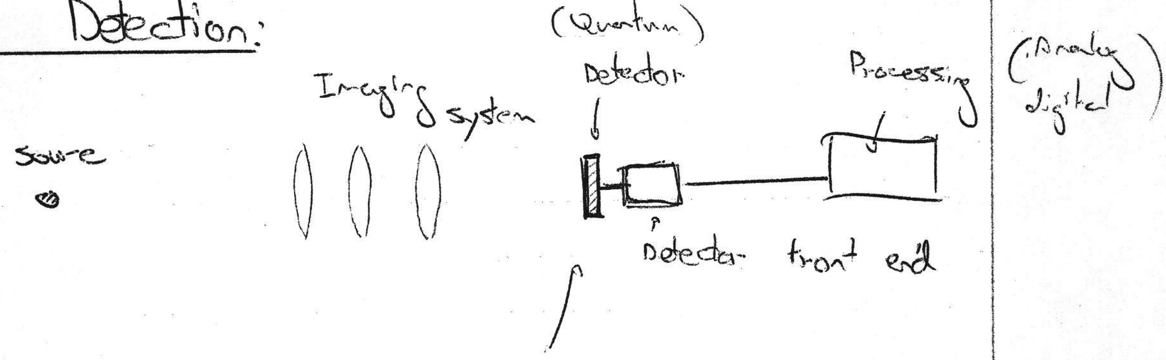
Optical Detection:



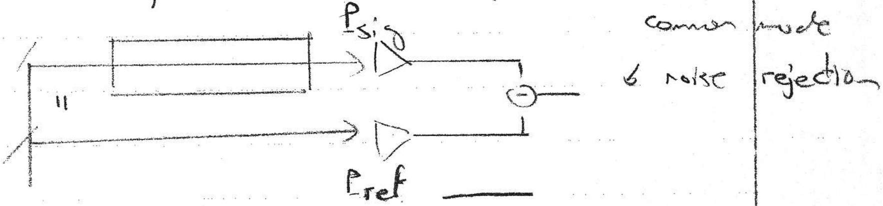
light ⇒ electricity

what you screw up at the optical part, cannot be retrieved at the electronics side.

i.e. the measurement minimum SNR is determined by the optical side. Can only be worsened by electronics

⇒ Photon budget:
How many photo-electrons will I get in a certain integration time?

example: Absorption spectroscopy.



$$N_{sig} = \left[\frac{P_{sig}}{\hbar\omega}\right] t_{meas} = \frac{1}{B}\left[\frac{P_{sig}}{\hbar\omega}\right] \qquad B = \frac{1}{t_{meas}}$$

$$N_e = \eta\,(N_{ref} - N_{sig}) = \frac{\eta}{B}\left[\frac{P_{ref} - P_{sig}}{\hbar\omega}\right]$$

quantum eff.

.Su

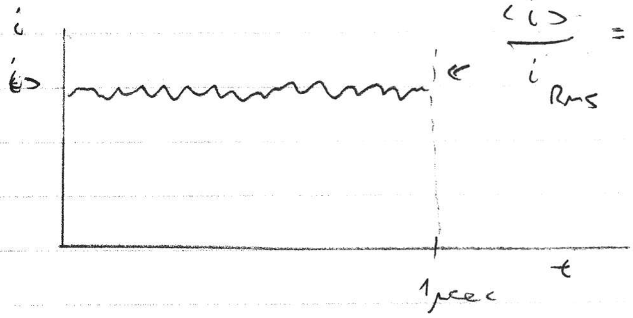
$$STD(N_e) = \sqrt{\eta (N_{ref} + N_{sig})} = \sqrt{\frac{\eta}{B} \frac{P_{ref} + P_{sig}}{\hbar\omega}}$$

$$\Rightarrow \text{photocurrent } SNR = \sqrt{\frac{\eta}{B\hbar\omega}} \frac{P_{ref} - P_{sig}}{\sqrt{P_{ref} + P_{sig}}}$$

Say we have a 1% absorption out of 100 μW beam, $B = 1\ MHz$ $\quad \lambda = 1\mu m \Rightarrow \omega = 2\pi \times 2 \times 10^{14}\ Hz$

$$\sqrt{\frac{0.4}{10^6 \times 6\times10^{-34} \times 2\times10^{14}}} \frac{10^{-6}}{\sqrt{2\times10^{-4}}} = \frac{10^9}{\sqrt{60}} 10^{-4} \simeq 10^4$$

$$\langle i \rangle = \frac{1}{t_0} \int_0^{t_0} i(t)\, dt$$



$$\frac{\langle i \rangle}{i_{RMS}} = \frac{\langle i \rangle}{(\langle i^2 \rangle - \langle i \rangle^2)^{1/2}}$$

$$\langle i^2 \rangle = \frac{1}{t_0} \int_0^{t_0} i^2(t)\, dt$$

SNR improves as $\sqrt{P}$; $\frac{1}{\sqrt{B}}$; $\sqrt{\eta}$

more photons — more integration time — higher quantum eff

to normalize out $\frac{1}{\sqrt{B}}$ the above result can be stated

as $SNR = 10^7 / \sqrt{Hz}$

why $\frac{1}{\sqrt{Hz}}$

- noise would average better as $\frac{1}{\sqrt{t_{meas}}}$
- noise power $i^2_{RMS} R_L$ is white. Noise energy linear with B.

SNR is usualy given in terms of

$\left(\frac{\text{Signal power}}{\text{Noise power}} \right)$

in photo-detection $\quad P_{opt} \propto i_{elect.} \quad P_{elect.} = (i_{elect.})^2 R_L$

$$Opt_{SNR} \propto \frac{1}{\sqrt{B}} \qquad Elect._{SNR} \propto \frac{1}{B}$$

very different

I belive that thinking in terms of photo-electrons rather than photons in optical measurement is healthier

1. η appears.

2. Easier to compare to other additional noises: Johnson etc.

3. Quantum optics - more correct, e.g. for a coherent state there is nothing noisy - pure state. Noise in the outcome of a Fock basis meass.

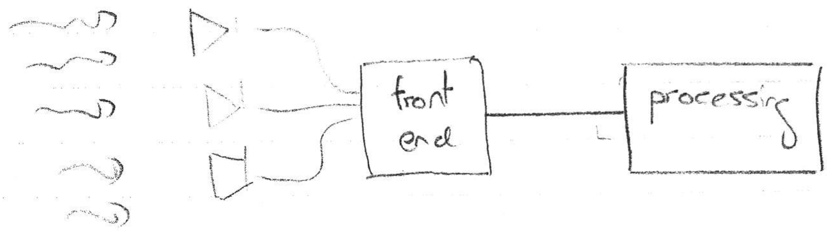
$$i_{N\,shot} = e\sqrt{2\eta N_{photo/sec}} = \sqrt{2e\, i_{sig}}$$

shot noise dominates when $V_{sig} > 50$ mV

$$P_{N\,shot} = i^2_{N\,shot} R_L = 2e\, i_{sig} R_L = 4KT \Rightarrow V_{sig} = \frac{2KT}{e} = 50 \text{ mV}$$

@ 300 K

$P_{Johnson}$ (thermal noise) in a resistor



photon Budget : Max SNR determined here.

Photon statistics : statistics of uncorrelated events with average occurance λ per unit time.

$$\underset{\text{N events / unit time}}{P(N)} = \frac{\lambda^N}{N!} e^{-\lambda}$$

For large λ looks Gaussian, with width $= \sqrt{\text{average}}$

$\Rightarrow$ called Gaussian Noise.

PSD - white. $I_d = \eta \frac{P}{\hbar\omega} e$

$\#$ of electrons $I_d T \equiv \frac{I_d}{2B}$

std of $\#$ electrons $= \sqrt{\frac{I_d}{2Be}}$

converting back to current i.e. $\times \frac{e}{T}$ $\Rightarrow$ $STD(i_D) = 2Be\sqrt{\frac{I_d}{2Be}} = \sqrt{2eI_d B}$

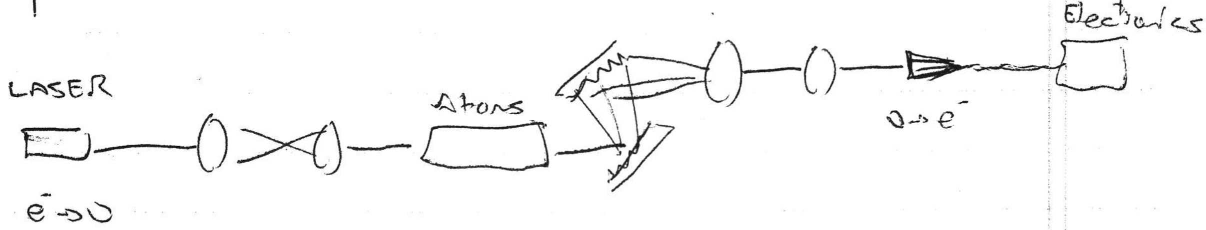
comparing to Johnson-(Nyquist) noise

$$i_n = \sqrt{\frac{4k_BTB}{R}}$$

we get $\quad I_d R = \frac{2k_BT}{e} = 51 \text{ mV}$

when the two contributions are equal!

Optical Detection

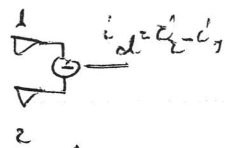


What you screw up at the optical part cannot be retrived at the electronics side.

$\Rightarrow$ work on effort,

$\Rightarrow$ All sensitive operations $\Rightarrow$ perform at detection

e.g. diff detection



before interference, Johnson etc.

Detector figures of Merit:

two types / Bolometers ← High powers:
\ quantum detectors ← i.e. photo-electric
photon ⇔ electron

- Aside: Superconducting transition edge bolometers are the best (number resolving) single photon detectors. $\eta > 0.9$.

- Here we discuss Q-det. / $std(I_d) = \sqrt{2eI_dB}$
\ $I_d^2 R = \frac{V_{th}^2}{R} \Rightarrow 2eI_dR = 4k_BT$
$\Rightarrow I_dR = \frac{2k_BT}{e} = 51$ mV

QE: Ratio of charge carriers to incident photons.

Responsivity R [A/W] $R = M \frac{\eta e}{\hbar\omega}$
↑ detector multiplicative gain   PP's → 1 (i.e. pmts → 10^4)

for $M=1$ (photo-diodes) $R \sim 1$ A/W in IR
~ 0.3 A/W in violet.

notice that for $\eta \neq \eta(\lambda)$ R is linear in λ

NEP   Given some intrinsic noise mechanism (independent of optical power).

NEP is the optical power req. for an SNR = 1

- Intrinsic noise is not necessarily flat → NEP would depend on how fast light is modulated.

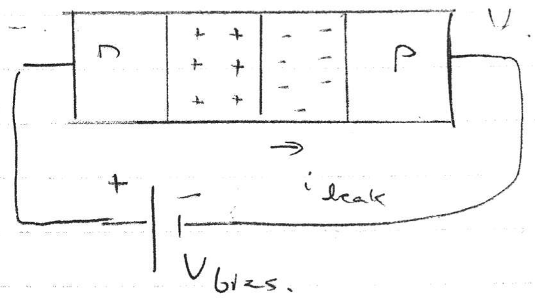
NEP (cont.)

Quoted close to d.c. where roughly flat in units of $W/\sqrt{Hz}$ ← the more you average the less optical power you need for SNR $=1$

Since often the intrinsic noise increases with detector linear size, sometimes quoted in units of $W/cm/\sqrt{Hz}$
compare between tech.

Back here after PDs are discussed!!
What is the intrinsic noise source?

example silicon photo-diode reversed biased:



$R_{shunt} \sim \dfrac{V_{bias}}{i_{leak}}$ (actually i_{leak} does NOT go to zero @ $V_{bias}=0$)

two intrinsic noise contributions:

i_{leak} shot noise & R_{shunt} Johnson noise.

$$i_N^2 = \frac{4\underbrace{k_B}_{1.38\times10^{-23}}TB}{R_{shunt}} + 2\underbrace{e}_{1.6\times10^{-19}\text{ coul}} i_{leak} B$$

Typical values: $i_{leak} \simeq 100\,pA$ @ 10_V reverse bias.

$\Rightarrow R_{shunt} \simeq 10\ G\Omega$

Johnson is x30 smaller compared to shot noise

$$i_{shot\ noise/\sqrt{B}}^{rms} = 6\ fA/\sqrt{Hz} \simeq \sqrt{3.2\times10^{-19}\cdot10^{-10}}\ A/\sqrt{Hz}$$

$$NEP = \frac{i_{shot\ noise}^{rms}}{\sqrt{B}}\left(\frac{\hbar\omega}{\eta e}\right) \simeq 10\ fW/\sqrt{Hz}$$

$\eta = 80\%$ @ 900 nm

what R do you need in order to be shot noise limited over 1 MHz

<u>Capacitance:</u> For photo-diodes typ. few pF → few 10's pF
(to geometry)

Important to determine bandwidth, simply measure

voltage across R_L (1 MΩ on scope)

$$B = \frac{1}{2\pi RC} \simeq \frac{10^{11}}{2\pi\times10^{6}} = 2\times10^{4}\ Hz$$

10 pF

not very fast

- can do better ⇒ front ends.

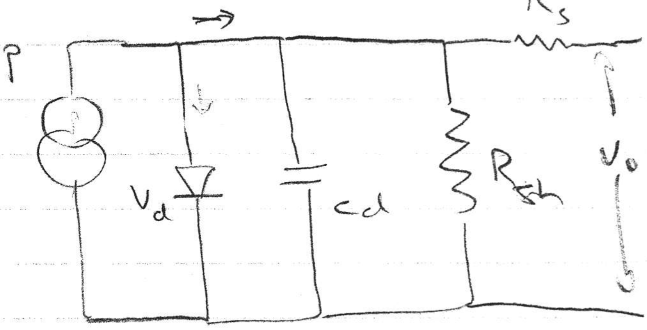
- spectral response: photo diode below band gap in IR
  UV – absorption before depletion region.
  – photon electron dispersion mismatch.

Si ~ 200 - 1100 nm

- spatial uniformity: 1-10% variation

Photodiode equivalent circuit:

low (few ohm) at most



in dep. out of dep.

$\tau_{pass} = \tau_{drift} + \tau_{diffusion}$

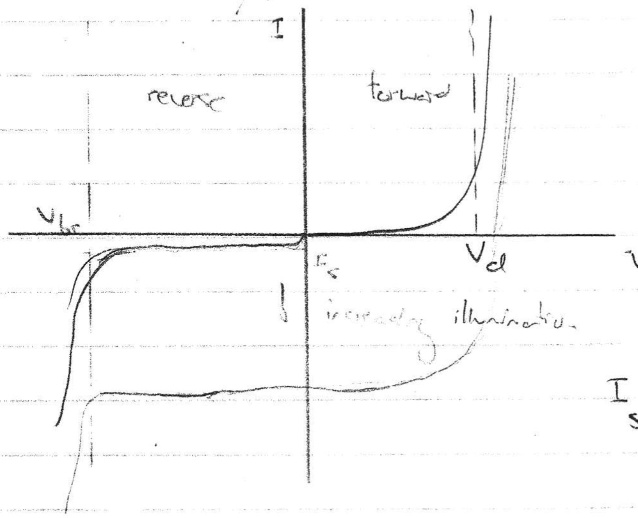
$$C_d = \frac{C_0}{\sqrt{1 + V_R / V_d}}$$ ← holes are ~2-3 slower than electrons

until full depletion

$V_R \to 0$ $\quad$ $R_{sh} \to \infty$ $\quad$ otherwise almost ind. of reverse bias

0.6 V

Shockley diode eq: $I = I_s \left(e^{V_d / \alpha V} - 1 \right)$



$I_s \propto e^{-\frac{E_g}{kT}}$

$I_{ph} = I_o - I_s$

always work in Reverse bias mode:
- faster ($C_d \to$ x to smaller drift charge depth)
- more efficient
- push non-linearity

Except when detecting at very low light levels ⇒ NEP ↓ ⇒ leakage current.

Hamamatsu 8\$ - 25\$.

<u>Photodiodes basics:</u> cheap, linear, fast, small with electronic front end.

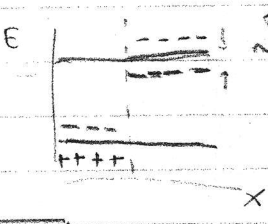
Example Si - 4 Valence electrons. Band gap = 1.1 eV

- doped with a low concentration 10^{13} - 10^{18} / cm^2

of:

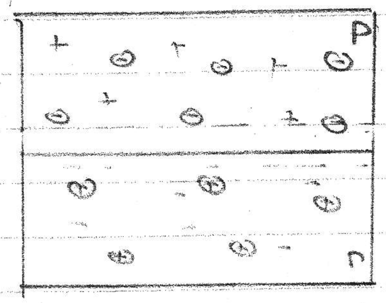
- donor = 5 valence electron atom: arsenic, phosphorus. n-type. the extra electron can be easily thermally excited to conduction band.
- acceptor = 3 Valence electron atom: Boron p-type. holes thermally formed in valence band.

Energy Distance of donor/acceptor from conduction/valence band ~ 20 - 50 meV, i.e. at 300K doped Si is conducting.



P-N junction

depletion region: carriers migrate and recombine.



+ mobile

⊖ fixed.

- mobile

⊕ fixed.

donor/acceptor ions form charge density - in p type and + in n type ⇒ electric field across junction.

Apply - n-type and + to p-type, charge carrier move into depletion region - diode conducting (after you over come the 0.6 V potential barrier)

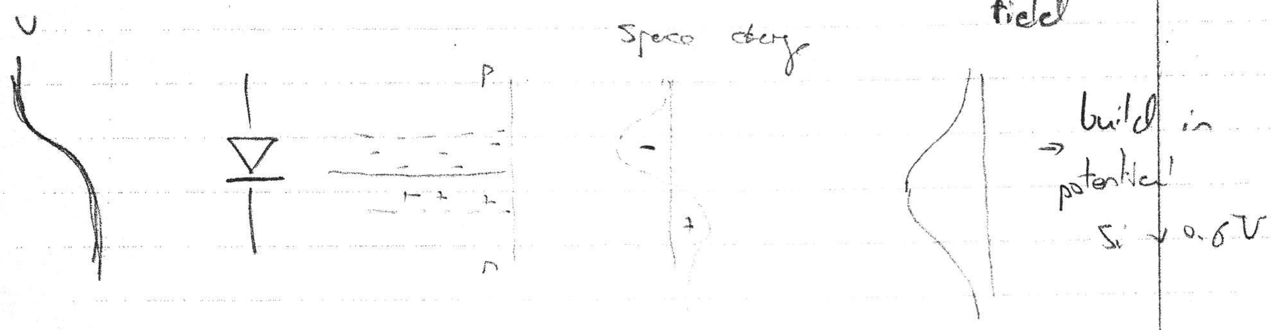
⇒ Forward bias.

- Apply + p type and - to n-type, carriers move away from depletion region

⇒ Reverse biased.

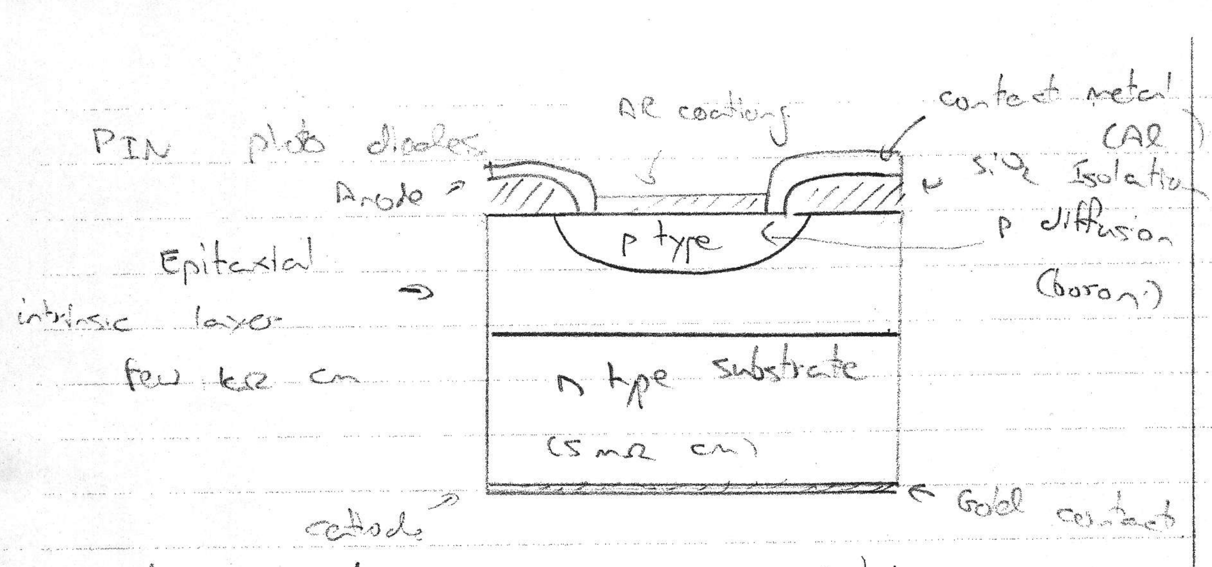
- Diode is not conducting and depletion region grows.

across the depletion region



a photon with $\hbar\omega > 1.1$ eV i.e. $\lambda \leq 1.1$ μm that is absorbed in depletion layer forms electron-hole pair. Electric field separate the two. If diode not connected p-type (anode) becomes positively charged. n-type (cathode) negatively charged. if connected, current flow from anode to cathode

PIN photo diodes.



Intrinsic layer: increases depletion area
more efficient ; higher photon bandwidth.
actually increases speed.
(large area with electric field)

responsivity: / cut off in IR
\ decrease in UV

IR No absorption when $\lambda > \lambda_g = \frac{hc}{E_g} \approx \frac{1.24}{E(V)}$

for Si ~ $\lambda_g = 1.11\,\mu m$. peak sensitivity (Si) ~ $0.96\,\mu m$.

- contact at cathode mirror to generate 2-pass through depletion region

UV - photons absorbed too soon, no large E field to separate them.

UV enhanced - depletion region as close as possible to surface.

other examples to detector materials:

	λ_g (nm)
C (diamond)	0.23
GaN	0.35
Si	1.11
$In_x Ga_{1-x} As$	1.7 - 2.6
Ge	1.88

- AR coating for Si $n = 3.5$.

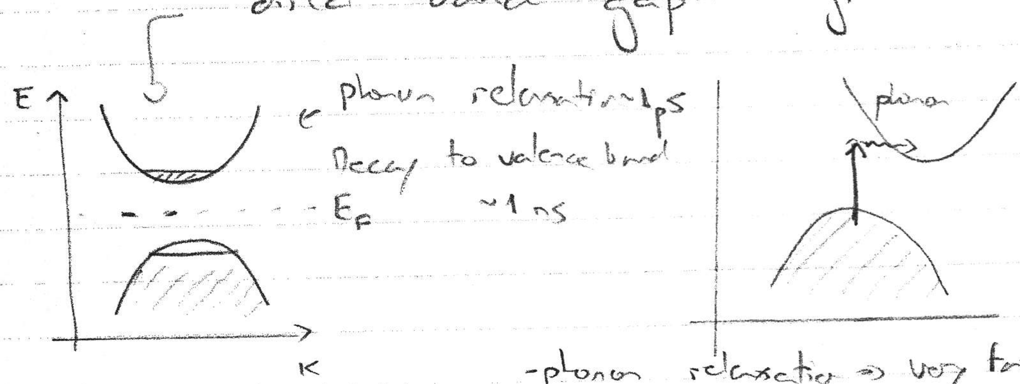
$\Rightarrow$ Fresnel reflection @ normal incidence $= 31\% = \left(\frac{n-1}{n+1}\right)^2$

$\Rightarrow$ w/o AR coating $\eta \sim 0.6$

w AR coating $\eta \sim 0.9$ possible

Non-direct band gap - e.g. Si

direct band gap - e.g. InGaAs



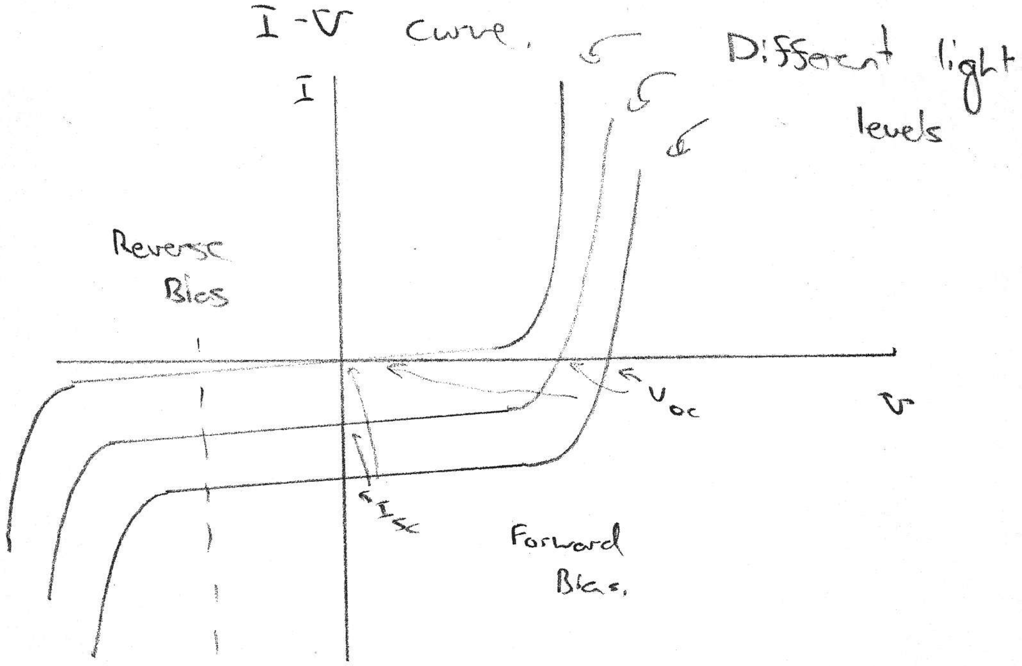
← phonon relaxation ~ ps

Decay to valence band ~1 ns

- phonon relaxation ⇒ very fast

- Direct band gap PD's more efficient e.g. InGaAs $\eta \sim 0.9$ .63

I-V curve.



← <u>Always</u> work in reverse bias mode.

→ More efficient (larger depletion region)

→ Faster ↗ C_d reduced by x 10 or so
↘

$$\tau_{pass} = \tau_{drift} + \tau_{diffusion}$$

τ_{drift}: in depletion region

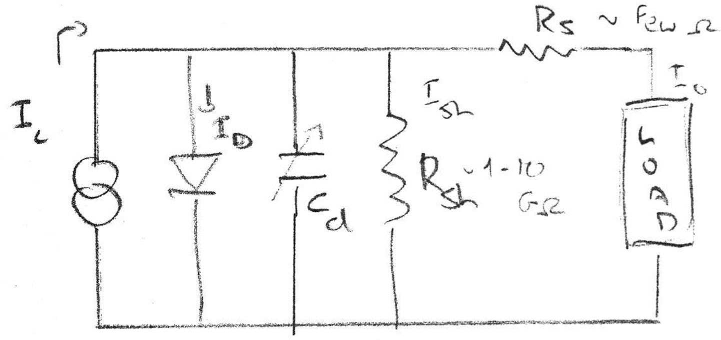
$\tau_{diffusion}$: outside depletion region.

→ Push non-linearity to higher light levels due to stronger field.

Exept when trying to low light levels

⇒ reduce I_{leak} and therefor I_{leak} shot noise

Yet again a trade-off between sensitivity and bandwidth.



$$C_d \simeq \frac{C_o}{\sqrt{1 + V_{RB}/V_d}} \quad \leftarrow \text{ until full depletion}$$

at d.c. (i.e. no current through C_d)

$$I_o = I_L - I_D - I_{sh} = I_L - I_s\left(e^{\frac{eV_D}{kT}} - 1\right) - I'$$

<u>Open circuit:</u> $I_o = 0 \quad (R_{load} \gg R_{sh})$

$$\Rightarrow V_{D_{oc}} = \frac{kT}{e} \ln\left[\frac{I_L - I_{sh}}{I_s} - 1\right]$$

if no light $I_L = 0 \quad I_{sh} = V_D/R_{sh}$

$$\Rightarrow V_{D_{oc}} = \frac{kT}{e} \ln\left[\frac{V_D}{I_s R_{sh}}\right] \quad \Rightarrow V_D = 0.6 - 0.7\ V$$

when $I_L \gg I_{sh}$

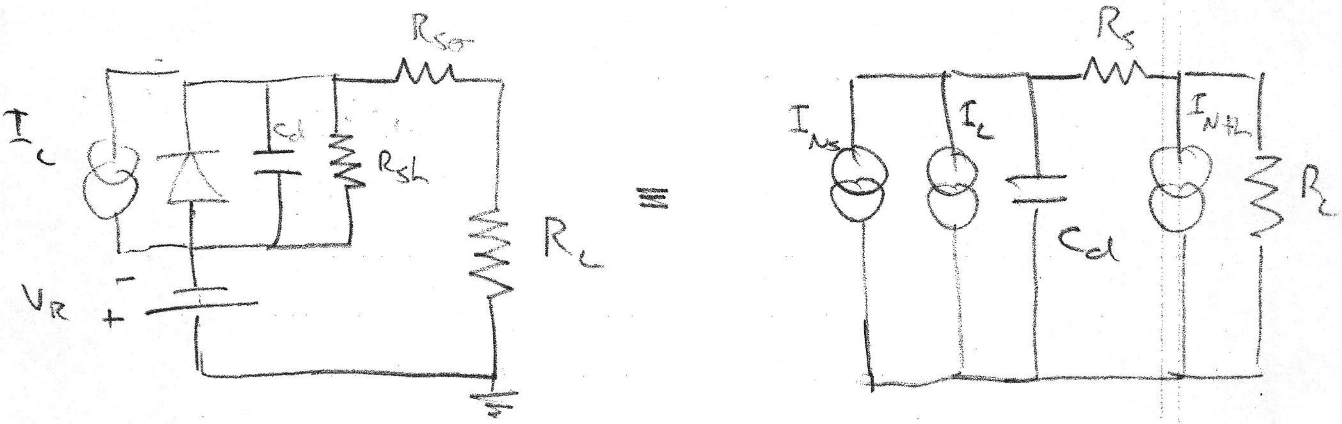
$$V_{D_{oc}} = 25_{mV} \ln\left[\frac{I_L}{I_s}\right] \quad \leftarrow \text{ non-linear in } P_{opt}.$$

<u>short circuit:</u> $V_o = 0$

$$I_{o_{sc}} = I_L - I_s\left(e^{\frac{e I_{sc} \cdot R_s}{kT}} - 1\right) - \frac{I_{sc} R_s}{R_{sh}}$$

Front ends again:

The simplest front end - a resistor:



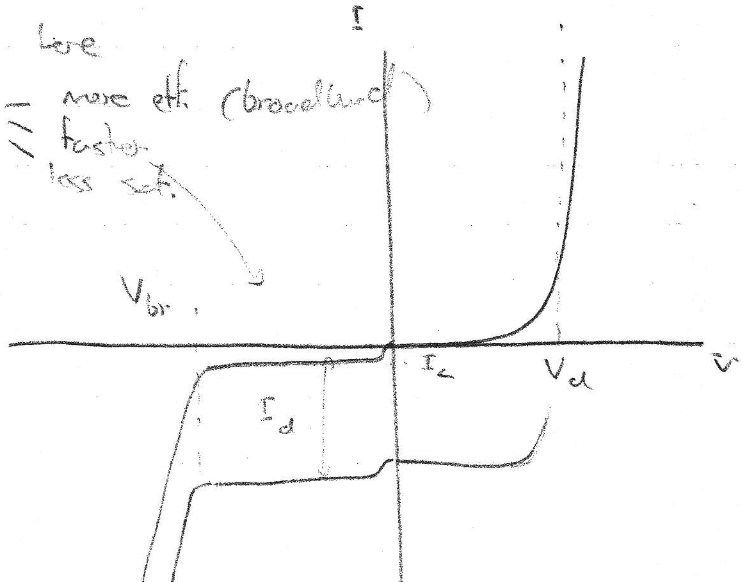
$$C_d = \frac{C_0}{\sqrt{1+ V_R/V_d}} \qquad V_d \sim 0.6\ \text{V for Si}$$

$$R_{sh} = \frac{V_R}{I_L} \sim 10\ \text{G}\Omega \qquad (V_R \sim 10\ \text{V},\ I_L \sim 100\ \text{pA})$$

typ ~ few → 100s pF

work here
reverse bias – more eff. (broadband)
– faster
– less sat.

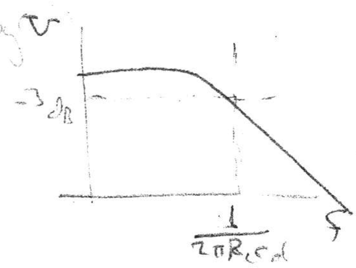
price at a small leakage current
⇒ NEP



- Neglect I_i (absorb into I_d) and R_s (absorb into R_L)

$$z^{-1} = i\omega C_d + \frac{1}{R_L} \Rightarrow z = \frac{R_L}{1 + i\omega R_L C_d}$$

$$\Rightarrow V = \frac{I_d R_L}{1 + i\omega R_L C_d}$$



for responsivity = 0.5 A/W P = 2 μW

$I_d = 1\ \mu A$

$\Rightarrow$ for 1V $\Rightarrow R_L = 1\,M\Omega$

assuming $C_d = 100$ pF (large area diode) 10 pF 1 pF

$f_{RC} \simeq 1.6$ KHz 16 KHz 160 KHz

- SNR does not depend on f Noise is being suppressed as the signal

- Makes sense to lower R_L until

$I_d R_L \simeq 2 \times 51$ mV or so

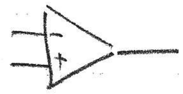
(Gain in terms of bandwidth as well as Dynamic range.)

Trans impedance amplifiers:

Idea: reduce voltage swing across C_d.

$\Rightarrow$ put C_d into virtual ground

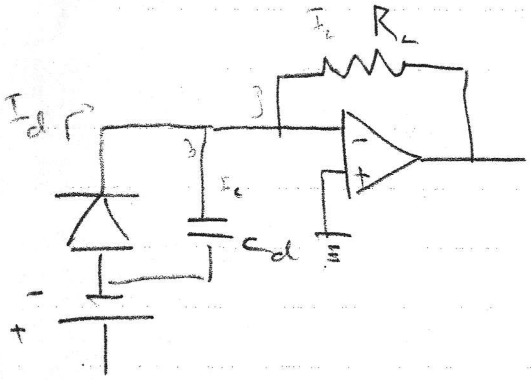
a) Ideal op-Amp:



$I_- = I_+ = 0$

$V_{out} = (V_+ - V_-)A$

open loop gain $A \to \infty$



- If $A \to \infty$ $(V_+ - V_-) = 0$ $\Rightarrow$ V_- is virtual ground

$\Rightarrow$ voltage across $C_d = 0$

$$I_d = I_L + I_c$$

$$I_c = -i\omega C_d V_- \qquad I_L = \frac{V_{out} - V_-}{R_L}$$

$$V_{out} = -V_- A = I_L R_L + V_- = (I_d - I_c) R_L + V_- =$$

$$= (I_d + iV_-\omega C_d) R_L + V_- = \left(I_d\ i\frac{V_{out}}{A}\omega C_d\right) R_L - \frac{V_{out}}{A}$$

$$\Rightarrow \quad V_{out} = \frac{I_d R_L}{1 + \frac{1}{A} + i\frac{\omega R_L C_d}{A}} \simeq \frac{I_d R_L}{1 + i\frac{\omega R_L C_d}{A}}$$

$C_d \Rightarrow C_d/A$

or $R_L \Rightarrow R_L/A$

a miracle: response wise: R_L

corner wise: R_L/A

For an ideal op-Amp $A \to \infty$ $\Rightarrow$ infinitely fast

Frequency Responce:

Non ideal op-Amp: $-\; A(f) = \frac{A_{dc}}{1 + i\, f/f_{Amp}}$

- Amp adds more capacitance to ground

$\Rightarrow$ effectively increases C_d by few pF

For large f $\quad A(f) \sim -i\,\frac{f_{GBP}}{f}$

$f_{GBP} = A_{dc} \cdot f_{Amp}$ = unity gain freq. characteristic in op Amp fab.

$$\Rightarrow \quad V_{out} = \frac{I_d R_L}{1 - \frac{2\pi f^2 R C_d}{f_{GBP}}} \equiv \frac{I_d R_L}{1 - \left(\frac{f}{f_{ti}}\right)^2}$$

where $f_{ti} = \sqrt{F_{RC} \cdot f_{GBP}}$ $\quad F_{RC} = \frac{1}{2\pi R_L C_d}$

Stability:

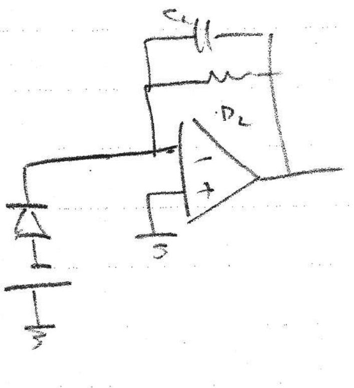
Notice that when $f = f_{ti}$

V_{out} diverges ⇒ 180° phase shift.

Negative feedback ⇒ positive feedback.

the circuit will mostly oscillate at the amplifier freq.

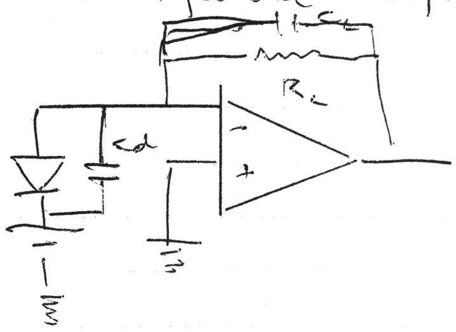
to regain stability: add another capacitor C_2 to roll-off the feedback gain at $f > f_{ti}$.



where $\frac{1}{2\pi R_2 C_2} \simeq f_{ti}$

last time:

trans-impedance amp.



$$f_{RC} = \frac{1}{2\pi R_c C_d}$$

$$f_{ti} = \sqrt{f_{RC} \cdot f_{GBP}}$$

$$f_{GBP} = A_{dc} \cdot f_c$$

choose C_c such that $\frac{1}{2\pi R_c C_c} \simeq f_{ti}$ (stability)

Next lectures Feb 1st, Feb 8th Tuesdays.

No lecture next monday. !!!

Offset

Non-ideal: op-amp do have input currents

$I_- \approx I_+$ (on the few % level)

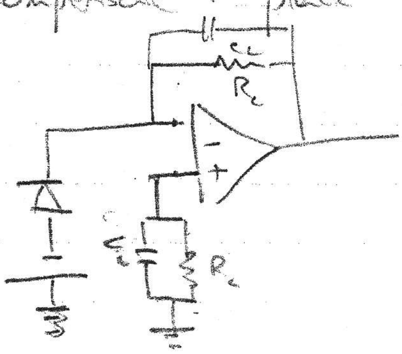
$I_d = I_L + I_-$

$V_{out} = \quad = (I_D - I_-) R_L + V_- \Rightarrow \underset{\text{virtual GND}}{0} = V_- = V_{out} - (I_D - I_-) R_L$

$V_{out} = (I_D - I_-) R_L$

$\Rightarrow \underset{I_D = 0}{=} -I_- R_L$

to compensate: place R_L between + and Gnd.



$V_{out} = (I_D - I_-) R_L + \overset{I_+ R_L}{\overset{\shortparallel}{V_-}}$

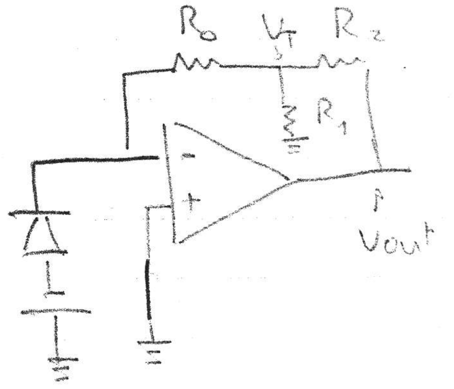
Same analysis:

$\Rightarrow V_{out} = (I_+ - I_-) R_L + I_D R_L$ ~ x100 smaller

This changes $V_R' = V_R - I_+ R_L$ - reverse bias voltage

usually not a problem unless want to work at $V_R \to 0$, and large R_L (low light collectors)

If want reduce the effect: Feedback Tee



$$I_1 = \frac{V_T}{R_1} \qquad I_D = \frac{V_T}{R_0} \qquad I_2 = I_D + I_1$$

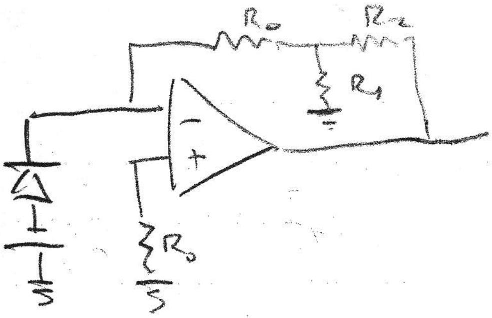
$$\Rightarrow V_{out} - V_T = I_2 R_2 = (I_D + I_1) R_2$$

$$V_{out} = I_D R_0 + (I_D + I_1) R_2 = I_D (R_0 + R_2) + \frac{\overset{= I_D R_0}{V_T}}{R_1} R_2 =$$

$$= I_D \left[R_0 \left(1 + \frac{R_2}{R_1}\right) + R_2 \right] \simeq I_D R_0 \left(1 + \frac{R_2}{R_1}\right)$$

$$R_0 \gg R_2 \gg R_1$$

effective load resistance: $R_0 \left(1 + \frac{R_2}{R_1}\right)$



$$V_- + (I_D - I_-) R_0 = V_T$$

$$V_{out} - V_T = (I_D - I_- + I_1) R_2$$

$$\Rightarrow \quad V_{out} = (I_D - I_- + I_1) R_2 + (I_D - I_-) R_0 + V_-$$

$$= I_D (R_0 + R_2) + I_1 R_2 - I_- (R_0 + R_2) =$$

$$= (I_D - I_-)(R_0 + R_2) + \frac{V_T}{R_1} R_2 =$$

$$V_T = (I_D - I_-) R_0 + V_-$$

$$V_T = V_- + (I_D - I_-) R_0$$

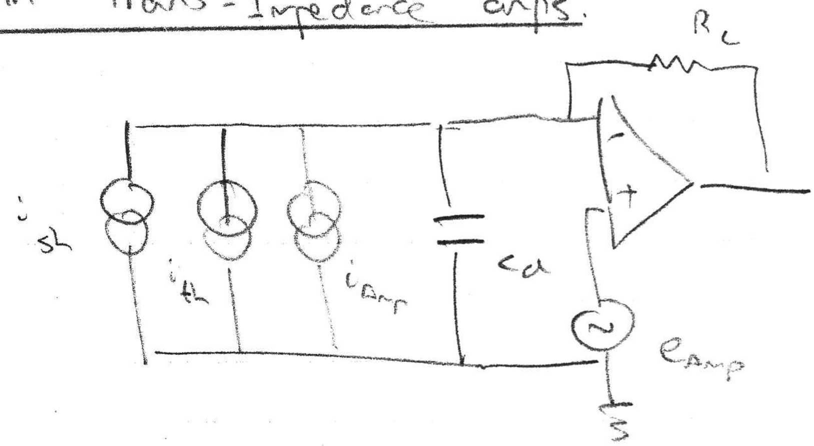
$$= (I_D - I_-)\left(R_0 + \cancel{R_2} + R_0 \frac{R_2}{R_1}\right) + V_- \frac{R_2}{R_1}$$

$$V_- = I_+ R_0$$

$$V_{out\ bias} \simeq (I_+ - I_-) R_0 \frac{R_2}{R_1}$$

But with a lower V_- reverse bias voltage

Noise in Trans-Impedance amps.



$$(e_{Amp} - V_-)A = V_{out}$$

$$(V_{out} - V_-) = IR_L \qquad V_- = I \cdot \frac{1}{i\omega C}$$

$$V_{out} = IR_L + V_- = V_- \, i\omega C R_L + V_- =$$

$$= V_-(1 + i\omega RC)$$

$$\Rightarrow V_- = \frac{V_{out}}{(1 + i\omega RC)}$$

$$\left(e_{Amp} - \frac{V_{out}}{1 + i\omega RC}\right)A = V_{out}$$

$$\Rightarrow Ae_{Amp} = V_{out}\left(\frac{A}{1 + i\omega RC} + 1\right)$$

$$\Rightarrow V_{out} = e_{Amp}\left[\frac{A}{1+\frac{A}{1+i\omega RC}}\right]$$

for $\omega > RC$

e_{Amp} is amplified, exactly at the freq all other noise sources start rolling off.