Fast Dynamical Decoupling of the Mølmer-Sørensen Entangling Gate

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(Received 28 June 2017; published 29 November 2017)

Engineering entanglement between quantum systems often involves coupling through a bosonic mediator, which should be disentangled from the systems at the operation's end. The quality of such an operation is generally limited by environmental and control noise. One of the prime techniques for suppressing noise is by dynamical decoupling, where one actively applies pulses at a rate that is faster than the typical time scale of the noise. However, for boson-mediated gates, current dynamical decoupling schemes require executing the pulses only when the boson and the quantum systems are disentangled. This restriction implies an increase of the gate time by a factor of $\sqrt{N}$, with N being the number of pulses applied. Here we propose and realize a method that enables dynamical decoupling in a boson-mediated system where the pulses can be applied while spin-boson entanglement persists, resulting in an increase in time that is at most a factor of $\pi/2$, independently of the number of pulses applied. We experimentally demonstrate the robustness of our entangling gate with fast dynamical decoupling to σ_z noise using ions in a Paul trap.

DOI: 10.1103/PhysRevLett.119.220505

High-quality on-demand generation of entanglement is a necessary condition for quantum information processing and quantum metrology. While for some physical platforms entanglement is generated by an inherent direct interaction between subsystems, various platforms of interest make use of a mediating boson with spin-dependent coupling. For instance, the interaction between trapped ions is carried via a vibrational phonon [1–14]; superconducting qubits are entangled via a microwave photon [15–17]; the interaction between distant NVs can be carried via a nanomechanical oscillator's phonon [18,19]; and a cavity photon carries the interaction between atoms in cavity quantum electrodynamics architectures [20–22]. The common Hamiltonian representing these quantum systems is of the form

$$H(t) = \tilde{\Omega} \sum_i \sigma_{\phi,i} (b^\dagger e^{i\varepsilon t} + \text{H.c.}), \tag{1}$$

with $\sigma_{\phi,i}$ representing the Pauli matrix in the ϕ direction of the ith spin, $b^\dagger$ the boson creation operator, and $\tilde{\Omega}$ the coupling strength between the spins and the bosonic mediator. ε denotes a detuning of the coupling term from the resonance of the combined spin-boson transition, and can often be controlled experimentally. This Hamiltonian thus represents a bosonic mode coupled off-resonantly to a number of spins. In the trapped ion case, this Hamiltonian allows one to execute the Mølmer-Sørensen (MS) gate [1]. After times $2\pi n/\varepsilon$ for an integer n, the boson is disentangled from the spins, leaving the spins entangled via a geometric phase which is proportional to the area of the closed circle traced by the boson trajectory in phase space [2,23,24].

Despite considerable progress in achieving high-fidelity entanglement in recent years, entanglement fidelity remains a primary obstacle for the performance of large-scale quantum information processing, and more particularly for fault-tolerant quantum computation. Attempts to improve the fidelity of entangling gates must overcome the limitations imposed by environmental noise as well as imperfections in the control apparatus. Dynamical decoupling (DD) is a common method for fighting the effects of noise. When utilizing DD pulses [25,26] during the entangling gate operation, one is required to consider the effect of the spin-dependent coupling on the mediating boson. In many experiments, a single DD pulse has been applied at a time $2\pi/\varepsilon$, exactly when the boson is disentangled from the spins [9–11,14]. However, a single pulse only eliminates the effect of the constant (dc) part of the noise and does not efficiently combat finite-frequency (ac) noise.

In order to improve the decoupling efficiency, the number of pulses and the frequency of dynamic decoupling should be increased. However, such an increase comes at a price: when applying N DD pulses, the constraints imposed by boson coupling—namely, applying the pulses only at times when the spin and boson are disentangled—impose an overall gate duration that is prolonged by a factor of $\sqrt{N}$. The prolonged time makes the gate more vulnerable to other uncompensated noise sources that reduce the gate fidelity. This can be contrasted with NMR schemes [27–29], where the gate time remains unaffected by the number of DD pulses.

The difficulty of adding DD pulses during gate operation is due to the need to apply the pulses in an orthogonal direction to the gate operator, denoted by ϕ in Eq. (1). This need often originates from the existence of noise that is parallel to the gate operator, such as the external parallel noise terms in the microwave gradient scheme [19,20,30] and in the single sideband protocols in the different platforms [12–18,21,22,24]. Note that in the case of a slow noise term that is orthogonal to the gate operator, it is sufficient to perform a small number of DD pulses along the direction of the gate operator. Since these pulses commute with the gate operation, they can be applied even when the spins and motion are entangled without affecting the structure of the gate or its duration. However, when the orthogonal noise is fast, and many parallel DD pulses are needed, the parallel pulse imperfections accumulate to an appreciable effect. In other words, these pulse imperfections result in parallel noise, which enforces the use of additional orthogonal DD pulses (like an XY4 or an XY8 sequence [31–33]) that create the difficulty.

In this Letter we present, and experimentally demonstrate with trapped ions, a DD scheme for boson-mediated systems that yields a refocused entangling gate, whose gate duration is increased by a reduced factor of $\sim\pi/2$. This scheme enables implementation of complex DD pulse sequences—such as CPMG and XY8—in boson-mediated systems. Furthermore, this scheme of using pulsed DD without significantly increasing the gate time can be integrated with pulsed schemes such as Refs. [34–36] for suppressing amplitude noise and timing inaccuracies that might result in entanglement between motional and internal degrees of freedom at the end of the gate.

For two qubits, the time evolution given by the MS Hamiltonian [Eq. (1)], obtained from either the MS gate or the single sideband gate (see Supplemental Material [37]), is

$$U_{\mathrm{MS}}(t) = \mathcal{D}\left(\frac{\tilde{\Omega}}{\varepsilon}\sum_{i=1,2}\sigma_{\phi,i}(1-e^{i\varepsilon t})\right)\cdot \times \exp\left[i\left(\frac{\tilde{\Omega}}{\varepsilon}\sum\nolimits_{i=1,2}\sigma_{\phi,i}\right)^2(\varepsilon t-\sin[\varepsilon t])\right], \qquad (2)$$

with $\phi = y,\ x$ denoting the MS or the single sideband gate, respectively, and i indexing the qubits. $\mathcal{D}(\alpha) = \exp(\alpha b^\dagger - \alpha^* b)$ is the displacement operator; therefore, the first term traces a circle in phase space with a radius which is proportional to $\tilde{\Omega}/\varepsilon$. At times $t_n = 2\pi n/\varepsilon$, for an integer n, the system returns to its original location in phase space, meaning the qubits and the boson mediator are disentangled and a pure two-qubit state is achieved. In the conventional DD scheme, one applies pulses at these times only, as naively applying DD pulses at any other time might decouple the qubit from the boson mediator or couple it in an uncontrolled way. N pulses require at least N such instances, giving a gate time that can be written as $T_N = 2\pi N/\varepsilon$. A maximally entangling gate is generated when the accumulated geometric phase is $4\tilde{\Omega}^2 T_N/\varepsilon = \pi/2$. Combined with the former condition, we arrive at $(T_N)^2 = (\pi/2\tilde{\Omega})^2 N$, or $T_N \sim \sqrt{N}$.

The restrictions imposed above undermine the efficiency of the conventional DD scheme for combatting ac noise. Since the DD time separation scales as $1/\sqrt{N}$ and thus the DD frequency scales as $\sqrt{N}$, and the gate time similarly scales as $\sqrt{N}$, a higher DD frequency will counter more of the noise spectrum, but will accordingly prolong the gate duration, causing the rest of the noise spectrum to be more damaging to the overall fidelity. Such a scheme is only effective for power spectrums that decay faster than $1/\omega$, meaning that due to the prolonged gate time the decoherence will scale as $\omega_{\mathrm{DD}} S_n(\omega_{\mathrm{DD}})$ instead of $S_n(\omega_{\mathrm{DD}})$, where $S_n(\omega)$ is the noise power spectral density and ω_{DD} is the frequency of the DD pulses.

Here we propose an alternative approach, in which the DD pulses are used for covering a larger area in the boson phase space. In this way, higher boson states are populated during the gate, which is, therefore, performed with low time overhead [2], regardless of the number of DD pulses involved. The π pulses alternate the sign of the $\sigma_{\phi,i}$ operators in the MS unitary [Eq. (2)], reversing the direction of the spin-dependent force and thus resulting in a greater effective spin-dependent displacement. For instance, a MS unitary for $t = \pi/\varepsilon$ duration gives rise to a spin-dependent displacement of $2\tilde{\Omega}/\varepsilon$, while a sequence of two such unitaries with an intermediate π pulse results in a double spin-dependent displacement of $4\tilde{\Omega}/\varepsilon$ [Fig. 1(b)]. Similar methods, relying on spin-dependent displacements, allow for ultrafast gates [38,39]. Although in this Letter we restrict ourselves to a two-spin case, the boson dynamics are identical for any number of spins as long as a single bosonic mode is in play, making a generalization to more spins straightforward.

By applying N equally spaced π pulses, with time separation $\Delta t(N) = \pi(2+N)/N\varepsilon$, a flower-shaped path in phase space is closed at the end of the gate operation

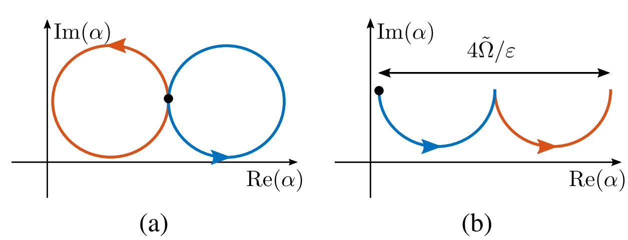

FIG. 1. Trajectory in phase space of a spin with initial state $|\uparrow,\alpha\rangle$, where α is the coherent state of the trap (marked as a bullet), due to the MS unitary [Eq. (2)]. (a) Applying a π pulse when the spin and the boson are disentangled, at times $t_n = 2\pi n/\varepsilon$, causes the spin to change between the blue and orange trajectories. (b) Applying a π pulse at time $t = \pi/\varepsilon$ causes the spin to change from the blue to the orange trajectory, resulting in an effective $4\tilde{\Omega}/\varepsilon$ displacement at time $t = 2\pi/\varepsilon$.

$T_{\mathrm{tot}} = \pi(N+2)/\varepsilon$, thus remaining decoupled from the boson field [Fig. 2(a)]. The geometric phase accumulated in the area enclosed by the flower's petals $4\tilde{\Omega}^2 T_{\mathrm{tot}}/\varepsilon$ is equal to the geometric phase accumulated in the slow-coupling regime, i.e., the area accumulated without applying the DD pulses. However, by applying the DD pulses, an additional geometric phase is accumulated in the polygon area $A = 8N(\tilde{\Omega}/\varepsilon)^2 \cot(\pi/N)$.

A maximally entangled state can be generated when the overall accumulated geometric phase is

$$A + \frac{4\tilde{\Omega}^2}{\varepsilon} T_{\mathrm{tot}} = \left(\frac{2\tilde{\Omega}}{\varepsilon}\right)^2 \left(\varepsilon T_{\mathrm{tot}} + 2N \cot\frac{\pi}{N}\right) = \frac{\pi}{2}, \quad (3)$$

with the gate duration being a monotonically increasing function of N

$$T_{\mathrm{tot}} = \frac{\pi}{2\tilde{\Omega}} \frac{\frac{N}{2}+1}{\sqrt{\frac{N}{2}+1+\frac{N}{\pi}\cot\frac{\pi}{N}}} \underset{N\to\infty}{\to} \frac{\pi}{2\tilde{\Omega}}\frac{\pi}{2}, \quad (4)$$

and with a pulse time separation limit $\Delta t(N) \to \pi^2/4\tilde{\Omega}N$. Hence, although applying a large number of DD pulses $N \gg 1$, the gate duration is prolonged only by a factor of less than $\pi/2$. Intuitively, this can be understood by the following observation: at this limit of $N \gg 1$, the geometric phase accumulated in the flower's petals is negligible relative to the polygon phase, which is approximately a circle. Every separation time of $\Delta t \approx \pi/\varepsilon$, we accumulate a $2\tilde{\Omega}/\varepsilon$ spin-dependent displacement [Fig. 2(b)]. Therefore, the effective displacement velocity in phase space, which is the angular velocity of the accumulated circle, is $2\tilde{\Omega}/\pi$. Comparing this to the regular strong coupling gate, where the effective displacement velocity is $\tilde{\Omega}$ [Eq. (1)], and taking into account that both gates should accumulate the same circle area in phase space, we find a factor of $\pi/2$ in the gate durations. Note that in our derivation we have considered instantaneous DD pulses; finite pulse time effects are discussed in the Supplemental Material [37].

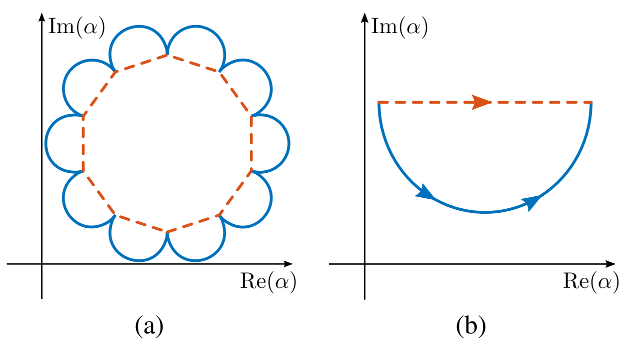

FIG. 2. Phase space trajectory of a spin under the MS Hamiltonian [Eq. (1)] dynamics and multiple DD pulses, where α is the coherent state of the trap. (a) Utilizing DD pulses for enlarging the area in phase space. A flower-shaped area is generated by applying N DD pulses with a specific time separation. The area enclosed by the flower shape, which circumscribes a polygon (orange dashed line) and flower petals (blue line), is proportional to the accumulated geometric phase. At the limit of many pulses $N \gg 1$, the polygon can be approximated as a circle, and the petals' area contribution nulls. (b) Every two π pulses separated by $\Delta t \approx \pi/\varepsilon$ give rise to a spin-dependent displacement (orange dashed line) $2\tilde{\Omega}/\varepsilon$, where the path effective velocity is $2\tilde{\Omega}/\pi$. In comparison to the regular strong coupling entangling gate where the path effective velocity is $\tilde{\Omega}$ [Eq. (1)], we find a factor of $\pi/2$ in the gate durations.

We experimentally implement the gate with fast DD using trapped ions and demonstrate its robustness to noise. Two $^{88}Sr^+$ ions are spatially confined in a linear Paul trap with an axial frequency of $\nu = 1.67$ MHz and radial frequencies of ~4 MHz [40]. A qubit is encoded on Zeeman-split sublevels of the $5S_{1/2}(m=-1/2) \to 4D_{5/2}(m=1/2)$ optical electric-quadrupole transition of each ion with a natural lifetime of ~0.4 s. This transition is driven using a narrow line width (~60 Hz) 674 nm laser locked to a stable Fabry-Perot cavity. State-selective fluorescence detection is performed by illuminating the ion with a 422 nm laser resonant with the $5S_{1/2} \to 5P_{1/2}$ dipole allowed transition and collecting the fluorescence signal with an EMCCD camera, enabling a nonambivalent readout of the two-ion state. The ions are Doppler cooled, followed by sideband cooling of the center-of-mass motion to the ground state. The Mølmer-Sørensen Hamiltonian [Eq. (1)] is enacted via bichromatic off-resonant driving of the $5S_{1/2} \to 4D_{5/2}$ transition with frequencies $\omega_{\mathrm{SD}} \pm (\nu + \varepsilon)$, where ω_{SD} is the resonance carrier frequency [41]. DD π pulses are implemented by halting the bichromatic field operation and pulsing a monochromatic field with frequency ω_{SD}. In protocols where more than a single pulse is needed, the phases of consecutive pulses are flipped in order to reverse coherent buildup of error due to pulse imperfections. The typical coupling constants for the bichromatic and monochromatic fields are $\eta\Omega_B \approx$ 3 kHz and $\Omega_M \approx 170$ kHz, respectively. A fast gate—with no DD—is performed at a detuning of $\varepsilon = 2\eta\Omega_B \approx 6$ kHz, giving a gate time of $T_{\mathrm{fast}} = 1/\varepsilon \approx 166$ μs. The bosonic mediator of interaction is the axial center-of-mass phononic mode.

In order to experimentally demonstrate the robustness of the dynamically decoupled entangling gate to σ_z-type noise, we detune the center frequency of the bichromatic and monochromatic fields, perform the entangling gate protocol, and measure the fidelity of the achieved state ρ with respect to the desired fully entangled state $|\psi\rangle \equiv (1/\sqrt{2})(|gg\rangle + i|ee\rangle)$ (Fig. 3). Varying the center frequency adds a constant σ_z term to the Hamiltonian similar to the effect of an external dc magnetic field. The fidelity at the end of the gate is calculated as $\mathcal{F} = \langle\psi|\rho|\psi\rangle$, which is the overlap squared of the measured state with the desired

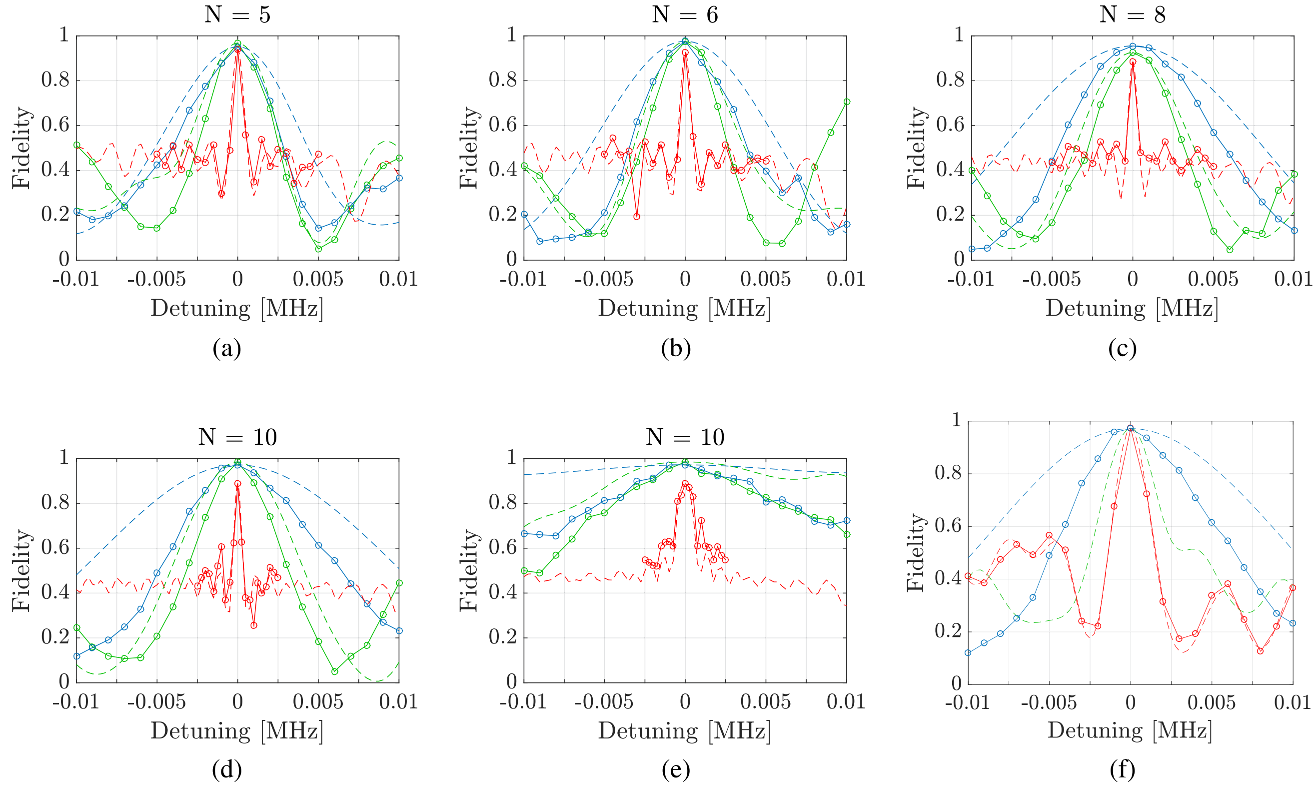

FIG. 3. An experimental and numerical comparison of σ_z robustness for three Mølmer-Sørensen entangling gate protocols: the fast DD scheme, as proposed in this Letter (blue), the slow DD scheme (green), and the slow DD scheme without DD pulses (red). Circles represent measurements connected by a solid line as a guide to the eye. Simulation results are presented in dashed lines. The number of pulses is denoted as N. The comparison is shown for differing pulse sequences enumerated by the number of DD arms. Two $^{88}Sr^{+}$ ions were entangled according to the appropriate protocol using a 674 nm laser and their final state measured via state-selective fluorescence. σ_z noise was implemented by detuning all driving lasers from their resonance frequencies. The fidelity with the specific fully entangled state, calculated from measurement results, is shown. Confidence intervals of 95% are under ± 0.03 and are not plotted. Numerical simulations were done for the MS Hamiltonian with the appropriate pulse sequence and a detuning term, alternating the dynamics between the MS Hamiltonian [Eq. (1)] with detuning $(\Delta/2)\sum_i \sigma_{z,i}$ and the detuned π pulse. Graphs (a)–(d) show the fidelity for a specific state. Graph (e) shows the fidelity for an entangled state up to an arbitrary phase; it shows that high-quality entangled states are achieved even at large detunings, albeit at a phase that differs from the zero-detuning case. Graph (f) compares the fast MS gate with $N = 0$ (red) and MS with $N = 1$ (green), the most popular schemes, vs the flower scheme with $N = 10$ (blue). The ten-pulse flower DD scheme offers a significantly more robust response with only a small overhead in time.

state. We execute this experiment with three distinct protocols: (a) The fast DD scheme, as detailed in the paragraphs above. (b) The slow scheme, in which a π pulse is applied only when the boson mediator is fully decoupled from the qubit subspace. (c) The slow scheme without executing the DD pulses. The latter acts as a reference to which one can compare the two DD schemes, thereby showing their meaningful impact. Furthermore, we execute all three protocols with different numbers of DD arms, ranging from five to ten. We compare the experimental results to a numerical simulation of the different protocols.

The gate with fast DD is shown to be more robust to σ_z noise than its slow counterpart with the same number of DD pulses. Increasing the number of pulses generates a marked robustness, particularly with the fast scheme. Measuring final-state fidelity with respect to some maximally entangled state at arbitrary phase shows that the generation of entanglement is fairly robust, and that a considerable portion of fidelity loss with respect to the specific required state at finite detuning is due to a phase shift of the entangled state. The reason for discrepancy between simulations and experiment is not known. Note that these measurements simulate dc noise only; for ac noise, the benefits of the fast scheme should be even more pronounced.

Discussion.—One interesting application of the gate with fast DD is in microwave-based trapped ion platforms. To overcome the negligible microwave photon recoil, the spin-motion interaction can originate from a static magnetic-field gradient. This gives rise to the MS Hamiltonian [Eq. (1)] $\tilde{\Omega}\sum_i \sigma_{z,i}(b^\dagger e^{i\epsilon t} + \text{H.c.})$, where $\tilde{\Omega} = \mu_B X_{i,n} dB_z/dz$ is the Rabi frequency, μ_B is the Bohr magneton, $X_{i,n}$ is the standard deviation of the nth vibrational mode and the ith ion, dB_z/dz represents the magnetic field gradient in the z axis, and $\epsilon = \nu_n$ is the vibrational frequency of mode n. As the microwave qubits have to be magnetic-field dependent, they are also sensitive to the ambient magnetic-field fluctuations. To compensate for this noise, pulsed DD has been considered in Ref. [42]; however, due to the very high detunings $\tilde{\Omega} \ll \varepsilon$, the gate was performed in the slow-interacting regime, and thus resulted in a very modest fidelity. By utilizing the presented scheme, the number of pulses and their duration could be adjusted such that the gate can be realized without this limitation. In comparison to current microwave-based entangling gates [43–46], the flower gate can be more than an order of magnitude faster, having a similar duration to Ref. [13].

Many quantum systems use a boson to mediate the interaction between different qubits. DD techniques can be used in order to mitigate the damage of noise on these systems. The naive approach of applying the DD π pulses, at times when the spins are disentangled from the boson, increases the gate duration by a factor of $\sqrt{N}$. To overcome this issue, we have proposed to apply DD pulses with a certain time separation, such that higher levels of the boson degrees of freedom are populated. In this way, the DD pulses not only suppress the main noise sources during the boson-mediated interaction, but also considerably reduce the DD time overhead, increasing the gate robustness to other uncompensated noises.

A. Rotem thanks Tomer Nussbaum for useful discussions. We acknowledge the support of the Israel Science Foundation (Grant No. 1500/13), the Marie Curie Career Integration Grant (CIG) IonQuanSense (No. 321798), the Niedersachsen-Israeli Research Cooperation Program, the U.S. Army Research Office under Contract No. W911NF-15-1-0250, the Crown Photonics Center, ICore-Israeli Excellence Center Circle of Light, the Israeli Science Foundation (Grant No. 986/15), the Israeli Ministry of Science Technology and Space, the Minerva Stiftung, and the European Research Council (Consolidator Grant No. 616919—Ionology).

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