Observation of Optomechanical Strain in a Cold Atomic Cloud

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We report the observation of optomechanical strain applied to thermal and quantum degenerate ^{87}Rb atomic clouds when illuminated by an intense, far detuned homogeneous laser beam. In this regime the atomic cloud acts as a lens that focuses the laser beam. As a backaction, the atoms experience a force opposite to the beam deflection, which depends on the atomic cloud density profile. We experimentally demonstrate the basic features of this force, distinguishing it from the well-established scattering and dipole forces. The observed strain saturates, ultimately limiting the momentum impulse that can be transferred to the atoms. This optomechanical force may effectively induce interparticle interactions, which can be optically tuned.

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Light-matter interactions are at the core of cold atom physics. A laser beam illuminating atoms close to atomic resonance frequency will apply a scattering force on them, and an inhomogeneous laser beam far from resonance will mainly apply an optical dipole force [1]. An intense, far detuned homogeneous laser beam does not exert a significant force on a single atom, though when applied on inhomogeneous atomic clouds, it will. This was pointed out [2] while studying lensing by cold atomic clouds in the context of nondestructive imaging.

The atom's electric polarizability makes atomic clouds behave as refractive media with an index locally dependent on the cloud density. An atomic cloud thus behaves as a lens that can focus or defocus the laser beam. The atoms recoil in the opposite direction to the beam deflection due to momentum conservation. In solid lenses, this optomechanical force causes a small amount of stress with negligible strain, due to their rigidity. An atomic lens, however, deforms, making the force on the atoms observable by imaging their strain. We refer to this optomechanical force as electrostriction, since it resembles shape changes of materials under the application of a static electric field. Electrostriction can be viewed as an optically induced force between atoms, since the force each atom experiences depends on the local density of the other atoms.

Optomechanical forces are applied in experiments on refractive matter mainly by optical tweezers, pioneered by [3], using structured light. Less commonly, such forces can be applied by homogeneous light using angular momentum conversion due to the material birefringence [4], or using structured refractive material shapes [5]. Optomechanical forces implemented by such techniques are used for optically translating and rotating small objects. By applying electrostriction on cold atoms we gain access to the effect of optical strain—an aspect in optomechanics not directly studied yet in spite of its importance in current research [6].

Interactions between cold atoms can appear naturally or be externally induced and tuned. Tuning is mostly done using a magnetic Feshbach resonance, which was used to demonstrate many important physical effects such as Bose-Einstein condensate (BEC) collapse and explosion [7], Feshbach molecules [8,9], BEC-BCS crossover in strongly interacting degenerate fermions [10–12], and Fermi superfluidity [13–16]. Interactions are also tuned by optical Feshbach resonance [17], optical cavities [18], or radio frequency Feshbach resonance [19]. Interactions can be induced by shining a laser beam on the atoms and creating a feedback mechanism to their response by an externally pumped cavity or a half cavity [20–25]. The electrostriction force reported here is a new kind of induced force between atoms, and may be useful in cold atoms and quantum degenerate atom experiments.

In this Letter we analyze and measure for the first time the optomechanical strain induced in a cold atomic cloud by a homogeneous laser beam far detuned from atomic resonance. We shine the beam on the cloud and directly observe the resulting strain after time of flight by absorption imaging. We show that this is a new kind of light-induced force acting on cold atoms. A saturation of the strain is observed, which depends only on the ratio between the momentum impulse applied to the atomic cloud and the initial momentum distribution width of the cloud. Possible implications for this new force are suggested, and, in particular, light-induced interaction tuning.

With respect to laser light far from resonance, an inhomogeneous atomic cloud behaves as a lens [2], as predicted by the optical Bloch equations. When a plane wave passes through the cloud, it acquires a position-dependent phase $\phi(\vec{r})$. If the phase is small, the Poynting vector direction changes [26] by an angle $|\vec{\nabla}_\perp \phi|/k_L$, where $\vec{\nabla}_\perp$ is the gradient along the two directions perpendicular to the laser beam propagation direction, and k_L, the wave number of the beam. As a backaction, the atomic momentum changes in the opposite direction. The momentum change of the atoms is associated with the electrostriction force, which takes the form [27]

$$\vec{f}_{\rm es} = \frac{\hbar\Gamma^2}{8\Delta}\frac{I}{I_s}\frac{\vec{\nabla}_\perp n}{n} = -\vec{\nabla}_\perp U_{\rm es}$$
$$U_{\rm es} = \frac{\hbar\Gamma^2}{8\Delta}\frac{I}{I_s}\ln\left(\frac{n}{n_0}\right), \qquad (1)$$

where n_0 is an arbitrarily chosen constant density that fixes the arbitrariness in defining a potential up to a constant, Γ, the width of the atomic transition, Δ, the detuning of the laser, I, its intensity, I_s, the ^{87}Rb saturation intensity, and n, the local density of atoms.

This force acts only in the directions transverse to the beam propagation and is derived from a potential in the transverse directions that scale logarithmically with the density. It is a collective force in the sense that it acts only on atoms consisting of an inhomogeneous atomic cloud. The laser induces interactions between the atoms and the resulting force is independent of the number of atoms. The force scales as I/Δ, similar to the dipole force, and unlike other light-induced interactions predicted before [31–33], which are second order in atom-light coupling. For convex clouds it is repulsive for red detuned laser $\Delta < 0$, and attractive for blue detuned laser $\Delta > 0$, opposite to the dipole force. Similar to the dipole force, changing the polarization has a small effect of coupling different atomic states, which effectively changes I_s.

In the experiment we typically trap 10^6 ^{87}Rb atoms in the $|F=1, m_F=1\rangle$ ground state of the $5^2S_{1/2}$ manifold at a temperature of $T = 400$ nK. Our crossed dipole trap has typical trap frequencies of $\omega_x = \omega_y = 2\pi \times 45$ Hz and $\omega_z = 2\pi \times 190$ Hz. The atomic cloud, when illuminated by a red detuned laser beam with $\Delta = -100$ GHz, is optically equivalent to a graded index lens of Gaussian profile $e^{-x^2/(2\sigma_z^2)-y^2/(2\sigma_y^2)-z^2/(2\sigma_z^2)}$. Its peak refractive index is $n_{\rm ref} = 1.0000093$ and its widths are $\sigma_x = \sigma_y = 22$ μm, and $\sigma_z = 5.2$ μm. To generate the electrostriction force we use a $\lambda = 780$ nm laser, 50–200 GHz detuned from the $|F=2\rangle \to |F'=3\rangle$ transition. The beam is coupled to a polarization maintaining single mode fiber and ejects with a waist of 1.1 mm. Under these parameters, the dipole force associated with the laser beam itself is suppressed by 10^{-3} compared to the electrostriction force, and the scattering probability is only a few percent. The dipole force that the light focused by the atoms exerts on the atoms is negligible. The electrostriction beam is shone from the $\hat{y}$ direction (see Fig. 1). The atomic cloud is optically extended ($\sigma \gg \lambda$), so a simple refractive media treatment is adequate. It is dilute ($nk^{-3} = 0.25$), so dipole-dipole interatomic interactions [34,35] do not affect our experiment. To measure the force we apply a short pulse of duration τ_p right after releasing the cloud, and image the momentum distribution after a long expansion time [18 ms, Figs. 1(a)–1(c)] by absorption imaging along the $\hat{z}$ direction. Since the force is anisotropic, the cloud expands more in the transverse directions and gains an aspect ratio (AR) larger than unity. If the atoms do not move during the pulse (impulse approximation, $\tau_p \ll \omega^{-1}$) we can calculate the atomic cloud size σ along the transverse ($\perp$) and axial ($\parallel$) directions after time of flight. For a cloud with initial temperature T and after expansion time t,

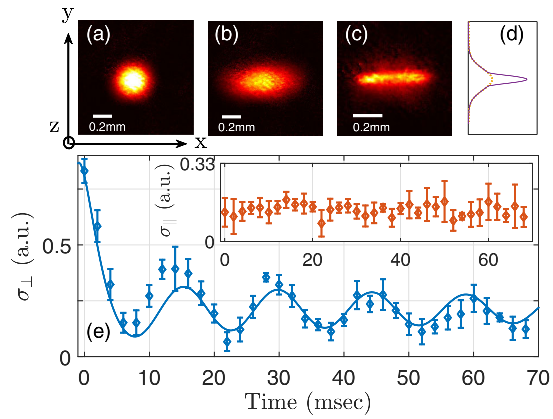

FIG. 1. Strain measurements. Absorption image of a thermal cloud after long expansion times with (b) and without (a) an electrostriction pulse. The cloud aspect ratio changes from unity to 2.1. We used a laser beam shone along the $\hat{y}$ axis with intensity 8×10^3 mW/cm^2, detuning 47 GHz and pulsed for 0.5 ms. (c) A BEC after an electrostriction pulse and long expansion time. Even for a strong impulse and large aspect ratio the BEC remains partly condensed, showing a bimodal distribution in the axial direction (d). (e) Oscillations in the cloud size along one transverse direction (axial direction shown in inset) induced by an electrostriction pulse as a function of a variable waiting time in the trap after applying the pulse. A pure transverse breathing mode is observed, fitting to a decaying oscillation (solid line) of twice the trap frequency.

$$\sigma_\perp = \sqrt{\frac{k_BT}{m\omega_\perp^2}}\sqrt{\left(1-\frac{\hbar\Gamma}{k_BT}\frac{\Gamma}{8\Delta}\frac{I}{I_s}\omega_\perp^2 t\tau_p\right)^2 + \omega_\perp^2 t^2},$$
$$\sigma_\parallel = \sqrt{\frac{k_BT}{m\omega_\parallel^2}}\sqrt{1+\omega_\parallel^2 t^2}. \qquad (2)$$

After a long expansion time the aspect ratio $\sigma_\perp/\sigma_\parallel$ of the cloud reaches an asymptotic value,

$$\mathrm{AR}^2 = 1 + \left(\frac{\hbar\Gamma}{k_BT}\frac{\Gamma}{8\Delta}\frac{I}{I_s}\omega_\perp\tau_p\right)^2 = 1 + \left(\frac{\sigma_P^{\rm es}}{\sigma_P^{\rm th}}\right)^2, \qquad (3)$$

where $\sigma_P^{\rm es} = (\hbar\Gamma\sqrt{m}/\sqrt{k_BT})(\Gamma/8\Delta)(I/I_s)\omega_\perp\tau_p$ is the momentum distribution width of the electrostriction impulse, and $\sigma_P^{\rm th} = \sqrt{mk_BT}$, the width of the initial cloud thermal momentum distribution.

The above derivation relies on the impulse approximation. In order to check its validity, we numerically solved the dynamics of the atomic cloud when applying electrostriction on it using a phase-space simulation. The simulation results coincide with our analytic predictions for all

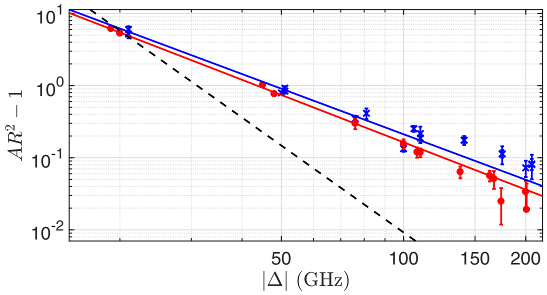

FIG. 2. Scaling of strain with detuning Δ. A thermal cloud AR after an electrostriction pulse and free expansion, red circles (blue crosses), correspond to a red (blue) detuned electrostriction laser. Fits to the data (solid lines) indicate a scaling of the force as $1/\Delta^\alpha$, with $\alpha = 1.09(5)$ [$\alpha = 1.05(9)$] for the red (blue) detuned electrostriction laser. A prediction (dashed line) based on a force that scales as $1/\Delta^2$ is shown as well. The error in α corresponds to a 95% confidence level. We used a cloud with a temperature of 1.1μK and a laser with intensity 1.1×10^4 mW/cm^2, pulsed for 0.5 ms.

measurements presented here, confirming the impulse approximation. Further theoretical considerations regarding the above derivation are detailed in [27].

Performing this experiment we observe that the electrostriction pulse neither changes the cloud size along the longitudinal direction nor the center of mass [Figs. 1(a) and 1(b)]. This indicates that our experiment suffers no significant scattering and demonstrates the transverse nature of the optomechanical strain. This is more dramatically demonstrated performing the same measurement on a BEC. In this case [Fig. 1(c)] the usually fragile bimodal distribution typical of a BEC along the axial direction is unaffected by the strong momentum impulse in the transverse directions. Similar results for pure condensates prove that the force acting on the atoms is different from that predicted in [33]. We nevertheless emphasize that our predictions in Eqs. (2) and (3) do not hold for a BEC, for which the equation has to be modified.

Applying an electrostriction pulse *in situ* generates a breathing mode oscillation, only in the transverse directions. This can be observed by letting the cloud evolve in the trap for some variable time, and imaging it after release [Fig. 1(e)]. The results in Fig. 1 did not depend on the laser polarization, in accordance with our theory. This observation also indicates that the interactions we induce between atoms are not dipole-dipole interactions.

We perform strain measurements after short electrostriction pulses for a large range of detunings $|\Delta| < 200$ GHz. The results (Fig. 2) are consistent with a $1/\Delta$ rather than a $1/\Delta^2$ scaling. This agrees with our prediction in Eq. (1) and rules out the scattering force and the forces in [32,33], which scale as $1/\Delta^2$, as a source of the strain observed. Imaging the cloud a short time after the electrostriction impulse we observe the effect of the detuning's sign as well [27].

To qualitatively compare our observations to the theoretical prediction [Eq. (2)], we carefully calibrate our experimental parameters. In particular, we measured the spontaneous Raman transition rate between the $|F = 1\rangle$ and $|F = 2\rangle$ hyperfine states due to the electrostriction laser. The measured rate was in accordance with the rate calculated [27] using the Kramers-Heizenberg equation [28,36], given the independently directly measured laser intensity and detuning values, and the atomic parameters [37]. After calibration, the observed effect is roughly 2.5 times weaker than expected. As we currently do not have an explanation for this discrepancy, we scale our predictions by this factor when comparing results to theory throughout this paper (Figs. 2–4).

We further investigated the dependence of the electrostriction force on the cloud parameters: total number of atoms N and cloud size. We measured the aspect ratio, N, and the cloud size, while applying the same strain pulse on the cloud (Fig. 3 and inset). As seen, the measured AR is independent of N, as expected from Eq. (1). On the other hand, the effect shows a strong dependence on the atomic cloud size. Decreasing the cloud size makes the cloud a stronger lens, causing the beam to focus stronger and impart more momentum on the atoms.

The dipole force might, in principle, cause dependence on the cloud size if the laser beam deviates from a plane wave, suffering intensity profile changes on length scales comparable with the cloud size. In order to avoid such situations, we work with a beam size about 100 times greater than our cloud size. We avoid speckles using a single mode fiber with a collimator and no other optical elements before the vacuum cell. We verified the absence of spatial sharp intensity changes by direct imaging of the beam. The strain we observed did not change after a slight misalignment of the beam, suggesting that indeed no significant local gradients appear. This shows that the

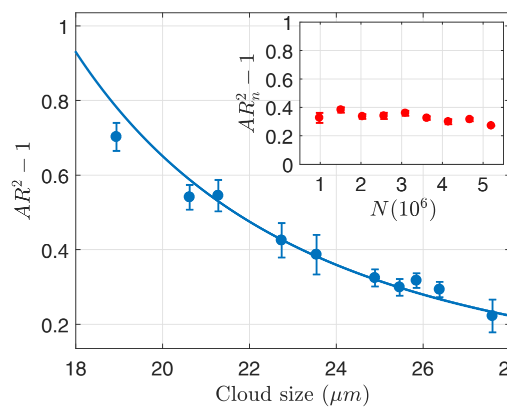

FIG. 3. Strain for clouds of different sizes. Measured cloud aspect ratio after an electrostriction pulse and free expansion for different cloud sizes (circles), and the theoretical prediction (line, scaled strain). All data points correspond to thermal clouds besides the first one, which includes a small condensed fraction. Smaller clouds consist of fewer atoms, but the AR_n (normalized AR [27]) is independent of the number of atoms as can be seen in the inset. We used a laser with intensity 7.4×10^3 mW/cm^2 and detuning 73 GHz, pulsed for 0.25 ms.

observed cloud size dependence is not due to a dipole force of the electrostriction beam.

In order to verify the linearity of the electrostriction force strength with intensity I, we measured the strain as a function of growing optical power and different pulse durations and detunings. As seen in Figs. 4(a) and 4(b), linearity is indeed evident for low intensities. However, a clear saturation of the strain [Figs. 4(a)–4(c)] occurs at high intensities, for various electrostriction pulse durations and detunings. We measured the dependence of saturation on the cloud temperature as well (not shown in Fig. 4), and found it appears to depend on the impulse applied to the atomic cloud $I\tau_p/(T\Delta)$. This is evident from the collapse of all data on a single curve as in Fig. 4. We note that the results presented in Figs. 2 and 3 were performed for unsaturated strain.

The saturation of the effect stems neither from changes in the internal state of the atoms nor from expansion of the cloud during the pulse. Our pulses are considerably short (up to 1 ms) compared with the trap oscillation period of typically 20 ms and the scattering rate of 20 Hz. We verified that there are no changes in the cloud density and internal state by imaging the cloud at short times and measuring the number of atoms in the $|F = 1\rangle$ hyperfine state. The only evident change is the momentum distribution of the atoms, which should not affect the strain via our theory. As is clear from Fig. 4(c), saturation occurs when the atoms have accelerated to a momentum roughly equal to their initial thermal velocity spread, $\sigma_P^{\rm es} = \sigma_P^{\rm th}$.

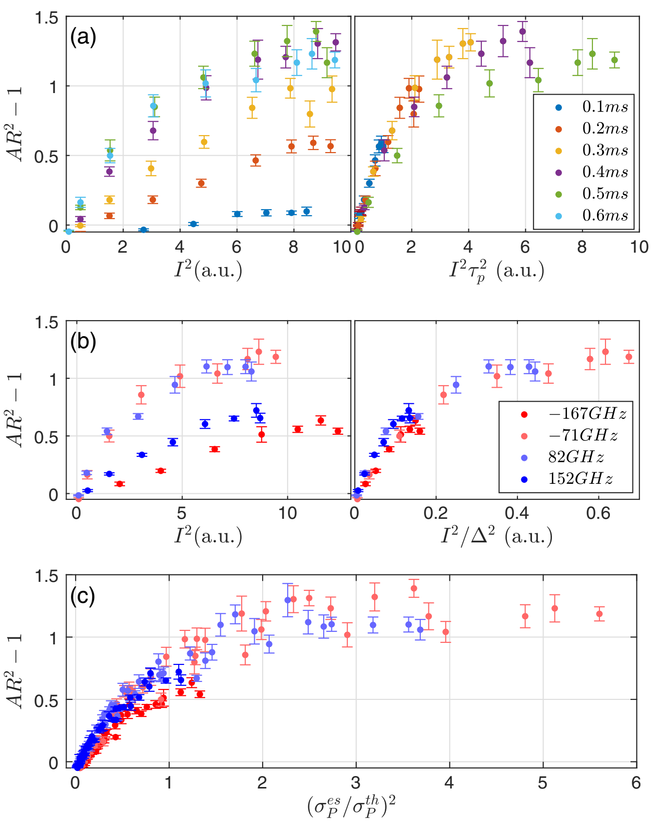

FIG. 4. Strain saturation with electrostriction laser intensity I, detuning Δ and pulse duration τ_p. (a) Saturation with laser intensity I for different pulse durations τ_p (left graph). After scaling the results by τ_p^2 (right) they collapse to a single curve. (b) Saturation with laser intensity I for different detunings Δ (left graph). After scaling the results by Δ^2 (right) they collapse to a single curve. (c) When plotted as a function of the momentum impulse $\sigma_P^{\rm es}$, all measurements collapse together. The laser intensity is changed between $0 - 9\times 10^3$ mW/cm^2, the detuning Δ between $(-167) - (+152)$ GHz, and the pulse duration τ_p between 0.1 and 0.6 ms. $\sigma_P^{\rm th}$ is the width of the thermal momentum distribution prior to the electrostriction impulse.

The observation that lensing saturates close to $\sigma_P^{\rm es} = \sigma_P^{\rm th}$ is reminiscent of a classical version of Einstein's recoiling-slit gedankenexperiment [38,39]. In this experiment an interference pattern of light that passed scatterers (slits) is dephased when the momentum imparted to the scatterers by the photons separates the scatterers in momentum space giving away the which-path information. In our experiment, lensing occurs due to coherent interference of light passing through different parts of the cloud. In an analogy to the above gedankenexperiment, the cloud would therefore cease to behave as a coherent lens after accumulating a momentum impulse $\sigma_P^{\rm es}$ comparable to their initial momentum distribution $\sigma_P^{\rm th}$.

The bound on the electrostriction momentum given to the atomic cloud may prevent application of electrostriction for long times. For short times, the optomechanical strain has some interesting features of potentially practical importance (details in [27]). An electrostriction laser beam applied to a BEC can effectively modify the interparticle interaction strength at the mean-field level, mimicking the effect of a Feshbach resonance, without really changing the scattering length. Interaction tuning was used before [40] for short times using an optical Feshbach resonance. A BEC with attractive effective interactions induced by an electrostriction laser is unstable to spatial density modulations seeded by initial noise in the density profile of the cloud, as in nonlinear optical fibers [41]. An atomic cloud with repulsive effective interactions works to smoothen out spatial density modulations. This can serve as an explanation to the unexplained red-blue asymmetry in [24]. The electrostriction potential [Eq. (1)] serves as a logarithmic nonlinearity, and thus a BEC under illumination can support stable solitons in any dimension [42], a nontrivial feature [43,44]. Finally, a thermal atomic cloud can be self-trapped by its own strain, resembling a bright soliton [45] in the transverse directions, incoherent and with arbitrary shape and size.

In summary, we report the observation of optomechanical strain applied to ^{87}Rb thermal and condensed atoms when illuminated by an intense, far detuned homogeneous laser beam. We experimentally demonstrate the basic features of electrostriction, distinguishing it from the well-established scattering and dipole forces, and proving that it is a new type of force acting on cold atoms. By the observed electrostriction characteristics, we point out that this force is distinct from theoretically predicted light-induced forces such as those discussed in [31–33] or collective forces measured in [46,47]. The experimental results are in qualitative agreement with our theory. Electrostriction has the potential to be an important tool in cold atom experiments as it effectively induces interparticle interactions, which can be optically tuned.

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[1] C. J. Foot, *Atomic Physics* (Oxford University Press, Oxford, 2005), Vol. 7.
[2] M. R. Andrews, M.-O. Mewes, N. J. van Druten, D. S. Durfee, D. M. Kurn, and W. Ketterle, Science **273**, 84 (1996).
[3] A. Ashkin, Phys. Rev. Lett. **24**, 156 (1970).
[4] M. Friese, T. Nieminen, N. Heckenberg, and H. Rubinsztein-Dunlop, Nature (London) **394**, 348 (1998).
[5] G. A. Swartzlander Jr., T. J. Peterson, A. B. Artusio-Glimpse, and A. D. Raisanen, Nat. Photonics **5**, 48 (2011).
[6] Z. Huang, K. Cui, Y. Li, X. Feng, F. Liu, W. Zhang, and Y. Huang, Sci. Rep. **5**, 15964 (2015).
[7] E. A. Donley, N. R. Claussen, S. L. Cornish, J. L. Roberts, E. A. Cornell, and C. E. Wieman, Nature (London) **412**, 295 (2001).
[8] E. A. Donley, N. R. Claussen, S. T. Thompson, and C. E. Wieman, Nature (London) **417**, 529 (2002).
[9] C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, Nature (London) **424**, 47 (2003).
[10] M. Greiner, C. A. Regal, and D. S. Jin, Nature (London) **426**, 537 (2003).
[11] C. A. Regal and D. S. Jin, Phys. Rev. Lett. **90**, 230404 (2003).
[12] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. **92**, 040403 (2004).
[13] D. E. Miller, J. K. Chin, C. A. Stan, Y. Liu, W. Setiawan, C. Sanner, and W. Ketterle, Phys. Rev. Lett. **99**, 070402 (2007).
[14] J. Siegl, W. Weimer, K. Morgener, K. Hueck, N. Luick, and H. Moritz, in *APS Division of Atomic, Molecular, and Optical Physics Meeting Abstracts* (2014), Vol. 1, p. 1016.
[15] I. Ferrier-Barbut, M. Delehaye, S. Laurent, A. T. Grier, M. Pierce, B. S. Rem, F. Chevy, and C. Salomon, Science **345**, 1035 (2014).
[16] M. Delehaye, S. Laurent, I. Ferrier-Barbut, S. Jin, F. Chevy, and C. Salomon, Phys. Rev. Lett. **115**, 265303 (2015).
[17] D. M. Bauer, M. Lettner, C. Vo, G. Rempe, and S. Dürr, Nat. Phys. **5**, 339 (2009).
[18] H. Ritsch, P. Domokos, F. Brennecke, and T. Esslinger, Rev. Mod. Phys. **85**, 553 (2013).
[19] Y. Ding, J. P. D'Incao, and C. H. Greene, Phys. Rev. A **95**, 022709 (2017).
[20] D. Nagy, J. Asboth, P. Domokos, and H. Ritsch, Europhys. Lett. **74**, 254 (2006).
[21] E. Tesio, G. R. M. Robb, T. Ackemann, W. J. Firth, and G.-L. Oppo, Phys. Rev. A **86**, 031801 (2012).
[22] E. Tesio, G. R. M. Robb, T. Ackemann, W. J. Firth, and G.-L. Oppo, Phys. Rev. Lett. **112**, 043901 (2014).
[23] G. R. M. Robb, E. Tesio, G.-L. Oppo, W. J. Firth, T. Ackemann, and R. Bonifacio, Phys. Rev. Lett. **114**, 173903 (2015).
[24] G. Labeyrie, E. Tesio, P. M. Gomes, G.-L. Oppo, W. J. Firth, G. R. Robb, A. S. Arnold, R. Kaiser, and T. Ackemann, Nat. Photonics **8**, 321 (2014).
[25] A. Camara, R. Kaiser, G. Labeyrie, W. J. Firth, G.-L. Oppo, G. R. M. Robb, A. S. Arnold, and T. Ackemann, Phys. Rev. A **92**, 013820 (2015).
[26] M. Born and E. Wolf, *Principles of Optics: Electromagnetic Theory of Propagation, Interference, and Diffraction of Light* (Elsevier, New York, 1980).
[27] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.119.163201, including Refs. [24,25,28–30], for detailed derivations, additional technical details, experimental results, and further research topics.
[28] R. Loudon, *The Quantum Theory of Light* (Oxford University Press, Oxford, 2000).
[29] P. Meystre and M. Sargent, *Elements of Quantum Optics* (Springer Science & Business Media, New York, 2013).
[30] H. Ritsch, P. Domokos, F. Brennecke, and T. Esslinger, Rev. Mod. Phys. **85**, 553 (2013).
[31] I. Mazets, Eur. Phys. J. D **8**, 371 (2000).
[32] G. Kurizki, S. Giovanazzi, D. ODell, and A. I. Artemiev, in *Dynamics and Thermodynamics of Systems with Long-Range Interactions* (Springer, New York, 2002), pp. 369–403.
[33] A. Kim, D. Cattani, D. Anderson, and M. Lisak, Sov. Phys. JETP **81** (2005).
[34] S. Balik, A. L. Win, M. D. Havey, I. M. Sokolov, and D. V. Kupriyanov, Phys. Rev. A **87**, 053817 (2013).
[35] L. Corman, J.-L. Ville, R. Saint-Jalm, M. Aidelsburger, T. Bienaimé, S. Nascimbène, J. Dalibard, and J. Beugnon, arXiv:1706.09698.
[36] R. Cline, J. Miller, M. Matthews, and D. Heinzen, Opt. Lett. **19**, 207 (1994).
[37] D. A. Steck, http://steck.us/alkalidata.
[38] P. A. Schilpp and H. Jehle, Am. J. Phys. **19**, 252 (1951).
[39] M. Jammer *The Philosophy of Quantum Mechanics: The Interpretations of Quantum Mechanics in Historical Perspective* (Wiley, New York, 1974).
[40] M. Cetina, M. Jag, R. S. Lous, I. Fritsche, J. T. Walraven, R. Grimm, J. Levinsen, M. M. Parish, R. Schmidt, M. Knap *et al.*, Science **354**, 96 (2016).
[41] K. Tai, A. Hasegawa, and A. Tomita, Phys. Rev. Lett. **56**, 135 (1986).
[42] I. Bialynicki-Birula and J. Mycielski, Bull. Acad. Polon. Sci. Cl **3**, 461 (1975).
[43] P. Pedri and L. Santos, Phys. Rev. Lett. **95**, 200404 (2005).
[44] J. W. Fleischer, M. Segev, N. K. Efremidis, and D. N. Christodoulides, Nature (London) **422**, 147 (2003).
[45] L. Khaykovich, F. Schreck, G. Ferrari, T. Bourdel, J. Cubizolles, L. Carr, Y. Castin, and C. Salomon, Science **296**, 1290 (2002).
[46] Z. Meir, O. Schwartz, E. Shahmoon, D. Oron, and R. Ozeri, Phys. Rev. Lett. **113**, 193002 (2014).
[47] J. Pellegrino, R. Bourgain, S. Jennewein, Y. R. P. Sortais, A. Browaeys, S. D. Jenkins, and J. Ruostekoski, Phys. Rev. Lett. **113**, 133602 (2014).