left(\vec{r}\right) = 0\right] \sim \frac{1}{r^2}$$

probability density of hitting the surface at distance *r* from the pinning point



OF, Phys. Rev. E **81**, 050902(R) (2010).

But …

PF of a membrane with 2 adhesion points

$$p\left[h\left(\vec{r}\right)=0\right]\equiv\frac{Z\left(0,\vec{r}\right)}{Z(0)}\equiv g\left(\vec{r}\right)\sim\frac{1}{r^2}$$

pair distribution function

PF of a membrane with 1 adhesion point

… and by definition

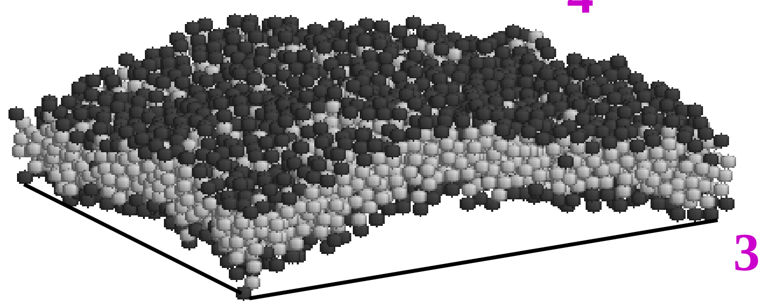
$$\phi\left(\vec{r}\right)=-k_BT\ln\left[g\left(\vec{r}\right)\right]=2k_BT\ln(r)$$

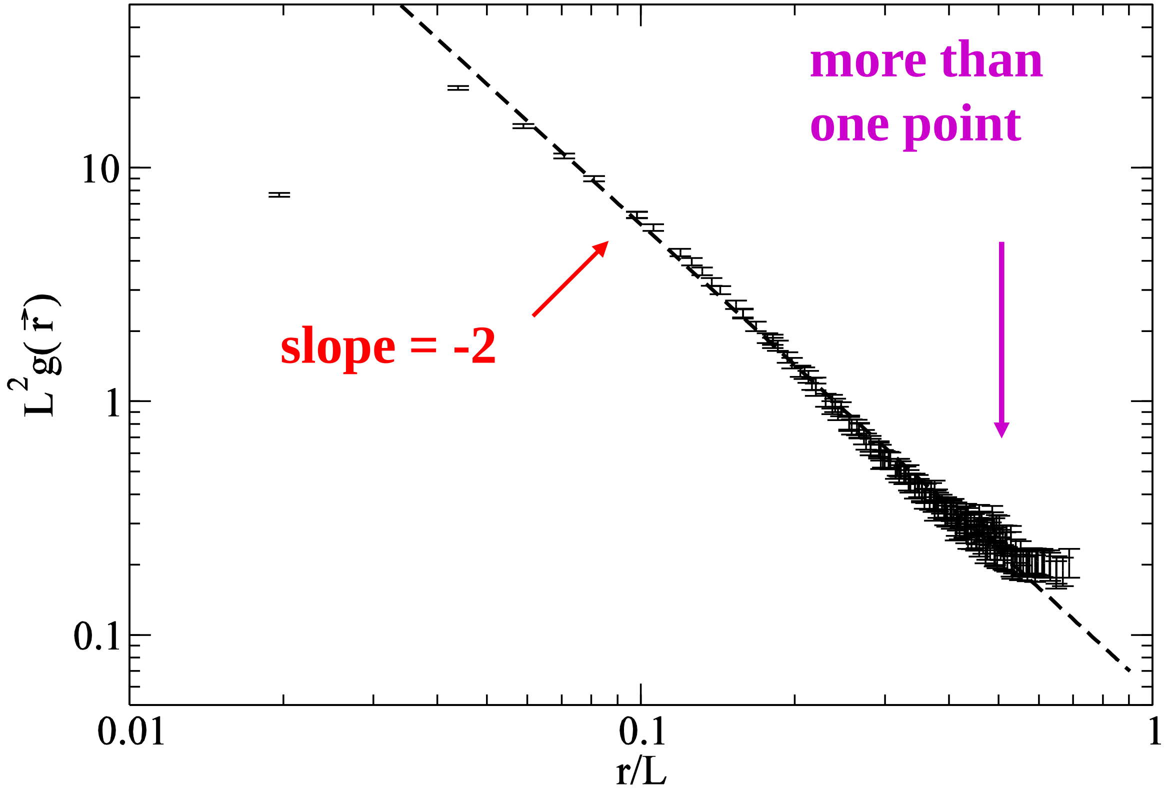
fluctuation –mediated pair potential of mean force

independent of κ !

Molecular simulations results

OF, Phys. Rev. E **81**, 050902(R) (2010).

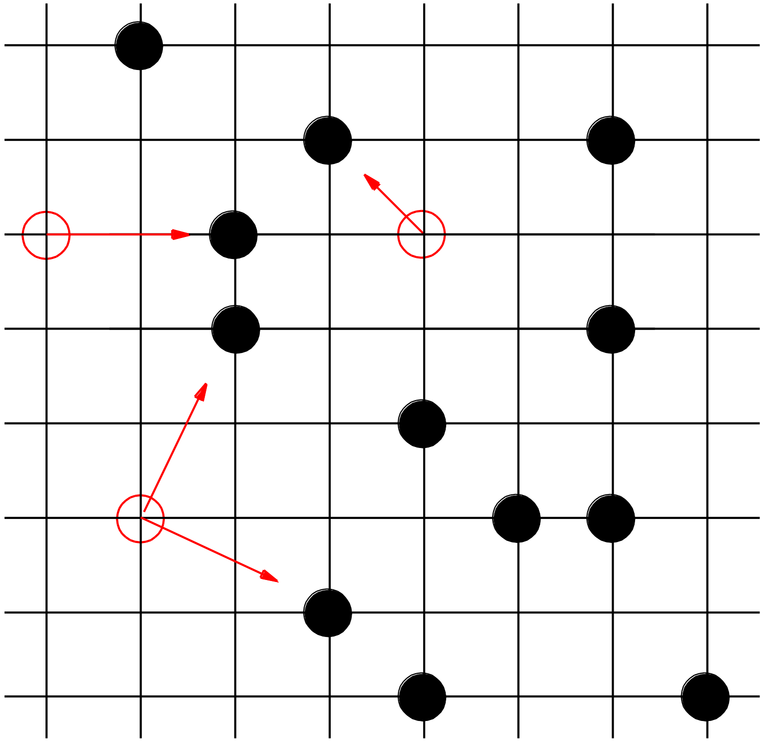




The interactions with the periodic images are weak

Formation of adhesion clusters

Suggestion: the extent by which the thermal fluctuations are limited is determined by the distance to the nearest adhesion point



$$V(r) = \frac{k_B T}{\pi r^2}$$ single adhesion point

$$V(\vec{r}) = \frac{k_B T}{\pi d^2}$$ multiple adhesion points

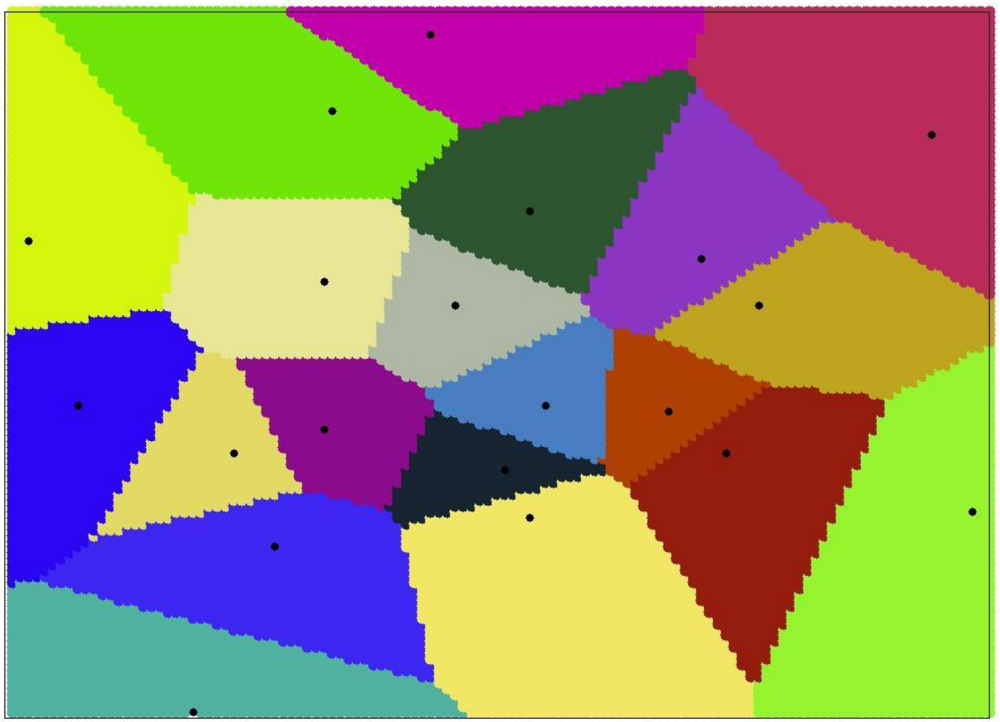
$$d = \min_{i=1,\ldots,N} \left(|\vec{r} - \vec{r}_i| \right)$$

N. Weil, OF, Submitted to EPJE (2010).

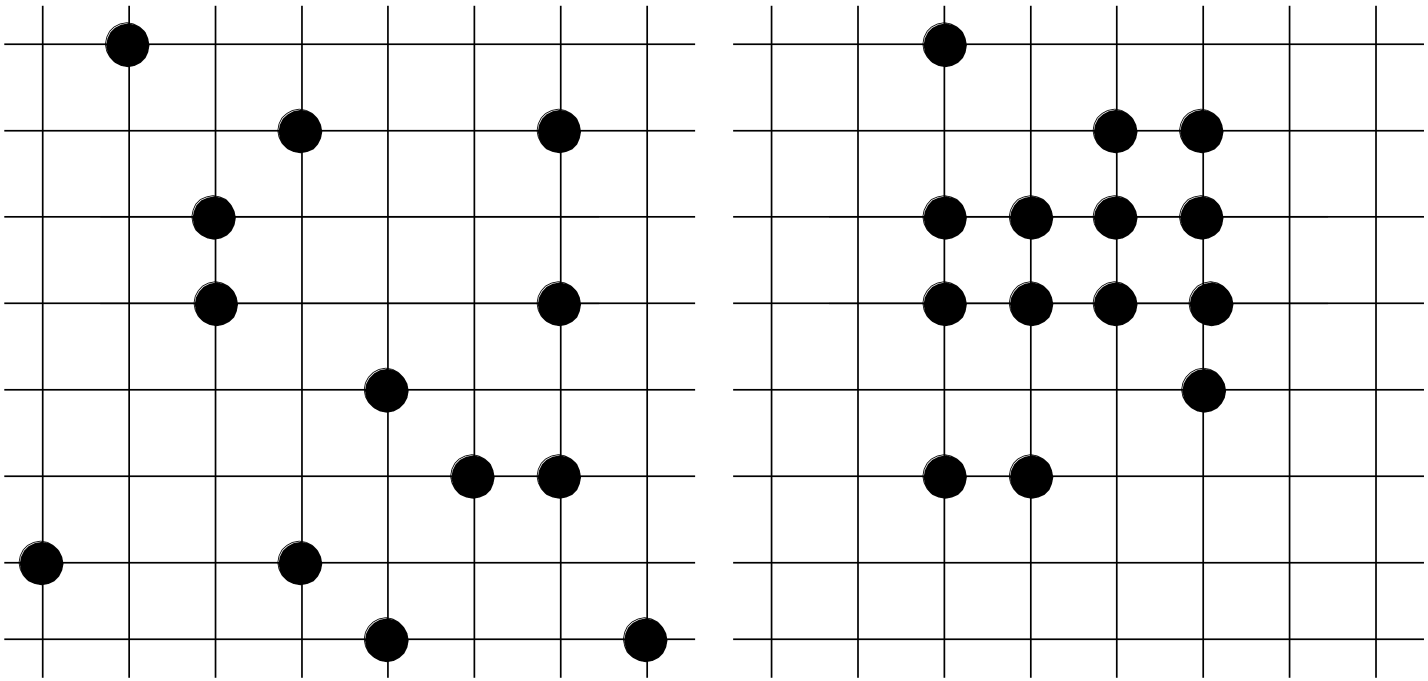
Voronoi diagram

The attachment free energy: $$F_{\text{attachment}} = \sum_{i=1}^{N} \int \frac{k_B T}{\pi d^2(\vec{r})} d^2\vec{r}$$

* excluding regions of size *l* around each attachment point



Lattice-gas model



$$\mathcal{H} = \sum_{\langle ij \rangle} -\epsilon s_i s_j + \sum_i U(\vec{r}_1, \ldots, \vec{r}_N)(1 - s_i)$$

$$\frac{k_B T}{\pi}\left(\frac{l}{d}\right)^2$$

direct interactions

fluctuation-mediated interactions

Mean-field analysis

N_s - lattice sites

N - adhesion ponits

N_c - clusters

$$\phi = N/N_s \ll 1$$
$$\phi^* = N_c/N_s \leq \phi$$

$$\frac{F_{\rm LG}}{N_s k_B T} = \phi^* \ln(\phi^*) - \phi^* + 2\phi\phi^* + \lambda\sqrt{\phi\phi^*}$$

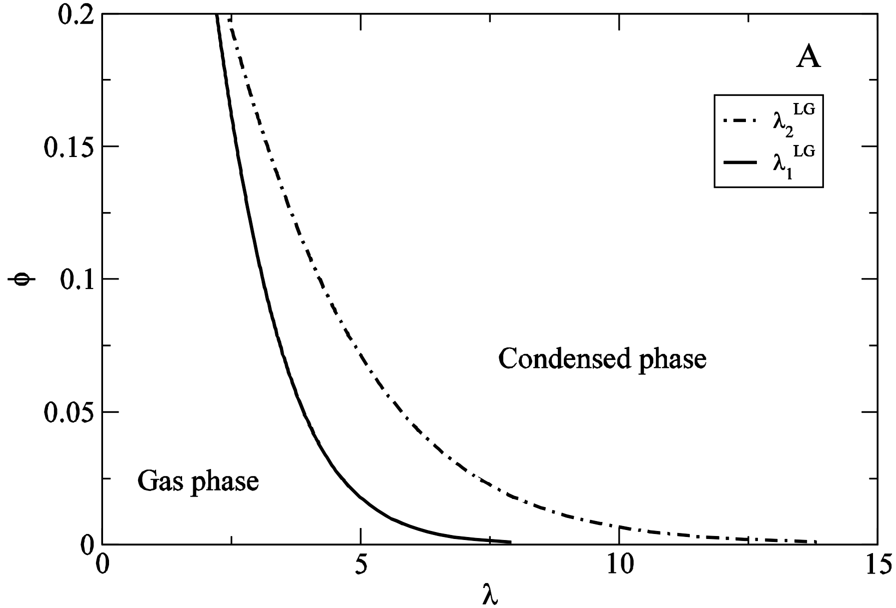
ideal gas mixing entropy , 2nd virial term , energy ($\lambda \propto \epsilon$ - line tension of clusters)

$$\frac{F}{N_s k_B T} = -\phi^* + 2\phi\phi^* + \lambda\sqrt{\phi\phi^*}$$

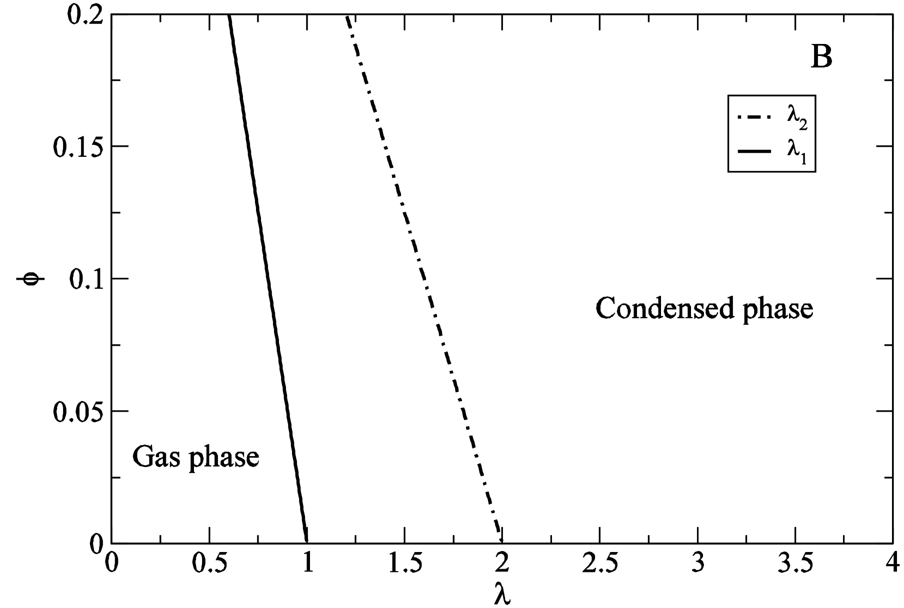
binodal ⟶ $\lambda_1^{\rm LG} = 1 - 2\phi - \ln(\phi)$ $\qquad$ $\lambda_1 = 1 - 2\phi$

spinodal ⟶ $\lambda_2^{\rm LG} = -4\phi - 2\ln(\phi)$ $\qquad$ $\lambda_2 = 2 - 4\phi = 2\lambda_1$

standard lattice-gas model



adhesion points



Notice the different scales of the axes !!!

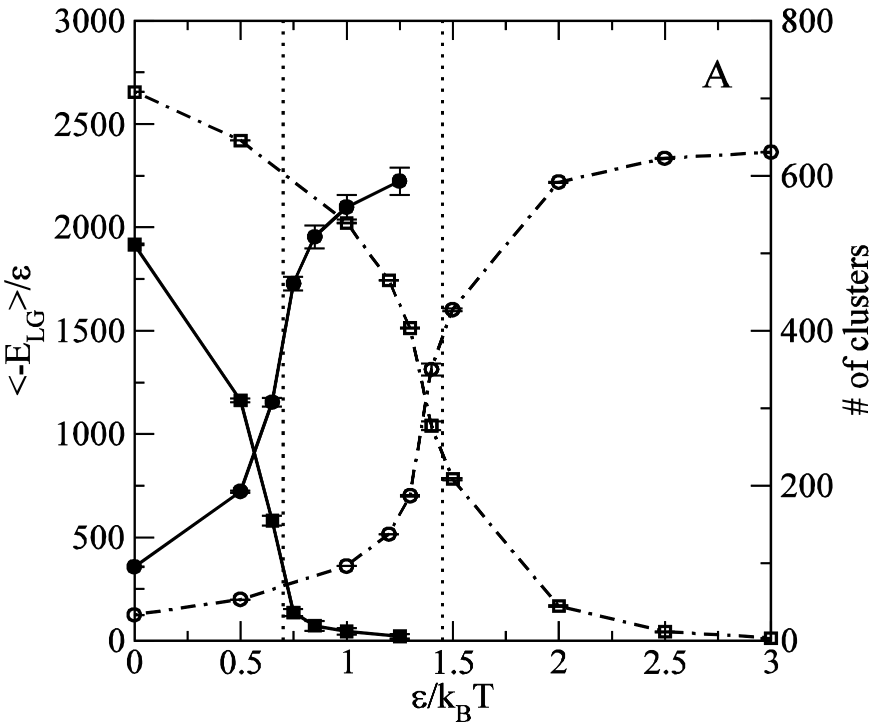
Memebrane fluctuations greatly reduce the strength of direct Interactions required to facilitate cluster formation !!!

See also:

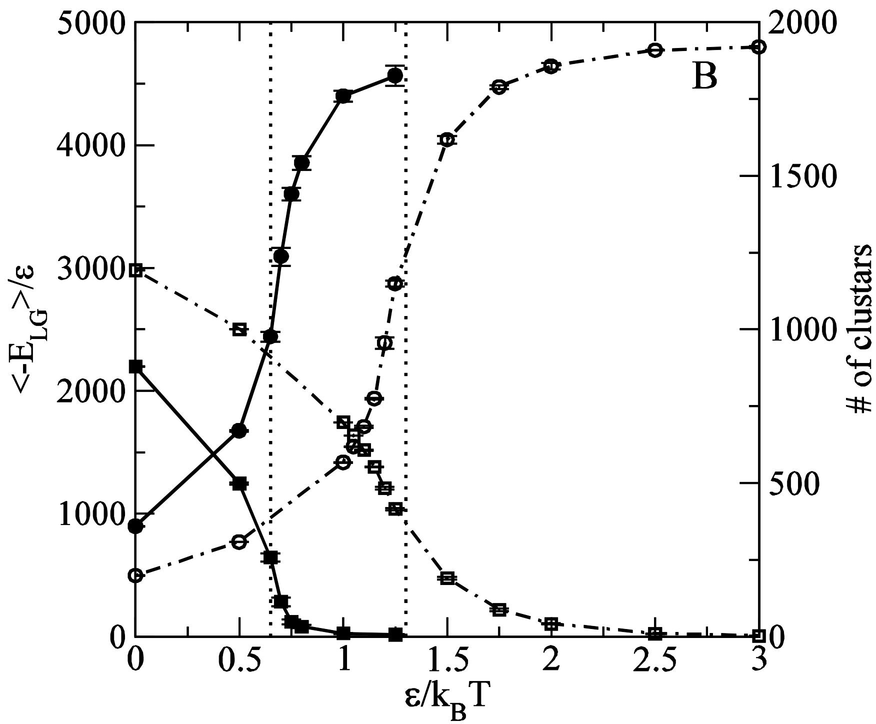
Weikl, Lipowsky, *Adv. Planar Lipid Bilayers and Liposomes* **5**, 63 (2007).

Speck, Reister, Seifert, cond-mat:1004.2696 (2010).

First-order condensation transition



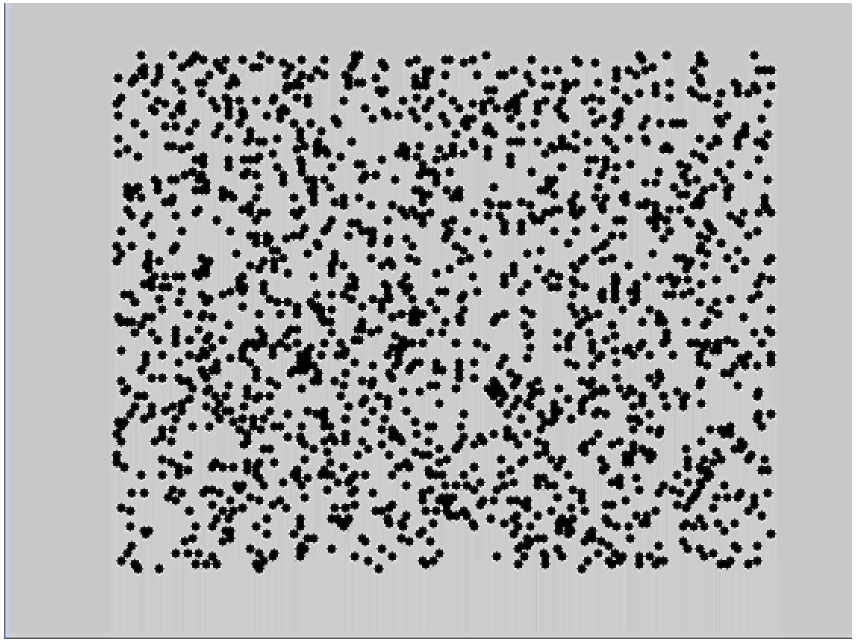
$\phi = 0.05$



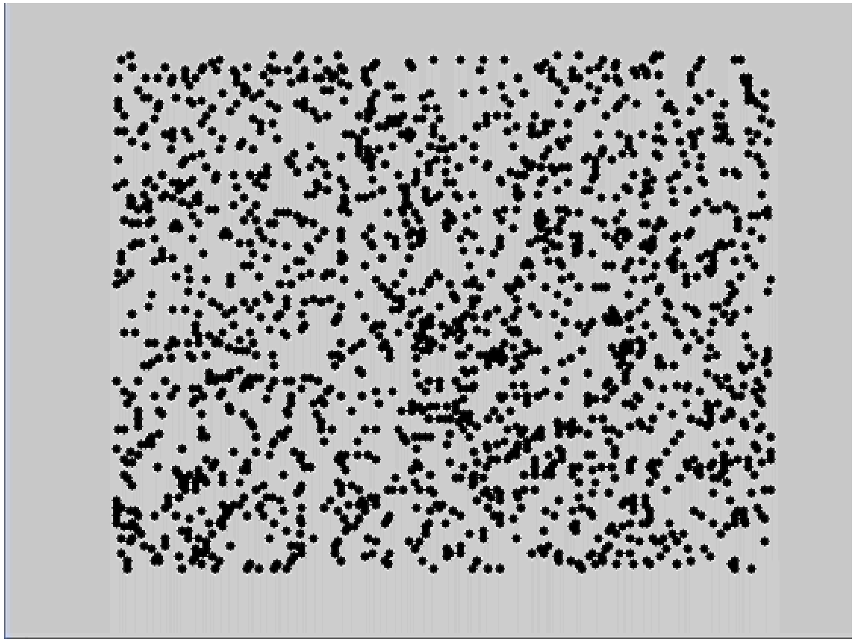
$\phi = 0.1$

Lattice-gas simulations

$$\phi = 0.1 \ , \ \epsilon = 1 k_B T$$



standard lattice-gas model



adhesion points

Summary

1. The thermal fluctuations of supported membranes induce attractive *many-body* interactions of *entropic origin* between the adhesion points.
2. The fluctuation-induced *pair* potential is *infinitely long-ranged* with a logarithmic dependence on the pair distance.
3. The fluctuation-induced interactions are *not sufficient* to allow the formation of adhesion domains, but they *greatly reduce* strength of the direct interactions required to facilitate cluster formation.