Problem set 1 : Laplace transform

1 Convolution theorem

The convolution of two functions $f_1(t)$ and $f_2(t)$ is defined as:

$$f_1(t) * f_2(t) \equiv \int_0^t f_1(t - \tau) f_2(\tau) d\tau$$

Prove that,

$$\mathcal{L}[f_1(t) * f_2(t)] = F_1(s) F_2(s)$$

where $\mathcal{L}[f(t)] = F(s)$.

2 Driven, undamped, harmonic oscillator

Using the Laplace transform find $x(t)$ and $p(t)$ for a driven undamped harmonic oscillator,

$$m\ddot{x}(t) + m\omega_0^2 x(t) = F_0 \cos(\omega_f t)$$

for the initial conditions $x(t = 0) = p(t = 0) = 0$ and for a force acting for a finite time T.